INTERACTION BETWEEN CONDUCTION ELECTRONS IN FERROMAGNETS

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It is shown that in ferromagnets there is an additional attraction between conduction electrons, because of spin-wave exchange.

1. If the interaction of electrons in a metal leads to an effective mutual attraction between them in the vicinity of the Fermi surface, then a pair of electrons can have, as is known,[1] a bound state with a negative energy of binding. This phenomenon forms the basis of a recently developed theory of superconductivity.[2,3]

In ordinary metals, the effective mutual attraction between electrons originates from virtual exchange of phonons. In the interaction energy, the matrix element dependent on this exchange approaches a finite limit as the phonon momentum approaches zero. This behavior of the matrix element is connected with the fact that it contains in its denominator the energy of a phonon, whereas the matrix elements of phonon absorption and emission are proportional to the square root of the phonon energy.

We should like to call attention to the fact that in ferromagnets there occurs an additional mutual attraction between conduction electrons, connected with the virtual emission and absorption of spin waves. Since the energy of a spin wave is proportional to the square of its momentum, and since the matrix elements of spin-wave emission and absorption contain no additional factors proportional to the square root of the spin-wave energy, therefore the matrix element of the effective energy of interaction between electrons resulting from spin-wave exchange is inversely proportional to the square of the spin-wave momentum, i.e., to the square of the momentum transferred.

The momentum transferred must, however, exceed a certain minimum value, since the energy of a conduction electron in a ferromagnet depends on the orientation of its spin,[4] and therefore to the two spin orientations there correspond Fermi spheres of different radii. This circumstance leads to a diminution of the effective interaction between electrons.

In this note we determine the nature of the effective interaction between electrons caused by spin-wave exchange.

2. We use a simplified model of a ferromagnet; we start from the concept of two groups of electrons — the conduction electrons (s electrons) and the ferromagnetic electrons (d electrons).

The operator $V(\mathbf{r})$ of interaction energy between s and d electrons must obviously contain linearly both the spin $\mathbf{s}$ of the s electron and the magnetic moment $\mathbf{M}(\mathbf{r})$ of unit volume of the ferromagnet, due to the d electrons. Therefore $V(\mathbf{r})$ may be represented in the form

$$V(\mathbf{r}) = \hat{C}\mathbf{s}\cdot\mathbf{M}(\mathbf{r}),$$

where $\hat{C}$ is a certain linear integral operator. Since the exchange forces fall off rapidly with distance, the size of $\hat{C}$ may be considered approximately constant. We write it in the form $C = \Delta a^3/\mu_0$, where μ_0 is the Bohr magneton, a is the lattice constant, and Δ is a certain energy connected with the Curie temperature Θ by the relation $\Delta \sim \sqrt{\Theta A}$. Here A has the order of magnitude of the atomic energy. Such an estimate of the magnitude of Δ corresponds to the idea that the exchange energy between d-electrons is caused by s-electrons and leads, appparently, to an increased value of Δ (cf. reference 5).

Thus the energy of interaction between s and d electrons has the form[5]

$$V(\mathbf{r}) = (\Delta a^3/\mu_0)\,\mathbf{s}\cdot\mathbf{M}(\mathbf{r}). \quad (1)$$

From this formula it follows that the energy of a conduction electron in a ferromagnet has the form

$$\varepsilon(\mathbf{p}, \sigma) = \varepsilon^0(\mathbf{p}) + 2\sigma\Delta,$$

where $\mathbf{p}$ is the momentum of the electron, $\sigma = \pm\frac{1}{2}$ is the projection of the electron spin on the z axis, and $\epsilon^0(\mathbf{p})$ is a quantity independent of σ; in the further calculations we consider for simplicity that $\epsilon^0(\mathbf{p}) = p^2/2m$.

The emission and absorption of spin waves is connected with the projection of the vector $\mathbf{M}(\mathbf{r})$ perpendicular to the axis of easiest magnetization (the z axis). This projection is determined by the formulas[6]

$$M_x = \sqrt{\mu_0 M_0 / 2\Omega} \sum_{\mathbf{k}} (a_{\mathbf{k}} + a^{*}_{-\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}};$$

$$M_y = i \sqrt{\mu_0 M_0 / 2\Omega} \sum_{\mathbf{k}} (a^{*}_{-\mathbf{k}} - a_{\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}}, \qquad (2)$$

where M_0 is the saturation magnetic moment, Ω is a normalizing volume, and a_k and a_k^* are the creation and annihilation operators for spin waves of momentum $\mathbf{k}$; they satisfy the relations

$$[a_{\mathbf{k}}, a^{*}_{\mathbf{k}'}] = \delta_{\mathbf{k}\mathbf{k}'}.$$

We now determine the matrix element of the energy of effective interaction between two s electrons, connected with spin-wave exchange. We denote the initial momenta and spins of the electrons by $\mathbf{p}_1$, $\mathbf{p}_2$, σ_1, σ_2 and the final by $\mathbf{p}_1'$, $\mathbf{p}_2'$. σ_1', σ_2'. The interaction of interest to us, like the known Møller interaction, is an outcome of the second approximation of perturbation theory. Since the matrix elements of the operators $s_x \pm is_y$ differ from zero only if $\sigma + \sigma' = 0$, the electrons interact only in the case in which their spins have opposite directions.

On taking account of the two types of intermediate state, corresponding to emission of a spin wave by the first and by the second electron, we get the following expression for the desired matrix element:

$$U_{if} = \frac{a^3}{\Omega} \Delta^2 \left\{ \frac{\delta_{\sigma_1, 1/2} \delta_{\sigma_2, -1/2}}{\varepsilon(\mathbf{p}_1, \sigma_1) - \varepsilon(\mathbf{p}_1 - \mathbf{k}, \sigma_1') - \theta(ak)^2} + \frac{\delta_{\sigma_2, 1/2} \delta_{\sigma_1, -1/2}}{\varepsilon(\mathbf{p}_2, \sigma_2) - \varepsilon(\mathbf{p}_2 + \mathbf{k}, \sigma_2') - \theta(ak)^2} \right\}, \qquad (3)$$

where $\theta(ak)^2$ is the energy of the spin wave.

Attraction between the electrons occurs in a triplet state with resultant projection of the spin equal to zero. From this it follows that the spatial part of the wave function of the electrons is antisymmetric. (In the simplest case it represents a p state.)

We introduce the radii p_+ and p_- of the Fermi spheres of electrons with different spin orientations. On assuming that $\epsilon^0(p) = p^2/2m$, we get

$$p_+ = \sqrt{2m(\mu - \Delta)}, \quad p_- = \sqrt{2m(\mu + \Delta)},$$

where μ is the chemical potential.

It is clear that if $p_1 \sim p_+$, then $p_1' \sim p_-$. From this it is easily deduced that the minimum value of the spin-wave momentum will be

$$k_{\min} = p_0 (\sqrt{1 + \Delta/\mu} - \sqrt{1 - \Delta/\mu}),$$

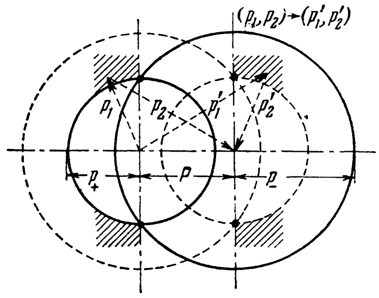
where $p_0 = \sqrt{2m\mu}$. If we consider that $\Delta \ll \mu$, we get from this $k_{\min} \sim p_0 \Delta/\mu$. Therefore the maximum value of a matrix element is equal to

$$(U_{if})_{\max} \equiv - a^3 U / \Omega, \quad U = \mu^2 / \Theta. \qquad (4)$$

To this value of the interaction energy corresponds, obviously, a resultant momentum of the electron pair equal to

$$P_0 = \sqrt{p_-^2 - p_+^2} = p_0 \sqrt{2\Delta/\mu}$$

(cf. the figure, in which the Fermi spheres of radii p_+ and p_- are shown displaced by $\mathbf{P}$. The greatest interaction will occur in the case when the circle of intersection is a great circle of the smaller sphere. The shaded areas in the figure are those in which the electron momenta are distributed before and after scattering.)



Comparison of the matrix element (4) with the matrix elements of interaction due to phonon exchange shows that the latter is smaller than the matrix element (4) by a factor of order of magnitude μ/Θ. The stronger interaction due to spin-wave exchange is compensated, however, by a diminution of statistical weight, connected with the fact that the resultant momentum of the interacting electrons is different from zero. Furthermore, the antisymmetry of the wave function contributes to a decrease of the effective attraction of the electrons. Consequently the problem of bound states of electrons resulting from spin-wave exchange requires a special study. As a general principle, however, the presence of an additional attraction between conduction electrons in ferromagnets can under suitable conditions contribute to the appearance of superconductivity. It is possible that such conditions may be realized in thin films, in which the critical magnetic field is increased.[7]

Note in proof (Feb. 12, 1959). In a recent paper [Matthias, Suhl, and Corenzwit, Phys. Rev. Let. **1**, 449 (1958)], the presence of superconductivity in the ferromagnetic alloys (Ce, Pr)Ru_2 and (Ce, Gd)Ru_2 has been established experimentally.

[1] L. Cooper, Phys. Rev. **104**, 1189 (1956).

[2] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[3] N. N. Bogolyubov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 58 and 73 (1958), Soviet Phys. JETP **7**, 41 and 51 (1958).

[4] A. A. Abrikosov and I. E. Dzyaloshinskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 771 (1958), Soviet Phys. JETP **8**, 535 (1959).

[5] C. Kittel and A. Mitchell, Phys. Rev. **101**, 1611 (1956).

[6] C. Kittel and E. Abrahams, Revs. Modern Phys. **25**, 233 (1953).

[7] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 202 (1956), Soviet Phys. JETP **4**, 153 (1957).

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