THE PROBLEM OF THE OPTICAL CONSTANTS OF CONDUCTORS

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We discuss the problem of determining the complete set of optical constants of a conductor. It is shown that for an isotropic conductor this set consists not only of the index of refraction and the absorption coefficient, but also of two real quantities corresponding to a complex boundary impedance. The real part of the boundary impedance determines the surface losses in the conductor, while the imaginary part of the dielectric constant determines the volume losses. We have formulated dispersion relations which connect the real and the imaginary parts of the complex surface conductivity. We have considered fluctuations in the electromagnetic field in the conductor and have obtained the correlation functions for the field components of a metal filling a half-space.

1. In a preceding paper by the author[1] the theory of the optical constants of conductors was considered for the case of radiation obliquely incident on the surface of a bulk, conducting body. We evaluated then the anomalous skin-effect by introducing a boundary condition at the surface of the conductor corresponding to the presence of a surface current. The surface conductivity evaluated in that way leads to additional losses corresponding to a diffusive scattering of the conduction electrons at the metal surface. In the following we shall discuss the problem of the optical constants of a conductor and we shall consider fluctuations in the electromagnetic field and pay attention to some electrodynamical relations, taking into account the presence of a boundary impedance $z = \gamma^{-1}$ which connects the surface current density $\mathbf{i}$ with the tangential components of the electrical field:

$$\mathbf{i} = \gamma(\omega)\{\mathbf{E} - \mathbf{n}(\mathbf{nE})\}, \tag{1}$$

$\gamma(\omega)$ is the complex surface conductivity, $\mathbf{n}$ the outward normal to the surface of the conducting body. We have taken here the time dependence in the form $e^{i\omega t}$. We have for an anisotropic conductor, and also for a crystal of cubic symmetry, instead of (1)

$$i_\alpha = \gamma_{\alpha\beta} E_\beta, \tag{2}$$

where $\gamma_{\alpha\beta}$ is the second-order conductivity tensor.

2. If we do not take the anomalous skin effect into account, the optical properties of conductors are characterized by a complex dielectric constant

$$\mathbf{D} = \varepsilon\mathbf{E}, \quad \varepsilon(\omega) = (n - i\varkappa)^2 = \varepsilon_1 + i\varepsilon_2. \tag{3}$$

Here n is the index of refraction and κ the absorption coefficient. With the same degree of accuracy with which we may neglect the anomalous skin effect, these quantities fully determine the optical properties of a conductor. In the region of the anomalous skin effect, on the other hand, when the mean free path is larger than or comparable to the penetration depth of the field, the complex dielectric constant, and thus n and κ, do no longer determine all the optical constants of the conductor. The problem therefore arises of determining the complete set of optical constants in that region. There is in the literature essentially no elucidation of this problem.

The majority of the papers considering the anomalous skin effect in the optical region are devoted to a microtheory and because of the complexity of the equations of the microfield they are restricted to a study of the case of bulk metals.* The discussion is appreciably simplified in the case when the absolute magnitude of the complex dielectric constant is appreciably larger than unity. One can then use Leontovich's boundary conditions[2] and they lead in such a limiting case to the fact that the optical properties of a bulk metal are completely characterized by the complex surface impedance (see reference 3). Moreover, it turns out that one can introduce an effective complex dielectric constant and thus n_{eff} and κ_{eff}.[4]

*See the papers quoted in reference 1.

These quantities characterize the conductor, however, only in the region where ϵ_{eff} is large, and only for the case where the dimensions of the conductor are large compared to the skin-depth δ, i.e., for bulk conductors. As an example we point out that the electrical polarizability of a sphere of radius R which determines the scattering and absorption of light in the case where R is small compared to the wavelength of the radiation both in a cavity and also inside the particle is equal to

$$\alpha^{(e)} = \frac{3}{4\pi}\frac{\tilde{\varepsilon}-1}{\tilde{\varepsilon}+2}, \qquad \text{where } \tilde{\varepsilon} = \varepsilon - \frac{8\pi i\gamma(\omega)}{\omega R}.$$

One can use the complex dielectric constant in optics in the region of the anomalous skin effect. This is because even in the infrared region the condition $v/\omega \ll \delta$, where v is the velocity of the conduction electrons, is satisfied. If we are interested also in terms of the order $(v/\omega\delta)^2$ we must take into account the spatial dispersion of the dielectric constant and instead of Eq. (3) use an equation of the form[5]

$$\mathbf{D} = \varepsilon\mathbf{E} + a\nabla^2\mathbf{E} + b\,\mathrm{grad\,div}\,\mathbf{E}. \tag{4}$$

Since the boundary impedance corresponds to an effect of the first order in $v/\omega\delta$ we need take the anomalous skin effect correction into account only when for some reason or other γ turns out to be excessively small. This can, in particular, occur in the case of specular reflection of the conduction electrons from the metal surface, a case which is of little practical interest. In metals, however, diffusive scattering takes apparently always place and γ is by no means abnormally small. Because of this one can with great accuracy use Eq. (3), i.e., neglect the spatial dispersion of the dielectric constant. If γ is not abnormally small the spatial dispersion can be appreciable only for large $v/\omega\delta$ when any expansion of (4) in powers of such a parameter becomes invalid.*

In essence the complete answer to the problem of the optical constants of a metal when the anomalous skin effect is taken into account is contained in what has been said. In addition to the complex dielectric constant there enters namely, into the complete set of optical constants also a complex boundary impedance, i.e., two real number γ_1 and γ_2 $(\gamma = \gamma_1 + i\gamma_2)$, which are, like n and κ, functions of the frequency of the electromagnetic field.

*The case when ε tends to zero is an exception. This case corresponds to the possibility of the propagation of plasma waves. The velocity of propagation of these waves is essentially determined by the spatial dispersion of the dielectric constant.

The optical properties of a conductor are then completely characterized by ϵ and γ, independent of whether or not ϵ is large. We note that the boundary impedance can in any case not be neglected (or rather, its real part cannot be neglected) when the imaginary part of ϵ is small, which occurs when $\delta \lesssim l$, where l is the conduction electron mean free path.

In the case of an anisotropic conductor we have instead of (3)

$$D_\alpha = \varepsilon_{\alpha\beta}E_\beta, \tag{5}$$

which gives us together with (2) the complete set of material equations which determine the optical properties of metallic crystals.

3. We shall consider some consequences of the Maxwell equations and the material equations given above which enable us to obtain useful relations for optics when the anomalous skin effect is taken into account. First of all we consider the boundary conditions at the interface of the conductor with a vacuum (the generalization to the case of an interface of a conductor with a dielectric or with another conductor is obvious). The presence of a surface current leads to the following boundary condition for the tangential components of the magnetic field (we shall take $\mu = 1$):

$$\mathrm{Curl}\,\mathbf{H} \equiv \mathbf{n}\times[\mathbf{H}^{\mathrm{med}} - \mathbf{H}^{\mathrm{vac}}] = 4\pi\mathbf{i}/c. \tag{6}$$

The component of the magnetic field normal to the surface as well as the tangential components of the electrical field are continuous

$$H_n^{\mathrm{vac}} = H_n^{\mathrm{med}}, \quad E_t^{\mathrm{vac}} = E_t^{\mathrm{med}}. \tag{7}$$

The normal component of the electrical induction, on the other hand, is discontinuous because the surface current density $\mathbf{i}$ leads by virtue of the equation of continuity to the presence of a surface charge density. If we take the time dependence in the form $e^{i\omega t}$, we get for the surface charge density $\sigma = (i/\omega)\,\mathrm{div}\,\mathbf{i}$. The last boundary condition is therefore of the form

$$D_n^{\mathrm{vac}} - D_n^{\mathrm{med}} = (4\pi i/\omega)\,\mathrm{div}\,\mathbf{i}. \tag{8}$$

Because the vector $\mathbf{i}$ lies in the surface, there is in the first part of this relation no normal derivative, and only the tangential derivatives of the tangential components of the electrical field enter therefore according to (1) or (2) and these are continuous because the normal components of the magnetic field are also continuous.

Using the boundary conditions (6) — (8) we can determine the heat released inside the conductor. The energy current flowing through the surface of the conductor is namely equal to

$$-\oint dS \frac{c}{4\pi}(\mathbf{n}[\mathbf{E}^{\mathrm{vac}}\times\mathbf{H}^{\mathrm{vac}}]$$

$$=\int dV \frac{1}{4\pi}\left(\mathbf{E}\frac{\partial\mathbf{D}}{\partial t}+\mathbf{H}\frac{\partial B}{\partial t}\right)+\oint dS\,(\mathbf{iE}), \qquad (9)$$

where the first integral is taken over the whole volume of the conducting body, and the second one over its surface. Taking the time average and using Eq. (1) we get for the average quantity of heat Q released in the conductor per unit time the following expression (we neglect the imaginary part of the magnetic susceptibility):

$$Q=-\frac{\omega\varepsilon_2}{4\pi}\int dV\,\overline{\mathbf{E}^2}+\gamma_1\oint dS\,\{\overline{\mathbf{E}^2}-\overline{(\mathbf{nE})^2}\}. \qquad (10)$$

We see that if we neglect the imaginary part of the magnetic susceptibility, the energy absorption is determined by the real part of γ (surface losses) and the imaginary part of ϵ (volume losses). It follows in particular from this that while $\epsilon_2 < 0$, γ_1 must be positive.

In the case of an anisotropic conductor we have instead of (10)

$$Q=-\frac{i\omega}{8\pi}\int dV\,(\varepsilon^*_{ik}-\varepsilon_{ki})\,\overline{E_i E^*_k}$$

$$+\frac{1}{2}\oint dS\,(\gamma^*_{ik}+\gamma_{ki})\,\overline{E_i E^*_k}. \qquad (11)$$

4. We considered above the surface current (1) for the components of the electromagnetic field with a well determined frequency ω taking into account that $\gamma(\omega)$ is some function of the frequency. The possibility of such a dispersion of γ follows, generally speaking, already from the fact that, for instance, in the isotropic case the surface current density at a given moment is determined in the usual way by the values of the field at the preceding moment (see, for instance, reference 3). One can thus show that $\gamma_1(\omega)$ is an even and $\gamma_2(\omega)$ an odd function of the frequency ω.

One can easily obtain the dispersion relations which connect the real and imaginary parts of the boundary impedance. To do this one can, for instance, follow the derivation given in reference 3 for obtaining the dispersion relations for the complex dielectric constant. The result is

$$\gamma_1(\omega)=\frac{2}{\pi}\int_0^\infty\frac{x\gamma_2(x)}{x^2-\omega^2}dx, \quad \gamma_2(\omega)=\frac{2\omega}{\pi}\int_0^\infty\frac{\gamma_1(x)}{x^2-\omega^2}dx. \qquad (12)$$

In the neighborhood of $x^2 = \omega^2$ one must take the principal values of the integral in (12) as is usual in dispersion relations. To obtain the dispersion relations it is necessary to take into account that $\gamma(\omega)$ has no singularity as $\omega \to 0$. This is in accordance with the fact that γ is finite in the static limit. Apart from this $\gamma(\omega)$ must decrease when the frequency increases. One can verify that this condition is also satisfied by using the free electron model which is, generally speaking, the better applicable, the higher the frequency of the variable field.

Indeed, the surface current density produced by the collisions of the electrons with the surface of the conductor and their subsequent diffusive scattering is as far as order of magnitude is concerned equal to $\mathbf{i} = enL\Delta\mathbf{v}w$, where e is the electron charge, n the number of electrons per unit volume, L the depth of the surface layer in which the electrons, which undergo noticeable collisions with the surface, lie. It is evident that under the conditions when the frequency of the variable field is large compared to the collision frequency of the electrons, L will be equal as to order of magnitude to the distance traversed by an electron during one period of the variable field, i.e., $L \sim v/\omega$ (where v is the root mean square velocity of the electrons). Furthermore, w, the probability for the diffusive scattering of the electrons, is never larger than unity. Finally, the change in the electron velocity under the influence of the variable field is as to order of magnitude (not taking phase amplification into account) equal to $\Delta\mathbf{v} \sim (e/m\omega)\mathbf{E}$ according to the equations of motion of a free electron. It is thus clear that $\gamma \lesssim e^2nv/m\omega^2$. A more rigorous evaluation, also based upon the free particle model but using the transport equation, gives $\gamma = 3e^2nv_0/16m\omega^2$, where v_0 is the velocity on the Fermi surface and m the effective mass.

The generalization of Eq. (12) for the anisotropic case is completely obvious.

5. We shall now consider the problem of the fluctuations in the electromagnetic field when there is a complex surface resistivity. It is usual when considering fluctuations in the electromagnetic field to introduce a "strange" fluctuating induction corresponding to spontaneous electrical and magnetic moments produced in the body as a result of the fluctuating charge oscillations (see reference 6 and Chap. XIII of reference 3). In accordance with this, the material equations take the form

$$D_\alpha(\omega)=\varepsilon_{\alpha\beta}(\omega)E_\beta(\omega)+K_\alpha(\omega),$$

$$B_\alpha(\omega)=\mu_{\alpha\beta}(\omega)H_\beta(\omega)+L_\alpha(\omega), \qquad (13)$$

and from the Maxwell equations we get

$$K_\alpha(\omega)=-\varepsilon_{\alpha\beta}E_\beta(\omega)-\frac{ic}{\omega}\,\mathrm{curl}_\alpha\mathbf{H}(\omega),$$

$$L_\alpha(\omega)=-\mu_{\alpha\beta}H_\beta(\omega)+\frac{ic}{\omega}\,\mathrm{curl}_\alpha\mathbf{E}(\omega). \qquad (14)$$

The correlation between the strange inductions is then determined by the equations (our notation differs from the one in reference 3 by the opposite sign of the frequency ω)

$$(K_\alpha(\mathbf{r}_1) K_\beta(\mathbf{r}_2))_\omega = i\hbar(\varepsilon_{\alpha\beta} - \varepsilon^*_{\beta\alpha})\,\delta(\mathbf{r}_2 - \mathbf{r}_1) \times \coth(\hbar\omega/2\kappa T), \text{ and so on} \tag{15}$$

Here κ is the Boltzmann constant. The formulae which we have written down solve in principle the problem of determining the electromagnetic fluctuations in any body, if there are no surface currents present.

It is also in our case natural to supplement the material Eq. (2) with a fluctuating surface current density $\mathbf{g}$:

$$i_\alpha(\omega) = \gamma_{\alpha\beta}(\omega) E_\beta(\omega) + g_\alpha(\omega). \tag{16}$$

Substituting this material equation into the boundary condition (6) we obtain the following relation in addition to Eqs. (14):

$$g_\alpha(\omega) = -\gamma_{\alpha\beta}(\omega) E_\beta(\omega) + (c/4\pi)\,\mathrm{curl}_\alpha \mathbf{H}(\omega). \tag{17}$$

One can say that the fluctuating surface current density $\mathbf{g}$ is caused by the fluctuations in the scattering of the electrons at the metal surface.

Equation (17) is analogous to the first equation of (14). In order that we can use the known results of the theory of electromagnetic fluctuations it is convenient to construct a symmetric scheme and to introduce a surface equation similar to the second equation of (14). The possibility of formulating such an equation can physically be connected, for instance, with the fact that the probability for an electron spin flip at the surface of the conductor is different from zero (see, for instance, reference 7). We can thus write

$$\mathrm{Curl}\,\mathbf{E} \equiv \mathbf{n} \times [\mathbf{E}^{\mathrm{med}} - \mathbf{E}^{\mathrm{vac}}] = -\frac{1}{c}\frac{\partial \mathbf{b}}{\partial t}, \tag{18}$$

where $\mathbf{b}/4\pi$ is the surface magnetization density. Introducing the strange fluctuating magnetic induction density $\boldsymbol{l}$, we have

$$b_\alpha = \nu_{\alpha\beta}(\omega)\tfrac{1}{2}(H^{\mathrm{vac}}_\beta + H^{\mathrm{med}}_\beta) + l_\alpha, \tag{19}$$

$$l_\alpha = -\tfrac{1}{2}\nu_{\alpha\beta}(H^{\mathrm{vac}}_\beta + H^{\mathrm{med}}_\beta) + \frac{ic}{\omega}\mathbf{n} \times [\mathbf{E}^{\mathrm{med}} - \mathbf{E}^{\mathrm{vac}}]_\alpha. \tag{20}$$

Since according to (18) the tangential components of the electrical field are discontinuous we must replace in Eqs. (16) and (17) $\mathbf{E}$ by, for instance, $(\mathbf{E}^{\mathrm{vac}} + \mathbf{E}^{\mathrm{med}})/2$. Moreover, it is convenient to introduce instead of the fluctuating surface current density $\mathbf{g}$ a fluctuating surface electrical induction $\mathbf{k}$. We get then

$$k_\alpha(\omega) = -\frac{1}{2}\eta_{\alpha\beta}(\omega)(E^{\mathrm{vac}}_\beta + E^{\mathrm{med}}_\beta) - \frac{ic}{\omega}\mathbf{n} \times [\mathbf{H}^{\mathrm{med}} - \mathbf{H}^{\mathrm{vac}}]_\alpha, \tag{21}$$

where

$$k_\alpha(\omega) = (4\pi/i\omega) g_\alpha(\omega), \qquad \eta_{\alpha,\beta} = (4\pi/i\omega)\gamma_{\alpha\beta}. \tag{22}$$

Relations (20) and (21) are completely analogous to the volume relations (14). The form in which they are written makes it therefore immediately possible to write down the following formulae for the correlations between the incidental surface inductions:

$$(k_\alpha(\mathbf{r}_1) k_\beta(\mathbf{r}_2))_\omega = i\hbar(\eta_{\alpha\beta} - \eta^*_{\beta\alpha})\,\delta([\mathbf{r}_2 - \mathbf{r}_1] \times \mathbf{n}) \coth(\hbar\omega/2\kappa T),$$

$$(l_\alpha(\mathbf{r}_1) l_\beta(\mathbf{r}_2))_\omega = i\hbar(\nu_{\alpha\beta} - \nu^*_{\beta\alpha})\,\delta([\mathbf{r}_2 - \mathbf{r}_1] \times \mathbf{n}) \coth(\hbar\omega/2\kappa T),$$

$$(k_\alpha l_\beta)_\omega = 0. \tag{23}$$

In Eqs. (23) there occur surface δ-functions and not volume δ-functions, in contradistinction to Eq. (15). This is, of course, connected with the fact that in the expressions for the change in energy $\mathbf{k}$ and $\boldsymbol{l}$ enter into a surface integral which is taken over the boundary surface of the body. Taking (22) into account we have the following expression for the correlation between the incidental surface currents:

$$(g_\alpha(\mathbf{r}_1) g_\beta(\mathbf{r}_2)) = \frac{\hbar\omega}{4\pi}(\gamma_{\alpha\beta} + \gamma^*_{\beta\alpha})\,\delta([\mathbf{r}_2 - \mathbf{r}_1] \times \mathbf{n}) \coth(\hbar\omega/2\kappa T). \tag{24}$$

From Eqs. (23), (24) and the Onsager relations in the case of bodies which do not possess a "magnetic structure"[3] and which are not in an external magnetic field the following equations follow

$$\nu_{\alpha\beta} = \nu_{\beta\alpha}, \qquad \eta_{\alpha\beta} = \eta_{\beta\alpha}, \qquad \gamma_{\alpha\beta} = \gamma_{\beta\alpha}. \tag{25}$$

If, however, the system is in a constant magnetic field $\mathbf{H}$, we have

$$\gamma_{\alpha\beta}(\mathbf{H}) = \gamma_{\beta\alpha}(-\mathbf{H}). \tag{26}$$

6. Equations (21) — (23) together with Eqs. (14) and (15) make it in principle possible to determine the fluctuations in the electromagnetic field in a conductor in the region where one can apply the material equations used by us. We shall in the following consider as an example the fluctuations in the electromagnetic field in an isotropic metal in the case where we can neglect the imaginary part of the dielectric constant and the magnetic susceptibility, but where it is necessary to take the real part of γ into account. Such a case corresponds just to the region of the anomalous skin-effect. One can then drop the strange fluctuating volume inductions, since it is evident that the fluctuations in the field will be caused only by the incidental surface current density **g**.

Assuming that the metal fills the half-space $z < 0$, we can write down the system of basic equations necessary to solve our problem in the following form

$$\operatorname{curl} \mathbf{E}(\omega) = i(\omega/c)\mathbf{H}(\omega),$$
$$\operatorname{curl} \mathbf{H}(\omega) = -i(\omega/c)\mathbf{E}(\omega), \quad z > 0,$$
$$\operatorname{curl} \mathbf{E}(\omega) = i(\omega/c)\mathbf{H}(\omega),$$
$$\operatorname{curl} \mathbf{H}(\omega) = -i(\omega/c)\varepsilon\mathbf{E}(\omega), \quad z < 0, \tag{27}$$
$$(c/4\pi)\,\mathbf{n}\times[\mathbf{H}^{\mathrm{med}} - \mathbf{H}^{\mathrm{vac}}] = \gamma(\mathbf{E} - \mathbf{n}(\mathbf{E}\mathbf{n})) + \mathbf{g}, \quad z = 0. \tag{28}$$

Solving the Maxwell equations (27) with the boundary conditions (28), using, for instance, Fourier transforms, we can express the electromagnetic field in the form of a functional of the fluctuating surface current density which is afterwards averaged by means of (24). We give the final relations which determine the spatial correlations of the components of the electrical field strength:

$$\overline{E_\alpha(\mathbf{r}_1,\omega)E^*_\beta(\mathbf{r}_2,\omega')} = \delta(\omega+\omega')(E^{(1)}_\alpha E^{(2)}_\beta)_\omega;$$

$$(E^{(1)}_\alpha E^{(2)}_\beta)_\omega = \frac{2\hbar\omega^3}{c^4}\coth\frac{\hbar\omega}{2\varkappa T}$$
$$\times \operatorname{Re}\gamma\int_0^\infty dq\,\frac{\exp\{i(\omega/c)[z_1\sqrt{\varepsilon(\omega)-q} - z_2\sqrt{\varepsilon(-\omega)-q}]\}}{|\sqrt{1-q}+\sqrt{\varepsilon-q}+(4\pi\gamma/c)|^2}$$
$$\times\left\{\left[\frac{1}{2}(1+|1-qA|^2)(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z}) + \frac{|1-qA|^2}{|\varepsilon-q|}q\delta_{\alpha z}\delta_{\beta z}\right]\right.$$
$$\times J_0\left(\frac{\omega}{c}R\sqrt{q}\right) - i\sqrt{q}\,|1-qA|^2$$
$$\times\left[\sqrt{\varepsilon(-\omega)-q}\,\frac{R_\alpha}{R}\delta_{\beta z}(1-\delta_{\alpha z})\right.$$
$$\left.+\sqrt{\varepsilon(\omega)-q}\,\frac{R_\beta}{R}\delta_{\alpha z}(1-\delta_{\beta z})\right]J_1\left(\frac{\omega}{c}R\sqrt{q}\right)$$
$$+(1-|1-qA|^2)$$
$$\times\left[\frac{R_\alpha R_\beta}{R^2}(1-\delta_{\alpha z})(1-\delta_{\beta z})\right.$$
$$\left.\left.-\frac{1}{2}(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z})\right]J_2\left(\frac{\omega}{c}R\sqrt{q}\right)\right\}, \qquad z_1,\ z_2 < 0;$$

$$(E^{(1)}_\alpha E^{(2)}_\beta)_\omega = \frac{2\hbar\omega^3}{c^4}\coth\frac{\hbar\omega}{2\varkappa T}\operatorname{Re}\gamma$$
$$\times\int_0^\infty dq\,\frac{\exp\{-i(\omega/c)[z_1\sqrt{1-q} - z_2(\sqrt{1-q})^*]\}}{|\sqrt{1-q}+\sqrt{\varepsilon-q}+(4\pi\gamma/c)|^2}$$
$$\times\left\{\left[\frac{1}{2}(1+|1-qA|^2)(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z}) + \frac{|1-qA|^2}{|1-q|}q\delta_{\alpha z}\delta_{\beta z}\right]\right.$$
$$\times J_0\left(\frac{\omega}{c}R\sqrt{q}\right) - i\sqrt{q}\,|1-qA|^2$$
$$\times\left[(\sqrt{1-q})^*\,\frac{R_\alpha}{R}\delta_{\beta z}(1-\delta_{\alpha z})\right.$$
$$\left.+\sqrt{1-q}\,\frac{R_\beta}{R}\delta_{\alpha z}(1-\delta_{\beta z})\right]J_1\left(\frac{\omega}{c}R\sqrt{q}\right)$$
$$+(1-|1-qA|^2)\left[\frac{R_\alpha R_\beta}{R^2}(1-\delta_{\alpha z})(1-\delta_{\beta z})\right.$$
$$\left.\left.-\frac{1}{2}(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z})\right]J_2\left(\frac{\omega}{c}R\sqrt{q}\right)\right\}, \qquad z_1,\ z_2 > 0;$$

$$(E^{(1)}_\alpha E^{(2)}_\beta)_\omega = \frac{2\hbar\omega^3}{c^4}\coth\frac{\hbar\omega}{2\varkappa T}\operatorname{Re}\gamma\int_0^\infty dq$$
$$\times\frac{\exp\{-i(\omega/c)[z_1\sqrt{1-q} + z_2\sqrt{\varepsilon(-\omega)-q}]\}}{|\sqrt{1-q}+\sqrt{\varepsilon-q}+(4\pi\gamma/c)|^2}$$
$$\times\left\{\left[\frac{1}{2}(1+|1-qA|^2)(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z})\right.\right.$$
$$\left.+\frac{|1-qA|^2}{\sqrt{1-q}\sqrt{\varepsilon(-\omega)-q}}q\delta_{\alpha z}\delta_{\beta z}\right]J_0\left(\frac{\omega}{c}R\sqrt{q}\right)$$
$$-i\sqrt{q}\,|1-qA|^2\left[\sqrt{\varepsilon(-\omega)-q}\,\frac{R_\alpha}{R}\delta_{\beta z}(1-\delta_{\alpha z})\right.$$
$$\left.+\sqrt{1-q}\,\frac{R_\beta}{R}\delta_{\alpha z}(1-\delta_{\beta z})\right]J_1\left(\frac{\omega}{c}R\sqrt{q}\right)$$
$$+(1-|1-qA|^2)\left[\frac{R_\alpha R_\beta}{R^2}(1-\delta_{\alpha z})(1-\delta_{\beta z})\right.$$
$$\left.\left.-\frac{1}{2}(\delta_{\alpha\beta}-\delta_{\alpha z}\delta_{\beta z})\right]J_2\left(\frac{\omega}{c}R\sqrt{q}\right)\right\}, \qquad z_1 > 0, \quad z_2 < 0. \tag{29}$$

Here $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$, J_n is a Bessel function, and A is defined by the formula

$$A = \left\{\frac{1}{\sqrt{1-q}} + \frac{1}{\sqrt{\varepsilon-q}}\right\}\left\{\frac{1}{\sqrt{1-q}} + \frac{\varepsilon}{\sqrt{\varepsilon-q}} + \frac{4\pi}{c}\gamma\right\}^{-1}.$$

There is no summation over repeated indices in Eqs. (29). One can find the correlation formulae for the magnetic field also by completely analogous means.

Equations (29) enable us, in particular, to obtain the following expression for the density of the energy of the electromagnetic field outside the metal:

$$w_\omega = \frac{1}{4\pi}(\mathbf{E}^2(\mathbf{r}) + \mathbf{H}^2(\mathbf{r}))_\omega = \frac{\hbar\omega^3}{\pi c^4}\coth\frac{\hbar\omega}{2\varkappa T}\operatorname{Re}\gamma\left\{\int_0^1 dq\right.$$
$$+\int_1^\infty dq\cdot q\exp\left(-2\frac{\omega}{c}z\sqrt{q-1}\right)$$
$$\times\left[\left|\sqrt{1-q}+\sqrt{\varepsilon-q}+\frac{4\pi\gamma}{c}\right|^{-2}\right.$$
$$\left.\left.+\left|1+\varepsilon\sqrt{\frac{1-q}{\varepsilon-q}}+\frac{4\pi\gamma}{c}\sqrt{1-q}\right|^{-2}\right]\right\}. \tag{30}$$

The integral over q from zero to unity gives the radiation energy density which does not depend on the distance and corresponds to the energy of the wave field. The second integral over q from unity to infinity in Eq. (30) corresponds to the energy of the quasi-stationary field which decreases away from the surface of the conductor. At large distances this diminution is determined by the relation

$$w_{\omega\,\text{quasi}} \approx \frac{\hbar\omega^3}{2\pi c^4}\coth\frac{\hbar\omega}{2\varkappa T}\,\mathrm{Re}\,\gamma \times\left\{1+\frac{1}{|\sqrt{\varepsilon-1}+4\pi\gamma/c|^2}\right\}(c/\omega z)^2. \quad (31)$$

In the case where the absolute magnitude of the dielectric constant is large compared to unity the expression for the energy density of the quasi-stationary field can essentially be obtained directly from Eq. (1.18) of Rytov's book[6] if one substitutes in that equation for the dielectric constant $\epsilon_{\text{eff}} = (\sqrt{\epsilon} + 4\pi\gamma/c)^2$. This possibility is caused by the fact that in the region of large values of ϵ the presence of a surface current can be taken into account by introducing just such an effective dielectric constant.[1]

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