NUCLEAR FORCES AND LEVELS OF THE LITHIUM ISOTOPES

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A refinement is introduced in the intermediate coupling model of the nuclear shell theory by taking into account paired spin-orbit interaction between the nucleons. Calculations carried out for Li^6 and Li^7 yield better agreement with experiment than the usual intermediate-coupling model which takes into account only single particle spin-orbit interaction. Some indications are obtained with respect to the existence of different types of radial dependence for nuclear forces of different exchange nature.

1. INTRODUCTION

DURING the last few years many papers have appeared on the analysis of the energy levels of light nuclei, their purpose being to obtain information on the forces acting between nucleons.[1-3] Of special interest from this point of view is Li^6, which contains two p-nucleons beyond the filled $S^4_{1/2}$ shell corresponding to the He^4 nucleus. The energy spectrum of this nucleus has been thoroughly studied[4] and the first six levels have been reliably identified (see Fig. 1). A detailed analysis of the levels for Li^6 and Li^7 has been carried out by Soper[1] and Meshkov and Ufford.[2] In both papers the calculation has been carried out for the intermediate-coupling approximation.

As shown in reference 5, a consistent description of the p-shell nuclei, involving no assumptions whatsoever about the actual form of the paired nucleon interaction, requires the introduction of 12 independent parameters to describe this interaction. The use of a smaller number of parameters is tantamount to setting down additional conditions for these independent quantities, associated with assumptions about the form of the potential of the nucleon interaction. In describing nuclei with the aid of an intermediate-coupling model, and among these descriptions we can cite the work of Soper and Meshkov and Ufford, other assumptions are made, based on considerations of convenience in computation, in addition to such a physical assumption about the properties of the two-nucleon potential. Upon analysis of these additional limitations, it was deemed interesting to dispense with them and to carry out this analysis consistently from an original physical point of view. Such a program requires more complicated computations than those involving the intermediate-coupling model but which, however, are readily performed by the use of computer techniques.

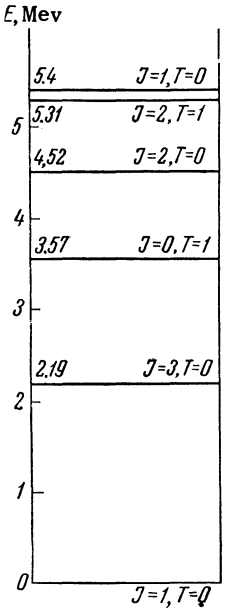

FIG. 1

In the present paper such an analysis is carried out on the basis of a computation for the nuclei of Li^6 and Li^7. We intend later to carry out computations for other p-shell nuclei, in particular for the nucleus of B^{10}, which has a very rich and thoroughly studied energy level spectrum.

2. CENTRAL NUCLEON INTERACTION

According to the intermediate coupling model the interaction of nuclei is described by paired central forces which have exchange terms and single-particle spin-orbit forces. Thus the Hamiltonian has the form

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} + a \sum \mathbf{l}_i \mathbf{s}_i, \quad (1)$$

where

$$V_{12} = [W + MP_x + BP_\sigma + HP_xP_\sigma] V(r_{12}). \quad (2)$$

Here P_X and P_σ are the operators of interchange of space and spin coordinates; W, M, B, H are the coefficients that characterize the contribution of the Wigner, Majorana, Bartlett and Heisenberg forces to the central-interaction potential, satisfying the normalization condition

$$W + M + B + H = 1. \quad (3)$$

In the analysis of the p-shell nuclei, all the integrals in $V(r_{12})$ can be expressed by two Slater integrals: direct (L) and exchange (K);[6] Therefore the central interaction as a whole is described by five independent parameters L, K and four coefficients W, M, B, H, connected by the additional relation (3).

The requirement of a common radial dependence of the various exchange terms of potential (2) is an additional limitation set on the central interaction of nuclei; this limitation follows neither from experiment nor from theory. Failure to use this limitation will lead to a substantial increase in the number of parameters when we set down the potential in its configurational presentation, while in the shell presentation[5] only one more parameter is required than in references 1 and 2. In this presentation the independent parameters of the central potential are matrix elements over the states of the two nucleons in the L-S coupling scheme, which are diagonal in L, S and T and are independent of the total momentum J:

$$F_1 = \langle p^2 : {}^1S_0^{T=1} | V_{12} | \rangle, \qquad F_2 = \langle p^2 : {}^3S_1^{T=0} | V_{12} | \rangle,$$
$$F_3 = \langle p^2 : {}^1P_1^{T=0} | V_{12} | \rangle, \qquad F_4 = \langle p^2 : {}^3P_J^{T=1} | V_{12} | \rangle,$$
$$F_5 = \langle p^2 : {}^1D_2^{T=1} | V_{12} | \rangle, \qquad F_6 = \langle p^2 : {}^3D_J^{T=0} | V_{12} | \rangle. \quad (4)$$

Here no assumptions are made about the exchange nature of the interaction, about its dependence on the velocity, or about the form of the radial functions of the nucleons in the nucleus.

Having expressed the six matrix elements (4) in terms of L, K, W, M, B, and H it is easy to show that the additional relation involving F_1, $F_2, \ldots, F_6$:

$$F_1/F_2 = F_5/F_6. \quad (5)$$

corresponds to the requirement of the common radial dependence of central forces of different nature.

3. SPIN-ORBIT INTERACTION OF NUCLEON

The dependence of the nucleon interaction on the velocity is shown in (1) in terms of single particle spin-orbit forces

$$V_{sl} = a \sum_i \mathbf{l}_i \mathbf{s}_i. \quad (6)$$

A general analysis of the dependence of the interaction on the velocity is given in references 7 and 8. The basic term that describes such an interaction consists of the two particle spin-orbital forces

$$V_{sl}(1,2) = f(1,2)(\sigma_1 + \sigma_2)[(\mathbf{r}_2 - \mathbf{r}_1)(\mathbf{p}_2 - \mathbf{p}_1)]/\hbar$$
$$= f(1,2)(\mathbf{S}_{12}\mathbf{L}_{12}), \quad (7)$$

where $\mathbf{S}_{12}$ and $\mathbf{L}_{12}$ are the total spin and orbital momentum for the relative motion of two nucleons. f(1, 2) consists of two exchange members — Wigner type and Heisenberg type.

The potential (7) leads to the same splitting of the interaction between the outer nucleon and the nucleons of the filled shell with $j = l + \frac{1}{2}$, as in (6).[9] Thus the single-particle spin-orbit forces (6) and the two-particle forces (7) are completely equivalent in the description of nuclei containing one nucleon or one "hole" in the unfilled shell. However, in the description of nuclei having a larger number of nucleons in the unfilled shell, there is no such simple correspondence. In this sense the accepted description of the nucleon interaction by means of potential (1), used in the intermediate-coupling model, where we account for the interaction of the external nucleons with the nucleons of the filled shells and neglect the spin-orbital interaction between them, is inconsistent, since this description does not correspond to any physical assumption about the nucleon interaction.

Going over to the center-of-mass system of two interacting nucleons, following Talmi,[10] it is easy to see that within the p-shell limits the potential (7) is characterized by two independent parameters α_1 and α_2, which correspond to the spin-orbit interaction of nucleons in the P and D states of relative motion:

$$\langle {}^3P_0^{T=1} | V_{sl}(1,2) | \rangle = 2\alpha_1, \qquad \langle {}^3D_1^{T=0} | V_{sl}(1,2) | \rangle = 3\alpha_2,$$
$$\langle {}^3P_1^{T=1} | V_{sl}(1,2) | \rangle = \alpha_1, \qquad \langle {}^3D_2^{T=0} | V_{sl}(1,2) | \rangle = \alpha_2,$$
$$\langle {}^3P_2^{T=1} | V_{sl}(1,2) | \rangle = -\alpha_1,$$
$$\langle {}^3D_3^{T=0} | V_{sl}(1,2) | \rangle = -2\alpha_2. \quad (8)$$

In the simplest case of oscillator wave functions,

TABLE I

Parameters	$\alpha_2 = 0$	$\alpha_2 = 0.2$	$\alpha_2 = 0.4$	$\alpha_2 = 0.6$
F_2	—7.869	—7.263	—6.746	—6.314
F_6	—4.333	—3.690	—3.135	—2.667
Δ	2.33	2.03	1.73	1.43

TABLE II

$F_1(\alpha_1, \alpha_2)$

α_1	$\alpha_2 = 0$	$\alpha_2 = 0.2$	$\alpha_2 = 0.4$	$\alpha_2 = 0.6$
0	—3.710	—3.170	—2.762	—2.341
0.4	—3.759	—3.201	—2.761	—2.403
0.8	—3.803	—3.254	—2.802	—2.445
1.2	—3.842	—3.292	—2.838	—2.477
1.6	—3.876	—3.326	—2.869	—2.504

TABLE III

$F_5(\alpha_1, \alpha_2)$

α_1	$\alpha_2 = 0$	$\alpha_2 = 0.2$	$\alpha_2 = 0.4$	$\alpha_2 = 0.6$
0	—2.160	—1.502	—0.897	—0.370
0.4	—2.092	—1.389	—0.727	—0.029
0.8	—2.007	—1.248	—0.443	+0.926
1.2	—1.898	—1.042	+0.132	7.817
1.6	—1.752	—0.707	1.909	—

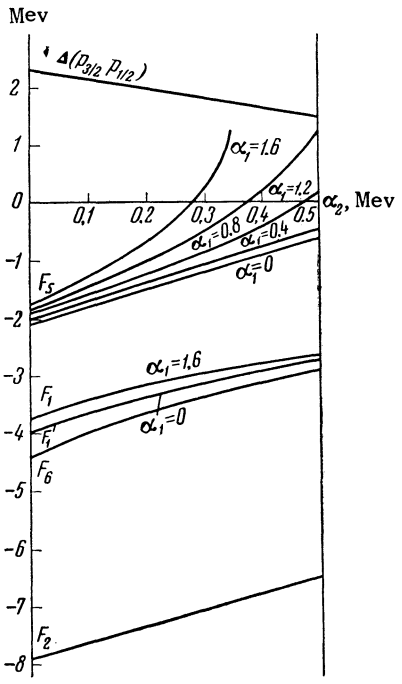

FIG. 2

which describe the nucleons of the s and p shells, the parameter a of the single-particle spin-orbital coupling is connected with α_1 are by the relation $a = 3\alpha_1$, and the magnitude of the spin-orbit splitting of the interaction of the p-nucleon with the $s^4_{1/2}$ shell can be expressed in terms of α_1:

$$\Delta(p_{3/2}, p_{1/2}) = {}^9/_2\,\alpha_1. \qquad (9)$$

In a more rigorous approach, relation (9) should be considered only as approximate: the magnitude of the spin-orbit splitting $\Delta(p_{3/2}, p_{1/2})$ may be more sensitive to the fine terms of the radial wave functions and nucleon interaction, right down to multiple effects, than the interaction of the nucleons in the p-shell with each other. For this reason we shall consider $\Delta(p_{3/2}, p_{1/2})$ as an additional independent parameter. In this way the problem of the accuracy of relation (9), obtained from the more rigorous model assumptions, becomes an experimental problem while relation (9) itself becomes a criterion for the correctness of these model assumptions.

4. CALCULATION OF THE Li^6 and Li^7 LEVELS

In Tables I — III and in Fig. 2 we present the values for the parameters $\Delta(p_{3/2}, p_{1/2})$, F_1, F_2, F_5, and F_6 (in Mev) as a function of α_1 and α_2, determined from the spectrum of the excited Li^6 levels (Fig. 1).

The data are given for the particular case of $F_3 = F_4 = 0$, which corresponds to neglecting the interaction of nucleons in the P state of relative motion as compared with the interaction in the S state.[11]

From the values of parameters thus obtained, we compute the magnitude of binding energy for Li^6 (relative to He^4). Here the energy for the Coulomb interaction of the external proton with the He^4 core is determined from the isotope multiplets, He^5—Li^5 and He^6 (J = 0, T = 1)—Li^6 (J = 0, T = 1), and comes to 0.66 Mev in both cases. The binding energy of the $p_{3/2}$ nucleon with the $S_{1/2}$ shell is equal to −0.81 Mev, i.e., the binding energy of He^5 relative to He^4. As can be seen from Fig. 3, $\alpha_2 \cong 0.2$ Mev corresponds to the experimental value E = 3.70 Mev.

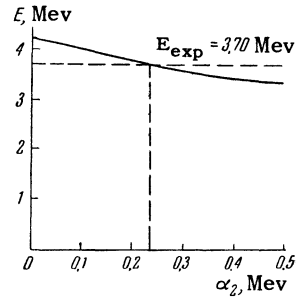

FIG. 3

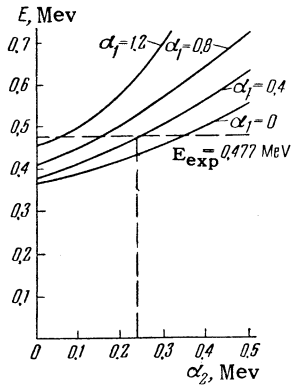

FIG. 4

In the energy-level spectrum of the Li^7 nucleus, identification has been made for levels with $J = \frac{1}{2}$, $T = \frac{1}{2}$ at $E = 0.477$ Mev and $J = \frac{5}{2}$, $T = \frac{1}{2}$ at $E = 7.46$ Mev. In Fig. 4 we give the results of the calculation for the energy of level $(\frac{1}{2}, \frac{1}{2})$ with the parameters determined for Li^6. We have chosen an energy scale for Li^7, after reference 1, by fixing the position corresponding to the 7.46-Mev level. $\alpha_1 \cong 0.4$ Mev corresponds to the experimental value for $E(\frac{1}{2}, \frac{1}{2}) = 0.477$ Mev; the ratio of the scales for Li^7 and Li^6 is 1.26. At this value of α_1, the first level with $J = \frac{7}{2}$, $T = \frac{1}{2}$ coincides with an unidentified level at $E = 4.61$ Mev, while the first level with $J = \frac{5}{2}$, $T = \frac{1}{2}$ corresponds to the level at $E = 5.5$ Mev (see Fig. 5).

The binding energy of Li^7 (relative to Li) for $\alpha_1 = 0.4$ and $\alpha_2 = 0.2$, comes to 7.06 Mev, while the experimental value is 7.15 Mev. On the other hand, the intermediate-coupling approximation gives a value of 9.15 Mev.

The calculation also includes, as a particular case, the intermediate-coupling approximation.

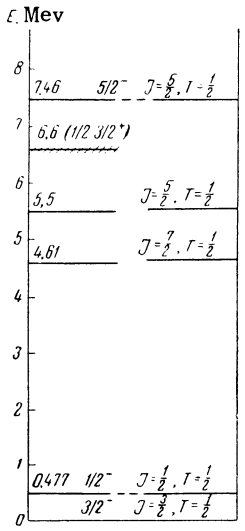

FIG. 5

For the latter the corresponding values are $\alpha_1 = 0$ and $\alpha_2 = 0$. The corresponding values $a = \frac{2}{3}$, $\Delta = 1.55$ Mev, $L = -(F_2 + 2F_6)/3 = 5.51$ Mev, and $K = -(F_2 - F_6)/2 = 1.18$ Mev coincide with those given by Soper.[1]

5. CONCLUSIONS

1. Accounting for the paired spin-orbit interaction between nucleons in the unfilled shell leads to a significant change in the parameters that describe the central interaction of the nucleons.

2. It follows from these calculations that a mixture of central (in a common form) and two-particle spin-orbital forces is a good approximation for the interaction between the nucleons in the nucleus. An analysis of the energy levels for the Li isotopes leads to the following values for the parameters of the paired nucleon interaction (Mev):

$$\Delta = 1.98, \quad \alpha_1 = 0.43, \quad \alpha_2 = 0.23,$$
$$F_1 = -3.12, \quad F_2 = -7.18,$$
$$F_5 = -1.28, \quad F_6 = -3.55. \qquad (10)$$

3. The values for the parameters $\Delta(p_{3/2}, p_{1/2})$ and α_1 are found to be in good agreement with relation (9) obtained by means of the oscillator functions for the s and p nucleons.

4. Relation (5), which holds for $\alpha_1 = 0$ and $\alpha_2 = 0$, does not hold for the parameters given by (10). This indicates the existence of a different radial dependence for nuclear forces of different exchange character. It should be pointed out that by the same token it becomes possible to explain the remarkably small value for the matrix element $\langle p_{3/2}: 21 | V | \rangle = 0.49$ Mev, obtained by the author[12] and Talmi and Thieberger[13] in connection with the analysis of the nuclear binding forces in the $p_{3/2}$ shell, which is completely impossible to obtain if relation (5) is satisfied.

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