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ON THE STABILITY OF NUCLEI

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The dependence of the interaction between a nucleon and the nucleus on N/Z was investigated. An estimate of the maximal A/Z is given; this ratio is found to vary between 3 and 3.8. It is shown that when N equals the number of neutrons in a closed shell, the minimal Z changes discontinuously by several units. It is suggested that these jumps in the values of Z_{min} on closing the neutron shell may be responsible for the exceptionally great abundance of certain isotopes, provided that the synthesis of atomic nuclei occurred in brief neutron bursts.

In a previous paper[1] we considered the single-particle levels of light nuclei obtained with the help of the optical model potential. The ordering and the energy of the levels can also be determined for the heavier nuclei. In comparing the obtained results with experiment it is necessary to make certain additional assumptions about the dependence of the nuclear radius and the well depth on A and Z. The comparison of theory and experiment with regard to the total cross sections, the angular distributions, and the polarization shows that the scattering from all nuclei can be described by a potential with one and the same depth, assuming that $R = (1.16\,A^{1/3} + 0.33) \times 10^{-13}$, where R is the distance at which the potential has fallen off to half its maximum value.

The agreement remains very good if we make the contrary assumption that the nuclear radius obeys the $A^{1/3}$ law, but the potential becomes smaller as the ratio N/Z increases. The well depth will then be less for heavy nuclei than for light ones. However, this effect should show up also in the different isotopes of the same element. It is therefore a natural step to add to the optical potential a term which takes account of the isotopic effect.*

Such a potential can be used for the calculation of the levels of nuclei with a closed shell plus one extra particle. In the framework of this model we regard the proton and neutron shells as independent, which is correct for heavy nuclei. We can then find the binding energies of closed-neutron-shell nuclei that differ from each other in the number of protons.

In studying these binding energies we can also determine the isotopic effect. It appears to be in agreement with the optical model data. The isotopic effect thus obtained may be extrapolated to the limits of nuclear stability in order to find the limiting value of N/A as a function of A. The answer to this problem permits certain conclusions about the creation of the elements.

*The author is grateful to L. A. Sliv for a conversation which stimulated the development of these ideas.

1. CHOICE OF THE POTENTIAL

We now take up the problem of the correct way of including the isotopic effect in the optical potential.

We chose the simplest model,

$$V = \frac{Z}{A} V_{np} + \frac{N}{A} V_{nn} \qquad (1)$$

where V_{np} and V_{nn} are the potentials for the interaction of the neutron with the proton and neutron matter, respectively. This means that we assume the additivity of the two interactions, both in the ordinary and in the spin-orbit potential. Furthermore, since the potential does not change very strongly, we retained the assumption $\alpha = k_0 = \sqrt{2mV_0}/h$, where α is the diffuseness parameter. This condition implies that α varies at most by 10%. This has no significant effect on the overall picture. We can therefore use the results obtained with a well of the same depth for all nuclei (i.e., which is independent of N/Z).

In the calculations we made use of the following potential:[2,3]

$$V = -\frac{V_0}{1 + e^{\alpha(r-R)}} - \frac{\varkappa}{r}\frac{d}{dr}\left[\frac{V_0}{1 + e^{\alpha(r-R)}}\right](\mathbf{l}\cdot\boldsymbol{\sigma}),$$

where $V_0 = \frac{Z}{A} V_{np}^0 + \frac{N}{A} V_{nn}^0$, with $2mV_0\kappa/h^2 = 0.66$. The nuclear radius was taken to be equal to $R = 1.25 \times 10^{-13}$.

We assumed that $V_0 = 49$ Mev for nuclei with $Z/N = 0.80$. This assumption gives good agree-

TABLE I

$E_b = 0$				$E_b = 0.16\ V_0 \approx 8$ Mev			
$X = k_0R$	l	j	Level	$X = k_0R$	l	j	Level
0.67	0	1/2	$1\,s$	2.19	0	1/2	$1\,s$
2.77	1	3/2	$1\,p_{3/2}$	3.80	1	3/2	$1\,p_{3/2}$
3.35	1	1/2	$1\,p_{1/2}$	4.34	1	1/2	$1\,p_{1/2}$
3.98	1	1/2	$2\,s$	5.30	2	5/2	$1\,d_{5/2}$
4.35	2	5/2	$1\,d_{5/2}$	5.66	0	1/2	$2\,s$
5.04	2	3/2	$1\,d_{3/2}$	6.00	2	3/2	$1\,d_{3/2}$
5.75	3	7/2	$1\,f_{7/2}$	6.74	3	7/2	$1\,f_{7/2}$
5.755	1	3/2	$2\,p_{3/2}$	7.24	1	3/2	$2\,p_{3/2}$
6.078	1	1/2	$2\,p_{1/2}$	7.46	3	5/2	$1\,f_{5/2}$
6.54	3	5/2	$1\,f_{5/2}$	7.52	1	1/2	$2\,p_{1/2}$
7.068	4	9/2	$1\,g_{9/2}$	8.13	4	9/2	$1\,g_{9/2}$
7.12	0	1/2	$3\,s$	8.78	2	5/2	$2\,d_{5/2}$
7.34	2	5/2	$2\,d_{5/2}$	8.89	4	7/2	$1\,g_{7/2}$
7.76	2	3/2	$2\,d_{3/2}$	9.08	0	1/2	$3\,s$
7.70	4	9/2	$1\,g_{7/2}$	9.16	2	3/2	$2\,d_{3/2}$
8.33	5	11/2	$1\,h_{11/2}$	9.474	5	11/2	$1\,h_{11/2}$
8.80	3	7/2	$2\,f_{7/2}$	10.278	3	7/2	$2\,f_{7/2}$
8.81	1	3/2	$3\,p_{3/2}$	10.29	5	9/2	$1\,h_{9/2}$
9.02	1	1/2	$3\,p_{1/2}$	10.73	1	3/2	$3\,p_{3/2}$
9.20	5	9/2	$1\,h_{9/2}$	10.74	3	5/2	$2\,f_{5/2}$
9.30	3	5/2	$2\,f_{5/2}$	10.91	1	1/2	$3\,p_{1/2}$

ment with the experimental interaction cross sections. It follows from the polarization data, which are very sensitive to k_0R, that the above-mentioned relations lead to the correct sign and magnitude of the polarization for Zr, Nb, and Mo. The data on the interaction cross sections for bismuth and lead show that, with $R = \eta_0 A^{1/3}$, V_0 for $A = 200$ must be 2.5 to 3 Mev less than for $A = 90$.

We chose $V_{np} = 3V_{nn}$. We then obtain $V^0_{np} = 78$ Mev and $V^0_{nn} = 26$ Mev. The potential V_0 is equal to 46.6 Mev for Pb^{208} and to 52 Mev for Ca^{40} and O^{16}. On the basis of these assumptions, which lead to good agreement with the experimental cross sections, we found the energies of the single-particle levels.

The order in which the levels are filled is given in Table I for $E_b = 0$ and $E_b = 0.16\,V_0$ (E_b is the binding energy). This table was used to find the ground states of a number of nuclei with a closed neutron shell plus one extra particle. The results are listed in Table II.

It is seen from Table II that the computed spin and parity* of the states are in agreement with experiment in all reliable cases. With regard to the binding energy there is a discrepancy of order 2 Mev for the shells $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, and 2s; but even in these cases the isotopic effect (the difference in the binding energies) comes out correctly.

The computed binding energy for the heavier nuclei agrees with experiment within the limits of 0.5 Mev (and much better in many cases). This shows that the potential as a whole as well as the isotopic effect have been treated correctly.

It was not possible to regard the state with 64 neutrons as a closed shell in view of the closeness of the neighboring states.

2. THE LIMITS OF STABILITY OF NUCLEI

We only considered neutral single-particle states, and left the calculation of the proton single-particle states for the future. Hence we can only study the neutron stability of the nuclei.

Experimental data on nuclei with neutron binding energies close to zero are available only for the beginning of the periodic system. This applies, in particular, to He^6 and Li^9. We see that $(N/A)_{max} = 2/3$ in the beginning of the periodic system (for H^3, He^6, and Li^9). The highest known value of N/A in the region of large A is N/A = 0.62 to 0.63 (0.618 for U^{240}).

We now ask, how does $(N/A)_{max}$ depend on A? We can attempt the answer on the basis of the data on the nuclear shells.

We have made the following assumptions: (1) the potential (1) holds for all nuclei, and (2) the diffuseness of the boundary and the spin-orbit interaction do not change with N/Z.

With these assumptions one can determine the number of protons for which the binding energy of the (N+1) th neutron is equal to zero, where N is the number of neutrons in the closed shell. One

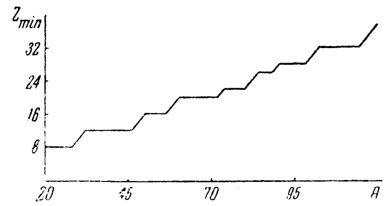
*The parity of the single-particle states is determined by the orbital angular momentum.

TABLE II.* Ground states of nuclei with a closed neutron shell plus 1 neutron

Nucleus	N − 1	Experimental state	Theoretical state	Experimental binding energy	Theoretical binding energy
O^{17}	8	$1d_{5/2}$	$1d_{5/2}$	4.14	5.4
Mg^{27}	14	$2s$	$2s$	6.43	8.7
Si^{29}	14	$2s$	$2s$	8.46	10.8
Si^{31}	16	$1d_{3/2}$	$1d_{3/2}$	1.6	8.6
S^{33}	16	$1d_{3/2}$	$1d_{4/2}$	8.64	10.6
Ca^{41}	20	$1f_{7/2}$	$1f_{7/2}$	8.37	8.53
S^{37}	20	$1f_{7/2}$	$1f_{7/2}$	5.01	4.90
A^{39}	20	$1f_{7/2}$	$1f_{7/2}$	6.64	6.84
Ca^{49}	28	$2p_{3/2}$	$2p_{3/2}$	5.15	5.55
Ti^{51}	28	$2p_{3/2}$	$2p_{3/2}$	6.43	6.76
Cr^{53}	28	$2p_{3/2}$	$2p_{3/2}$	7.93	8.04
Fe^{55}	28	$2p_{3/2}$	$2p_{3/2}$	9.29	9.41
Ni^{57}	28	$2p_{3/2}$	$2p_{3/2}$		
Zn^{71}	40	$1g_{9/2}$	$1g_{9/2}$	6.12	6.04
Ge^{73}	40	$1g_{9/2}$	$1g_{9/2}$	6.64	7.05
Se^{75}	40	$5/2^+$	$1g_{9/2}$	7.96	8.14
Kr^{87}	50	$2d_{5/2}$	$2d_{5/2}$	5.51	5.43
Sr^{89}	50	$2d_{5/2}$	$2d_{5/2}$	6.52	6.24
Zr^{91}	50	$2d_{5/2}$	$2d_{5/2}$	7.16	7.02
Mo^{93}	50	$2d_{5/2}$	$2d_{5/2}$	7.88	7.85
Sn^{115}	64	$3s$	$3s$	7.91	9.05
Xe^{137}	82	$2f_{7/2}$	$2f_{7/2}$	4.45	4.32
Ba^{139}	82	$2f_{7/2}$	$2f_{7/2}$	4.66	4.86
Ce^{141}	82	$2f_{7/2}$	$2f_{7/2}$	5.40	5.36
Nd^{143}	82	$2f_{7/2}$	$2f_{7/2}$	6.02	5.88
Pb^{209}	126	$2g_{9/2}$	$2g_{9/2}$	3.87	4.45
Po^{211}	126	$2g_{9/2}$	$2g_{9/2}$	4.55	4.80

*The data on the binding energy for the nuclei Xe^{137} to Nd^{143} are taken from Johnson and Nier;[5] the other data are from Wapstra.[4]

only has to find the nucleus for which k_0R at a given N is smaller than the value given in column 1 of Table I. We carried through this calculation using the empirical values of $V_{np} = 3V_{nn}$. The results obtained are shown in the figure (only the even Z are given).



It is seen that Z_{min} changes by from 26 to 37.5% as a function of A. These data may, of course, be incorrect in the region of light nuclei, as here there is no good agreement with the experimental values for the binding energies. But in the region of intermediate and heavy nuclei, where one has to extrapolate to 38 to 30% of the protons, our results should come rather close to the truth.

Here we assume, however, that the instability is due to transitions to the next shell, and that inside a given shell the binding energy of the third, fifth, . . . nucleon is positive if that of the first nucleon is positive.

In Table III we give the values A for which Z_{min} changes.

TABLE III

Z	A_{max}	N_{max}	Z	A_{max}	N_{max}
8	28	20	14	48	34
10	30	20	16	56	40
12	46	34	18	58	40
20	72	52	52	178	126
22	80	58	54	180	126
24	82	58	56	182	126
26	88	62	58	184	126
28	98	70	60	198	138
30	100	70	62	200	138
32	114	82	64	220	156
34	116	82	66	222	156
36	118	82	68	252	184
38	132	94	70	254	184
40	136	96	72	256	184
42	138	96	74	258	184
44	156	112	76	260	184
46	158	112	78	262	184
48	174	126	80	264	184
50	176	126	82	266	184
			84	282	198

We see from Table III that $Z/A_{max} \approx 0.3$, varying between 0.261 and 0.33. Of special interest is the behavior of Z_{min} as a function of N. For N = 82, Z_{min} increases from 32 to 38, for

TABLE IV. Neutron binding energies for the isotopes of Ca with odd A in filling one shell

A	Shell	E_n	A	Shell	E_n
41	$(1f_{7/2})_1$	8.37	61	$(1f_{5/2})_2(2p_{3/2})_3$	7.77
43	$(1f_{7/2})_3$	7.94	63	$(1f_{5/2})_4(2p_{3/2})_3$	6.84
45	$(1f_{7/2})_5$	7.41	65	$(1f_{5/2})_5$	6.13
63	$(1f_{5/2})_2(2p_{3/2})_3$	9,25	91	$(2d_{5/2})_1$	7.16
65	$(1f_{5/2})_4(2p_{3/2})_3$	7.98	93	$(2d_{5/2})_3$	6.610
67	$(1f_{5/2})_5$	7.03	95	$(2d_{5/2})_5$	6.42

N = 126, it changes from 48 to 60, and for N = 184, from 68 to 82.

One may ask, how sensitive are these results to the initial assumptions?

In the region of light nuclei the potential may be different from (1), and the assumption of j-j coupling may break down. Therefore our picture is certainly not correct for nuclei with $A < 30$.

In the region of heavier nuclei one can vary the potential parameters somewhat without changing the results of Table III appreciably. For example, changing the width of the surface region by 25% has only a slight effect on the results for Z/A_{max}. The dependence on the isotopic effect is more significant. However, big changes in the value of V_{np}/V_{nn} destroy the agreement with the experimental binding energies, while changing V_{np}/V_{nn} by from 10 to 15% does not make any appreciable difference. The width of the surface region for nuclei with large N/Z can be somewhat greater than for β-stable nuclei, since the binding energy of the neutrons is small and the wave function drops slowly outside the nucleus. It is difficult to estimate this effect, but one should expect that it slightly lowers A_{max} for a given Z. Finally, the assumption that the binding energy of all the neutrons in the shell is positive if that of the first neutron is positive, presumably is correct in the majority of cases. Indeed, the binding energy within the shell changes slowly and without jumps, as can be seen from the example of the isotopes of Ca (Table IV) and of certain other elements. The scheme given in Table III can therefore be regarded as a preliminary estimate of the limits of stability of nuclei with the maximum number of neutrons.

It should be noted that estimates of this kind cannot possibly be made on the basis of the Weizsaecker formula. For the semi-empirical formula includes only the first term of the expansion in terms of (N-Z). But together with the quadratic term, all the even power terms in the expansion should be present and should make a contribution as one gets away from the region of β stable nuclei. The binding energy of Ca^{48} computed by the semi-empirical formula is wrong by 6 Mev (if the errors for Ca^{40}, Ca^{42}, and Ti^{48} are discounted). The inaccuracy is the same for Sn^{124} and Xe^{136}. This shows that even close to the limits of stability the Weizsaecker formula is completely inadequate. It always underestimates the binding energy. Furthermore, since the shell structure is of decisive significance in the study of the stability, it is altogether impossible to use statistical formulas.

3. A FEW REMARKS ON THE ORIGINS OF THE ELEMENTS

The capture of neutrons plays an important role in the existing theories of the creation of the elements. The theory of Alpher and Herman[7] explains the creation of all nuclei by successive neutron capture.

We assume today that the neutron capture plays a role in the synthesis of the nuclei heavier than oxygen or neon.[8] It seems plausible that the mechanism of neutron capture may not have anything to do with the initial stage of the evolution of the universe. For example, if the creation of the elements is continuous, the most suitable "cauldrons" are the red giants and the supernovae. There may exist shortlived energetic neutron currents in the supernovae. If these currents act at the moment of the explosion of the star and last for about a second or a fraction of a second, the formation of the nuclei should go through a minimal number of β decays, i.e., through a minimal Z for a given A or through a Z which is close to the minimal value.

It should be noted that the synthesis of uranium and thorium from lighter nuclei in quantities comparable to those of the other elements is possible only in shortlived neutron bursts. The most probable synthesis is via nuclei with a Z which is lower than in the region of β stability. In this case the synthesis should take no longer than the periods of the β decay of the nuclei Bi^{215}, Bi^{216}, and Bi^{217}, which are estimated to be of order 1 min.

We also call attention to reference 9, where it is assumed that the luminosity curve of the supernovae of the first type* can be interpreted with the help of the decay of Cf^{254}.

Then those nuclei which have an atomic weight A for which the minimal Z increases abruptly, will have a particularly strong tendency to accumulate. Indeed, for these values of A, β decay must occur before the capture of the next neutron is possible. These nuclei will thus accumulate in much greater than average quantities. As a rule, the nucleus remaining after β decay can capture another neutron; however, during the time of the β decay the neutron current may have gone down appreciably, which means that there will be an accumulation of nuclei with the given A. After the capture of one neutron we again obtain a closed shell, further neutron capture may be impossible, and at the mass number $A+1$, β decay occurs again and leads to the accumulation of nuclei with the mass number $A+1$, etc. The mass distribution curve of the nuclei will then have peaks followed by deep dips. In general, this is what actually happens. The position of the peaks will now be determined by the closed neutron shells. Here we must not take the closed neutron shells for β-stable nuclei, but those for nuclei with the minimal Z.

The list of mass numbers in Table III should then give an approximate idea of the position of the peaks in the distribution of the elements. The peaks should appear at the beginning of the sharp increases, i.e., for A = 28, 46, 56, 72, 80, 98, and 114. In the region of higher A one should, obviously, consider only N = 126 and N = 184, since in the intervals N = 82 to 126 and N = 126 to 184 the nuclei are nonspherical, and have no separate closed shells. The mass numbers A = 174 and 252 therefore play a special role in our model. However, this picture is only very approximate, and it is entirely possible that the value of A deviates by several units. Of special interest among the values of A considered are A = 28 and especially A = 56. The first of these values corresponds to the basic isotope of Si, which is one of the most abundant elements in the world.

Even more significant is the fact that our model predicts the great abundance of Fe^{56}. It is well known that this nucleus has the highest abundance of all nuclei with $A > 28$ in nature. All previous hypotheses were not able to explain the great abundance of this nucleus. It is true that the maximum at A = 46 is unwarranted, but a slight change in the parameters may change the position of this peak to A = 40, in accordance with observation.

From the nuclear point of view, these considerations presuppose the fulfilment of the following conditions: (1) the basic maxima are unaffected by changes in the parameters, (2) the chain of captures does not break up until the shell is filled, and (3) arguments can be found to explain away the spurious maxima.

From the cosmological point of view, everything depends on the intensity and the duration of the neutron bursts. There is no upper limit to the intensity of the burst if it is impossible to add not only one, but even two neutrons to the closed shell. If the pairing energy is such that capture of a neutron pair is possible, and if the probability for three-body collisions is high, the breaks in the chain of captures will not be important any more. For example, assume that two neutrons can be added to a nucleus with Z = 16, A = 56; this makes the nucleus Z = 16, A = 58 possible; after β decay we obtain Z = 17, A = 58, which is unstable and emits a neutron, giving Z = 17, A = 57. This nucleus then undergoes β decay. The privileged role of A = 56 disappears. The currents, therefore, must be sufficiently weak to make three-body collisions improbable. The lowest estimate for the currents is determined from the relative abundance of burnt out and intact nuclei for different A.

The duration of the neutron burst is determined by the period of the β decay of the nuclei with minimal Z. The latter can be estimated roughly. It is natural to assume that the transitions in nuclei with minimal Z are allowed, because there are sufficiently many possibilities for β decay to excited levels. The lowest estimate for ft in the region of intermediate and heavy nuclei is approximately $ft = 10^4$. However, β decay can occur not only in the ground state, but also in excited states. In actual fact the period may therefore be several times smaller than that determined from the ft value. The energies of the decays should be of order 10 to 15 Mev. As is well-known, f is proportional to E_0^5, so that an error of 20% in E_0 does not change the order of magnitude of the decay period. The half-life will be within the limits 10^{-3} sec $< T < 10^{-1}$ sec.

In many cases the decay goes mainly into excited states, so that the emission of a neutron may follow the β decay. As a rule, the excitation energies are not high and will be essentially removed by the emission of the first neutron. This implies a spreading of the peak, which will then include the

*The supernovae of the first type have exponentially decaying luminosity curves with a half-life of 55 days. Explosions of this kind are observed in our galaxy about once in 300 years.

mass numbers preceding the break in the basic chain.

The author plans a more detailed study of the separate neutron chains for the near future.

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