THE TRANSFORMATION $K_2^0 \to K_1^0$ BY ELECTRONS

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The E0 transformation of K_2^0 to K_1^0 by their interaction with electrons is examined. An estimate is given of the cross section for the process and its angular distribution. The interference of the electron and nuclear interactions in the transformation of K_2^0 to K_1^0 in the unscattered beam is considered.

1. INTRODUCTION

As is well known,[1,2] the neutral mesons K_1^0 and K_2^0 have only approximate masses, and their linear combinations K^0 and $\overline{K}^0$, which have definite strangeness, are not neutral in the true sense: that is, for charge conjugation C (or, more exactly, for the combined transform CP[3]) K^0 transforms into $\overline{K}^0$ and $\overline{K}^0$ into K^0. Therefore, it is possible in principle for the interaction with the electromagnetic field to have opposite signs for K^0 and $\overline{K}^0$. In the representation K_1^0, K_2^0 this interaction does not have any diagonal elements by virtue of the true neutrality of K_1^0 and K_2^0. However, such an interaction can have nondiagonal elements capable of producing the transformation of K_2^0 into K_1^0 and K_1^0 into K_2^0. The question has been considered before under the assumption of a K-meson spin different from zero.[4] However, nowadays the K-meson spin is held to be zero. In such a case, as Feinberg showed,[5] the only K interaction linear in the electromagnetic field is the monopole E0 interaction, corresponding to the spherical-condenser model.

The interaction is proportional to div **E**, that is, to the charge density ρ at the point where the K meson is. This interaction, examined in the present paper, is a basic kind of interaction for electrons with neutral K's and significantly predominates over the weak (Fermi) interaction of electrons with K's.

By virtue of the small mass difference between K_1^0 and K_2^0, the E0-interaction with the field gives only a negligibly small probability for the spontaneous transformation of K_2^0 into K_1^0 in vacuo with a photon emitted. K^0 and $\overline{K}^0$ interact strongly with nucleons, and the effect considered here is only a small correction in the nucleon case.

2. ESTIMATE OF THE E0-MOMENT AND INTERACTIONS WITH ELECTRONS

For an estimate of the E0 moment, equal to $\Sigma e_i r_i^2$, we look at a model in which the K^0 meson is represented by the system $(K^+ + \pi^- + \pi^0)$, where K^+ is located at the center, and the π^- is spread over a sphere of radius $\hbar/m_\pi c$.

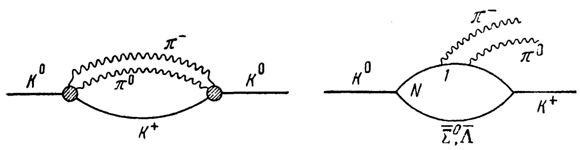
The K^0 transformation in such a system satisfies the selection rules for strong interactions, in particular conservation of strangeness, which forbides the transformation K^0 into $K^- + \pi^+ + \pi^0$. The transformation K^0 into $K^+ + \pi^-$ is forbidden by the pseudoscalarity of the π meson in strong interactions. Other possible virtual states, in particular those considered by Feinberg, require greater expenditures of energy.

Evidently, the estimate obtained, according to which

$$\sum e_i r_i^2 = -e(\hbar / m_\pi c)^2,$$

represents an upper limit, and a most likely excessive one.

In Feynman diagram language we consider a type A diagram (see figure) together with a photon which interacts with one or the other of the charged particles. If there is no direct $K-\pi$ interaction, then the hatched circles in A are taken to be Type B diagrams. It is essential that the sign of the contribution of the diagram considered (see figure) does not



depend on the Σ-nucleon mass difference or the difference in their interactions with π's.

However, for the estimate we do not use perturbation-theory calculations, but look instead directly at the picture in coordinate space.

Let us find the K-electron interaction energy.* In the field of a potential ϕ, the energy of a spherical condenser is

$$E' = \frac{1}{6}\Delta\varphi\cdot\sum e_i r_i^2.$$

Substituting $\Delta\phi = -4\pi\rho$, we find

$$E' = (2\pi/3)\rho e(\hbar/m_\pi c)^2.$$

The charge density owing to the electron is $\rho = -e|\psi_e|^2$, where e is the absolute value of the elementary charge.

In the center-of-mass system of the colliding K and e, the wavelength of the electron is of the order of $\hbar/m_e c$ or more, that is, significantly larger than the dimensions of the condenser. Therefore, the interaction can be taken to be a point one, with the potential

$$U(\mathbf{r}_K - \mathbf{r}_e) = -\delta(\mathbf{r}_K - \mathbf{r}_e)(2\pi/3)(e^2/\hbar c)(\hbar/m_\pi c)^3 m_\pi c^2$$
$$= -B\delta(\mathbf{r}_K - \mathbf{r}_e).$$

The coefficient B of the delta function has dimensions $cm^3 \times$ erg.

3. K SCATTERING ON THE ELECTRON

The interaction described satisfies all the conditions required for the Born approximation to be applicable. Inasmuch as we are only getting estimates, we limit ourselves to the nonrelativistic approximation, and only S waves are taken to be excited.

The solution has the form

$$e^{i\mathbf{p}_0\mathbf{r}} - \frac{m_e}{2\pi\hbar^2}\frac{e^{i\mathbf{p}_b\mathbf{r}}}{r}\int U dV = e^{i\mathbf{p}_0\mathbf{r}} + \frac{m_e B}{2\pi\hbar^2}\frac{e^{i\,r}}{r},$$
$$\mathbf{r} = \mathbf{r}_K - \mathbf{r}_e, \qquad m = m_K m_e/(m_K + m_e) \approx m_e.$$

The scattering cross section is equal to

$$\sigma = m_e^2 B^2/\pi\hbar^4 = (4\pi/9)(e^2/\hbar c)^2(m_e/m_\pi)^2(\hbar/m_\pi c)^2$$
$$\approx 2\cdot 10^{-35}\ \text{cm}^2.$$

Feinberg[5] estimates a possible cross section, substituting in his formula a quantity of the dimensions of the pion Compton wavelength. His last formula, in the nonrelativistic limit ($E_e = m_e c^2$) and after multiplying by h and c where necessary, differs from ours only by the factor $9\pi^2/8$. Therefore, the numerical value $10^{-30}\,\text{cm}^2$, given in reference 5, is evidently a misprint and should read $10^{-36}\,\text{cm}^2$.

The expression derived above for the wave function and the cross section, equal in our work to $2 \times 10^{-35}\,\text{cm}^2$, pertain to the K meson. For the energy of the interaction of a $\bar{K}$ with an electron, the constant and the amplitude of the scattered wave have the opposite sign, but the quantity σ is the same as for K.

For the scattering of K_2, which is represented as a linear combination of K and $\bar{K}$, the ratio of the phases of K and $\bar{K}$ in the scattered wave changes sign relative to that in the incident wave. Therefore, if the incident wave is a current of long-lived K_2, the wave scattered by the electron becomes a pure K_1 current (short-lived, decaying into two pions) without any K_2 admixture. The reaction cross section or, more accurately, its upper limit is given in the formula derived above.

We note that the cross section depends neither on the energy (mass) difference between K_2 and K_1 nor on the kinetic energy of the K in the nonrelativistic approximation. The cross section for the transformation of K_2 into K_1 by collision with nucleons is the order of a few millibarns,

In real substances, there is one electron for about every two nucleons. We shall take it that the nuclear cross section for the process $K_2 \to K_1$, per electron, is on the order of $5 \times 10^{-27}\,\text{cm}^2$. This quantity is 2.5×10^8 times greater than the cross section for the process examined, for electron interactions.

In the laboratory system, the K interactions with nucleons and with electrons have essentially different angular distributions. For K energies of the order of 100 Mev the wavelength is on the order of 1.5×10^{-13}, and the forward differential scattering cross section for nuclei is not more than a few times greater than the average. For the order of magnitude for one electron

$$d\sigma_{\text{nuc}}/d\Omega|_{\theta=0} \sim 2\cdot 10^{-28}\ \text{cm}^2/\text{steradian}.$$

For scattering by electrons the average energy transfer from 100 Mev K mesons is on the order of 50 kev; almost all the electrons in the atom can be regarded as being free. The spherically symmetric scattering of the center of mass system becomes very strongly peaked in the forward direction on going over to the laboratory system, the maximum angle of inclination of the K meson (in the nonrelativistic approximation) being equal to $\theta_m = m_e/m_K = 10^{-3}$. From this, the differential cross section for forward scattering with no energy loss

$$d\sigma_{\text{el}}/d\Omega|_{0=0} = (m_K/m_e)^2\sigma_{\text{el}}/4\pi = 1.5\cdot 10^{-30}.$$

To this must be added the same cross section for the particles which have transferred the maximum energy (the fraction $4m_e/m_K$) to the electrons. Consequently, the forward cross section for scat-

*From now on we omit the index 0 everywhere, since we only consider neutral K mesons.

tering by electrons, taking the most favorable estimate, is a hundred times less than that for nuclei. When the angle is increased from zero to the maximum inclination θ_m, the cross section increases and the average differential cross section in the interval $0 < \theta < \theta_m$ is four times greater than the one given above.

We note that in the exact forward direction there is a stream of K_1 mesons appearing in the unscattered beam according to the mechanism observed by Case[6] and especially Good,[7] which sharply hinders the observation of the K_1's obtained in the scattering from electrons. From another side, this mechanism leads to effects linear in the scattering amplitude from electrons because of interference with the nuclear interaction. These questions will be considered in detail in the next section.

The remark of Feinberg[5] about the possibility of detecting the electromagnetic scattering of a K^0 by the deviation from charge independence in the scattering of K^0's by protons and neutrons is founded on a misapprehension. The fact is that the K meson has isotopic spin 1/2, K^0 has the isotopic spin projection $-1/2$, and therefore the scattering of K^0 by a proton ($t_z = +1/2$) and on a neutron ($t_z = -1/2$) should not be unique in the theory of the isotopic spin invariance of interactions. The isotopic spin invariance theory establishes connections only between K^0 scattering by a proton and K^+ scattering by a neutron, but for electromagnetic effects the interactions of the K^+ charge with the magnetic moment of the neutron is greater than the E0-interaction of K^0 with the proton. Obviously the experiments are carried out not with free neutrons, but with nuclei, and the K^+ interaction with the charge of the nucleus also enters.

4. CREATION OF K_1 MESONS IN THE UNSCATTERED BEAM

In the very elegant paper of Good[7] there is an exhaustive treatment of the question of the appearance of K_1's in the unscattered beam, a process depending on A(0), the scattering amplitude for zero angle, that is, on the quantity determining the refractive index of the medium n, given by

$$n = 1 + 2\pi N k^{-2} A(0),\quad n' = 1 + 2\pi N k^{-2} A'(0),$$
$$w_1(0) = w_2 k^2 |n - n'|^2 x^2 \delta(\theta)$$
$$= w_2 |A(0) - A'(0)| (\pi N x / k)^2 \delta(\theta).$$

In these formulas n and A are related to K, n′ and A′ are related to $\bar{K}$, N is the number density of the scattering nuclei, k is the wave vector (inverse wavelength) of the mesons, w_2 is the incident K_2 current, and $w_1(0)$ is the K_1 current formed in the undeflected beam, that is, having precisely the same direction as the primary K_2 beam (the factor $\delta(\theta)$ on the right side of the formula). The peculiarity of the formula is that w_1 is proportional to the square of the thickness of the scatterer.

This peculiarity is connected with the fact that K_1 and K_2 are two very close states whose creation takes place only through weak interactions. The quadratic growth of $w_1(0)$ continues only as long as the time of flight for the path x is less than the time corresponding to a certain mass difference, $\hbar/(m_1 - m_2)c^2$. This time is of the order of magnitude of the decay lifetime of K_1, 10^{-10} sec, which gives $x < 1$ cm for K's with energies of 100 Mev.

The K_1 current, got by means of the elastic scattering into the solid angle $d\Omega$ at an angle θ to the direction of the primary beam K_2, gives the expression

$$dw_1 = {}^1/_4 w_2 |A(\theta) - A'(\theta)|^2 N x d\Omega.$$

In such a way Good establishes the relation between $w_1(0)$ in the direct current and $dw_1/d\Omega$ in the elastically scattered current:

$$w_1(0) = 4\pi^2 N x k^{-2} (dw_1 / d\Omega)_{\theta\to 0} \delta(\theta).$$

This relationship is general and for small x is fulfilled for arbitrary (electronic, nuclear) mechanisms of the process $K_2 \to K_1$.

The requirements for the electron process are confined to the fact that the only contribution to $dw_1/d\Omega$ is for small θ, $< 10^{-3}$. Therefore, if $dw_1/d\Omega$ is experimentally determined for angles, say, of $\theta = 0.05$, 0.02, and 0.01, extrapolation to zero does not disclose the contribution from the electron process. Besides this, we have to do experimentally with scatterings on two different kinds of particles, nuclei and electrons; so one of the conditions of Good's results is not met.

Let us take a fresh look at the question and compute directly the contribution to A(0), A′(0) and n, n′ from the interaction with the electrons. In Sec. 3 the scattering amplitude of K and $\bar{K}$ in the center of mass system was found. The amplitude of the wave function of the K_1 formed in the collision of a K_2 with an electron is equal to the same quantity, $m_e B/2\pi\hbar^2$. For the transition from the center of mass system to the laboratory system, the forward differential cross section, proportional to the square of the amplitude, is multiplied by the factor $(m_K/m_e)^2$. Consequently, the forward amplitude in the laboratory system is

$$A(0) = -A'(0) = \frac{m_K}{m_e}\frac{m_e B}{2\pi\hbar^2} = \frac{1}{3}\frac{e^2}{\hbar c}\frac{m_K}{m_\pi}\frac{\hbar}{m_\pi c}.$$

From this

$$(n - n')_{\rm el} = 2\pi N_{\rm el} k^{-2} (A - A')$$
$$= m_K \cdot 2BN_{\rm el} / \hbar^2 k^2 = BN_{\rm el} / E_K.$$

The part of the refractive index depending on the interaction between the K and the electron can be easily found directly, without examining the scattering problem. The potential energy of the interaction with an individual electron is $B\delta \times (r_e - r_K)$, which means that the average potential energy of a K in a medium with an electron density N_{el} is $U = N_{el} B$. For the transition of a K from the vacuum into the medium, its kinetic energy changes as a function of the electrons by the ratio $1 + \bar{U}/E_K$, while the wave vector changes by the ratio $\sqrt{1 - \bar{U}/E_K} \approx 1 + \bar{U}/2E_K$, which corresponds to a refractive index

$$n = 1 + N_{\rm el} B / 2E_K, \qquad n' = 1 - N_{\rm el} B / 2E_K$$

in complete correspondence with the formula written above. For iron (density 8 g/cm^3) and 100 Mev K mesons we get numerically $(n - n')_{el} = 1.4 \times 10^{-16}$. Now we find the value of $(n - n')_{nuc}$ corresponding to the chosen nuclear differential cross section 2×10^{-27}cm^2/steradian-nucleon, i.e., for iron, 10^{-25}cm^2/steradian-nucleus. We get

$$|A(0) - A'(0)|^2 = 4d\sigma / d\Omega = 4 \cdot 10^{-25},$$
$$n - n'|_{\rm nuc} = 2\pi N_{\rm nuc} k^{-2} |A(0) - A'(0)|$$
$$= \pi \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{m_K c^2}{E_K} N_{\rm nuc} |A(0) - A'(0)| = 1.2 \cdot 10^{-15}.$$

The electronic part of the refractive index is real. The relation between the real and imaginary parts of the nuclear difference $|n - n'|_{nuc}$ is unknown. In the most favorable case of real $(n - n')_{nuc}$, the electronic correction can reach ± 25% of the observed number of K_1's in the unscattered beam (proportional to $|(n - n')_{el} + (n - n')_{el}|^2$).

Thus, from Good's general statement the examination of the connection he found between the creation of K_1's at small angles (but always larger than θ_m) and the creation of K_1's in the unscattered beam can give new interesting information about neutral K mesons; the actual divergences between Good's formula and experiment would be shown in the interaction of K's and electrons.

We must remember here that the numerical estimates were made with assumptions that lead to excessive values and that in reality the possibility cannot be excluded that the E0 moment of the K is several times smaller than the chosen value. The effect is lessened in direct proportion.

We shall clear up a possible misunderstanding here. The quantity $(n - n')_{el}$ depends on the density of negative charge from electrons in the substance, but on the average the substance is electrically neutral. Should we not include the electromagnetic interaction of the K with the protons, which are of opposite sign and will compensate for the electron interaction? Actually, in the formation of K_1's on nuclei, the influence of the electromagnetic field of the protons is already included. Because of the local character of the E0 interaction, it acts at all angles, and not at just very small angles. (Above, in the electron case, the small angles arose because of the smallness of the electron mass.)

Consequently, if there were an electromagnetic interaction between the K's and the protons, but not the electrons, Good's formula would be exact. Deviations from it come only because of the interactions with electrons, since $(n - n')_{nuc}$ is taken from experiment.

The experimental investigation of the question poses great difficulties and will be possible only with K_2^0 sources of high intensity and with great accuracy in determining the direction of flight of the K_1, evidently by adding the momenta in the π^+ and π^- decays.

I want to take this opportunity to thank L. D. Landau and L. B. Okun' for their comments.

RESULTS

An estimate of the possible cross section for the process $K_2 + e = K_1 + e$. The ratio of this cross section to the nuclear one is of the order of 10^{-8}, and therefore direct observation of the process is excluded practically. The ratio increases to 1/100 for K_1's traveling in the direction of K_2, that is, forward.

The contribution from the E0-interaction of K_2 with electrons to the refractive index of a substance for the "high" estimate reaches 10%, and so gives a correction of up to 25% in the described relation of Good between the creation of K_1's for small angles and in the unscattered beam. The observation of this correction, although difficult, cannot be excluded.

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