GEODESICS IN FRIEDMAN-LOBACHEVSKY SPACE

I. G. FIKHTENGOL' TS

Leningrad Institute of Precision Mechanics and Optics

Submitted to JETP editor January 6, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1322-1323 (April, 1959)

WE consider the equations for the geodesics in the space in which the square of the line element ds has the form

$$ds^2 = H^2(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2), \quad (1)$$

where H is some function of the variables x_0 and $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

Taking the time coordinate x_0 as the independent parameter, the equations for the geodesics are then written in the form (see, e.g., reference 1)

$$\ddot{x}_i - \dot{x}_i\Gamma^0_{\alpha\beta}\dot{x}_\alpha\dot{x}_\beta + \Gamma^i_{\alpha\beta}\dot{x}_\alpha\dot{x}_\beta = 0 \quad (i = 1, 2, 3). \quad (2)$$

Here $\Gamma^\nu_{\alpha\beta}$ is the Christoffel symbol of the second kind; the dot denotes the derivative with respect to the variable x_0; greek indices run through the values 0, 1, 2, 3, and identical indices are understood to be summed from 0 to 3.

Starting with formula (1), we obtain the following expressions for the Christoffel symbol of the second kind:

$$\Gamma^0_{00} = \frac{1}{H}\frac{\partial H}{\partial x_0}, \quad \Gamma^i_{00} = \Gamma^0_{0i} = \frac{1}{rH}\frac{\partial H}{\partial r}x_i, \quad \Gamma^i_{0k} = \Gamma^0_{ik} = \frac{1}{H}\frac{\partial H}{\partial x_0}\delta_{ik},$$

$$\Gamma^l_{ik} = \frac{1}{rH}\frac{\partial H}{\partial r}(x_i\delta_{kl} + x_k\delta_{il} - x_l\delta_{ik}). \quad (3)$$

In these expressions $\delta_{ik} = 1$ for $i = k$, and $\delta_{ik} = 0$ for $i \neq k$; the Latin indices i, k, l run through the values 1, 2, 3.

With the expressions (3), the equations in (2) now take the form

$$\ddot{x}_i + \frac{1 - \dot{r}^2}{H}\left(\frac{1}{r}\frac{\partial H}{\partial r}x_i + \frac{\partial H}{\partial x_0}\dot{x}_i\right) = 0. \quad (4)$$

If we now set[1] $H = H(S)$, where $S = \sqrt{x_0^2 - r^2}$, then we finally obtain, according to (4),

$$\ddot{x}_i + \frac{\dot{r}^2 - 1}{S}\frac{H'}{H}(x_i - \dot{x}_i x_0) = 0, \quad (5)$$

where the prime denotes the derivative with respect to the variable S.

It is seen immediately that the relations

$$x_i = \dot{x}_i x_0 \quad (6)$$

yield $\dot{x}_i = \text{const}$; the functions x_i defined by them are therefore particular solutions of (5).

The relations (6) are used in a well-known way for the explanation of the phenomenon of the "recession of the galaxies," by regarding the quantities $\dot{x}_i$ as the coordinates of the corresponding mass in the accompanying system of coordinates.

[1] V. A. Fock, Теория пространства, времени и тяготения (The Theory of Space, Time, and Gravitation), GTTI, 1955.

Translated by R. Lipperheide
254

ON THE PRODUCTION OF PION AND MUON PAIRS BY THE ANNIHILATION OF HIGH-ENERGY POSITRONS

A. I. NIKISHOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 8, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1323-1324 (April, 1959)

THE study of the processes $e^+ + e^- \rightleftarrows \mu^+ + \mu^-$ and $e^+ + e^- \rightleftarrows \pi^+ + \pi^-$ is of interest in connection with the possibility of detecting deviations from local theory at distances $\sim 10^{-13}$ cm. If we describe the deviation by a factor $F(q^2)$ in the expression for the transition current, we get the cross-sections for these processes from the corresponding expressions of local theory by simply multiplying by the squares of the form factors $F(q^2)$ for the particles in question. Since in the annihilation of two particles $q^2 = -4E^2$ (in the center-of-mass system), the introduction of the form factors does not change the angular distributions for these processes. The values of the form factors for $q^2 < 0$ (annihilation of particles) cannot be obtained from the values of $F(q^2)$ for $q^2 > 0$ (scattering).*

The absolute squares of the matrix elements in the center-of-mass system, averaged over the initial spin states and summed over the final (for those particles having spin), are as follows:†

$$|M|^2(e^+ + e^- \rightleftarrows \mu^+ + \mu^-) = \frac{1}{16E^4}\left\{1 + \left(\frac{\mu}{E}\right)^2 + \frac{p_\mu^2}{E^2}\cos^2\vartheta\right\},$$

$$|M|^2(e^+ + e^- \rightleftarrows \pi^+ + \pi^-) = \frac{p_\pi^2}{32E^4}\sin^2\vartheta.$$

We note that the matrix elements for the processes involving π mesons are small for nonrela-