ANISOTROPIC FISSION OF ROTATING NUCLEI

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WHEN the probability of fission depends on nuclear angular momentum,[1] in order to determine the dependence of the anisotropy on $x = (Z^2/A)/(Z^2/A)_{cr}$, where Z and A are the charge and mass of the fissioning nucleus, we must take other competing processes into account (specifically here, neutron evaporation). The fission probability of a nucleus with angular momentum j and its projection K on the symmetry axis of the nucleus, for the direction $d\omega = \sin\vartheta\, d\vartheta$, where ϑ is the angle between the incident beam direction and the direction in which a fragment come off, is given by

$$dw_f(j, K, \vartheta) = \frac{\gamma_f(j, K, \vartheta)\, d\omega}{\Gamma_n + \Gamma_f(j)}, \quad \Gamma_f(j) = \int_0^j dK \int_0^\pi \gamma_f \sin\vartheta d\vartheta, \quad (1)$$

where Γ_n is the neutron width and Γ_f is the fission width for the given angular momentum. When $j_{cr} \gg j \gg 1$ (where j_{cr} is the angular momentum for which the nucleus is not stable against fission[1]) for γ_f we can use a statistical formula such as[2]

$$\gamma_f(j, K, \vartheta) = \frac{\Gamma_f(0)}{j\pi}\left(\sin^2\vartheta - \frac{K^2}{j^2}\right)^{-1/2} \exp\left\{\frac{\hbar^2 j^2}{2\tau}\left(\frac{1}{I_0} - \frac{1}{I_A}\right) - \frac{\hbar^2 K^2}{2\tau}\left(\frac{1}{I_C} - \frac{1}{I_A}\right)\right\}, \quad (2)$$

where $\Gamma_f(0)$ is the fission width for zero angular momentum, $\tau = \sqrt{10U/A}$ is the temperature, U is the nuclear excitation energy, I_0 is the moment of inertia of a spherical nucleus, I_A is the moment of inertia about an axis perpendicular to the symmetry axis and I_C is the moment of inertia about the symmetry axis of the nucleus. I_A and I_C are taken for a deformation corresponding to the top of the fission barrier. For a moment of inertia equal to that of a rigid body we have

$$\frac{1}{I_0} - \frac{1}{I_A} = \frac{1}{I_0}(1.2z + 5.6z^2) = \frac{\beta^2 2\tau}{\hbar^2};$$

$$\frac{1}{I_C} - \frac{1}{I_A} = \frac{1}{I_0}(3.5z + 8.8z^2) = \frac{\alpha^2 4\tau}{\hbar^2},$$

where $z = 1 - x$. Inserting (2) into (1), summing over all K and averaging over j, we obtain the angular distribution

$$w_f(\vartheta)\, d\omega = d\omega \frac{\Gamma_f(0)}{j_{max}^2} 2 \times \int_0^{j_{max}} \frac{I_0(\alpha^2 j^2 \sin^2\vartheta)\exp(j^2\beta^2 - \alpha^2 j^2 \sin^2\vartheta)}{\Gamma_n + \Gamma_f(0)\,\Phi(j\alpha)\exp(j^2\beta^2)}\, jdj = \int_0^{j_{max}} jdj \cdot \varphi(j), \quad (3)$$

where

$$\Phi(j\alpha) = (j\alpha)^{-1}\int_0^{j\alpha} dt \cdot e^{-t^2},$$

$$\hbar^2 j_{max}^2 = 2m(E - B)(R + R_1)^2 = 2mE\sigma/\pi;$$

I_0 is a Bessel function with imaginary argument; m, E, B, R are the mass, energy, Coulomb barrier and radius of the incident particle; R_1 is the radius of the target nucleus; σ is the cross section for the formation of a compound nucleus.

Equation (3) gives the angular distribution for one stage of the cascade process of neutron evaporation. For $\Gamma_f \gg \Gamma_n$ fission occurs without neutron evaporation and $w_f(0)/w_f(\pi/2)$ determines the anisotropy. In this case (3) agrees with Strutinskiĭ's formula.[2] For $\Gamma_f \ll \Gamma_n$ the entire neutron evaporation cascade must be taken into account. Therefore the anisotropy is given by

$$\sigma_f(0)/\sigma_f(\pi/2) = \sum_{i=0}^N w_{fi}(0) \Big/ \sum_{i=0}^N w_{fi}(\pi/2),$$

$$w_{fi} = \int_0^{j_{max}} jdj\varphi_i(j) \prod_{s=0}^i \Gamma_{ns} \Big/ (\Gamma_{ns} + \Gamma_{fs}(j)),$$

where i is the number of the cascade stage. In w_{fi} $\exp(j^2\beta^2)$ remains in the numerator; this represents the increased weight of states with high angular momentum and, consequently, increased anisotropy. The essential point is that when $\Gamma_f \gg \Gamma_n$ fission is equally probably for different values of the angular momentum, while when $\Gamma_f \ll \Gamma_n$ mainly nuclei with high angular momentum will fission.

If it is assumed that the dependence of the nuclear moment of inertia about the symmetry axis (I_C) on z is determined just as for a rigid body, where $\xi < 1$, then for nuclei with $\Gamma_f \gg \Gamma_n$ we can obtain ξ; then assuming that ξ is identical for all nuclei and is independent of temperature, we obtain the anisotropy for nuclei with $\Gamma_f \ll \Gamma_n$. Using experimental data[3] on fission induced by α particles, we obtain $\xi = 0.46$ for Pu^{239}, Np^{237}, and U^{235}. If for Ra^{226} $\Gamma_f \ll \Gamma_n$ the anisotropy is 1.87 (compared with the experimental value 2.03 ± 0.05).

We note that the theory can be tested for both $\Gamma_f \gg \Gamma_n$ and $\Gamma_f \ll \Gamma_n$ by studying $\sigma_f(\vartheta)$ $\sigma_f(\pi/2)$. For $\vartheta \neq 0$ this ratio approaches a constant limit as $\alpha^2 j^2_{max}$ increases. Thus $\sigma_f(\pi/4)/\sigma_f(\pi/2) \to \sqrt{2}$ as $\alpha^2 j^2_{max} \to \infty$ and is independent of the nuclear parameters.

In conclusion I wish to thank D. P. Grechukhin for a discussion and I. Halpern for his kindness in making experimental data available.

[1] G. A. Pik-Pichak, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 341 (1958), Soviet Phys. JETP **7**, 238 (1958).

[2] V. M. Strutinskiĭ, Атомная энергия (Atomic Energy) **2**, 508 (1957). (Translation, Consultants Bureau, Inc., p. 621).

[3] C. T. Coffin and I. Halpern, Phys. Rev. **112**, 536 (1958).

Note added in proof (January 27, 1959). $\sigma(0°)/\sigma(90°)$ was also calculated for Ra, assuming $\xi = 1$ but with a more exact value of the angular momentum of the compound nucleus (See I. Halpern and V. M. Strutinskiĭ, Report P/1315 at the Second Geneva Conference on the Peaceful Uses of Atomic Energy, 1958). It was assumed that $\tau = \sqrt{a(U - E_f)}$, where E_f is the fission threshold and the constant a was determined from the anisotropy for Pu. The initial temperature of the nucleus Pu + α particle at the saddle point is 1.3 to 1.5 Mev. The anisotropy for Ra is calculated to be 2.0. For Bi, taking into account the dependence of fissionability on excitation energy, we obtain $\sigma(0°)/\sigma(90°) = 1.9$ ($\sigma(0°)/\sigma(90°)_{exp} = 2.02$; $\Gamma_f^{(0)}/\Gamma_n \sim \exp[(E_n - E_f)/\tau]$, where $E_f - E_n \approx 8$ Mev and $E_f \approx 15$ Mev. Agreement of the calculated and experimental anisotropy for Ra shows that we have no reason to assume the absence of correspondence between the nuclear moment of inertia and that of a rigid body, including the cases of excitations below 10 or 12 Mev, as was suggested by Halpern and Strutinskiĭ on the basis of neutron experiments.

The results given here were obtained in collaboration with V. M. Strutinskiĭ.

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ON THE THERMODYNAMICS OF HELIUM

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In the paper by Esel'son, Kaganov, and Lifshitz[1] it is shown that the lambda transition in a solution of helium isotopes is a phase transition of the second order. Using the condition for equilibrium between phases and the equations for the chemical potentials of the components in the gaseous phase, the authors obtain the equation

$$kT\ln P_3 + \chi(T) = \varphi + (1 - x_{liq})\partial\varphi/\partial x_{liq},$$
$$kT\ln P_4 + \chi(T) = \varphi - x_{liq}\partial\varphi/\partial x_{liq} \qquad (2)$$

(the equation numbers are those of reference 1). If we take the total derivative of both sides of Eq. (2), we get, according to the authors, the expression

$$-S_3 + kT\left(\frac{d}{dT}\ln P_3\right) = \frac{\partial\varphi}{\partial T} + (1 - x_{liq})\frac{\partial^2\varphi}{\partial x_{liq}\partial T} \qquad (3)$$

and so on. It is further asserted that at the temperature T_λ one obtains

$$kT_\lambda\Delta\left(\frac{d}{dT}\ln P_3\right) = (1 - x_{liq})\Delta\left(\frac{\partial^2\varphi}{\partial T\partial x_{liq}}\right) \qquad (4)$$

and so on, since the first derivative with respect to the thermodynamic potential has a break and the second derivative a jump at a phase transition of the second order. Substituting, according to Eqs. (6) and (9), for the quantities which occur here the authors obtain the following equation for the total pressure

$$kT_\lambda\Delta\left(\frac{d}{dT}\ln P\right) = (x_{vap} - x_{liq})\frac{\Delta C_p}{T_\lambda}\frac{\partial T_\lambda}{\partial x_{liq}}. \qquad (10)$$

Since, as is shown in reference 1, $\partial T_\lambda/\partial x_{liq} < 0$, $\Delta C_p > 0$, $x_{vap} > x_{liq}$, we have $\Delta\{d\ln p/dT\} < 0$ which is in accordance with experimental data.[2]

The authors conclude from this that the lambda transition is a phase transition of the second order.

We must draw attention to an error which has crept in in the process of this proof. If we take into account that along the equilibrium curve the total derivative with respect to the temperature is given by the expression $d\mu/dT = \partial\mu/\partial T + (\partial\mu/\partial P)\,dP/dT$ one must write the basic equations (3) and (4) in the form

$$\frac{d}{dT}[kT\ln P_3 + \chi(T)] = \frac{\partial\varphi}{\partial T} + (1 - x_{liq})\frac{\partial^2\varphi}{\partial T\partial x_{liq}}$$
$$+\left[(1 - x_{liq})\frac{\partial^2\varphi}{\partial P\partial x_{liq}} + \frac{\partial\varphi}{\partial P}\right]\frac{dP}{dT}, \qquad (3')$$