FISSION OF NONSPHERICAL NUCLEI

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The direct fission of a nonspherical nucleus, which occurs when the absorbed particle transfers a large amount of angular momentum to the nucleus, is considered.

1. It is well known that the fission of stable nuclei under the action of fast particles is possible if the excitation energy of the compound nucleus formed in the process turns out to be larger than the critical energy for fission. The critical energy for fission can be found from the conditions of unstable equilibrium of a nucleus under the action of forces of surface tension and forces of Coulomb repulsion.[1,2] However, in the case of nonspherical nuclei which have a rotational degree of freedom such a discussion turns out to be incomplete.

Indeed, since the moment of inertia of a nonspherical nucleus differs from zero a collective rotation of the nucleus is possible.[3,4] This rotation leads to the appearance of centrifugal forces which in the case of sufficiently large angular momentum may lead to nuclear fission. The condition for fission of a rotating nonspherical nucleus may be obtained by investigating its stability under rotation with respect to small deformations. The angular momentum of rotation which may lead to fission can be communicated to the nucleus by some fast particle absorbed by it if the impact parameter of this particle with respect to the center of the nucleus exceeds a certain critical value. Indeed, as a result of the law of conservation of angular momentum the mechanical angular momentum of the absorbed particle is transferred to the nucleus, and such a transfer will be accompanied by fission as a result of the instability of the rotating nucleus with respect to small deformations.

The fission of nonspherical nuclei arising as a result of the absorption of particles with large amounts of mechanical angular momentum will evidently occur without the intermediate stage of a compound nucleus. Therefore the indicated fission mechanism for nonspherical nuclei may be regarded as a mechanism of direct fission.*

*Pik-Pichak[5] has investigated fission with formation of a rotating compound nucleus.

The angular distribution of fission products in the case of such a direct mechanism will have properties characteristic of distributions pertaining to direct processes.

2. First let us obtain the condition for the fission of a rotating nonspherical nucleus. For the sake of simplicity we shall assume that the nonspherical nucleus has the shape of an elongated ellipsoid of revolution. We denote the length of the semi-axes of the ellipsoid by a (axis of symmetry) and b ($a \geq b$). We denote the eccentricity of the ellipsoid by ϵ:

$$\varepsilon^2 = 1 - b^2 / a^2.$$

The total energy of the rotating nonspherical nucleus $\mathscr{E}$ may be written in the form of a sum of the surface energy $\mathscr{E}_s$, the Coulomb energy $\mathscr{E}_q$ and the rotational energy $\mathscr{E}_{rot}$.

The surface energy of the nucleus is equal to

$$\mathscr{E}_s = 2\pi O ab \left\{\sqrt{1-\varepsilon^2} + \frac{1}{\varepsilon} \arcsin \varepsilon\right\}, \qquad (1)$$

where O is the phenomenological surface tension coefficient.

On the assumption that the electric charge is distributed in the nucleus with the constant density $\rho = Ze/V$ the Coulomb energy of the nucleus may be written in the form

$$\mathscr{E}_q = \frac{1}{2} \rho \int \varphi(\mathbf{r})\, dV,$$

where $\varphi(\mathbf{r})$ is the electrostatic potential inside the nucleus and the integration is taken over the volume of the nucleus. By utilizing for the electrostatic potential of a uniformly charged ellipsoid the expression[6]

$$\varphi(\mathbf{r}) = \pi\rho ab^2 \int_0^\infty \left\{1 - \frac{x^2}{a^2+s} - \frac{y^2+z^2}{b^2+s}\right\} \frac{ds}{\sqrt{a^2+s}\,(b^2+s)},$$

we will obtain the Coulomb energy of the nucleus in the form

$$\mathscr{E}_q = \frac{3}{10} \frac{(Ze)^2}{a\varepsilon} \ln \frac{1+\varepsilon}{1-\varepsilon}. \qquad (2)$$

The rotational energy of the nucleus is equal to

$$\mathscr{E}_{rot} = L^2/2I, \qquad (3)$$

where L is the angular momentum of rotation of the nucleus, I is the moment of inertia of the nucleus which depends on the eccentricity ϵ. For a spherical nucleus the moment of inertia I must reduce to zero. Therefore the nucleus cannot be regarded as a solid. In order to determine the moment of inertia of the nucleus we can make use of the model of potential flow of an ideal fluid inside a rotating nonspherical shell.[4,7] In this case the moment of inertia will be determined by the following expression

$$I = \frac{Ma^2}{5}\cdot\frac{\varepsilon^4}{2-\varepsilon^2}\gamma, \qquad (4)$$

where M is the mass of the nucleus and γ is a numerical coefficient ($\gamma \sim 4$), which must be introduced in order that for a given nuclear deformation ϵ a value for the moment of inertia will be obtained which corresponds to experimental data.

Let us consider infinitesimal deformations of the nucleus of the form $a \to a + \delta a$, where $\delta a > 0$. By assuming that nuclear matter is incompressible we obtain from the condition of constancy of nuclear volume during deformations ($V = \text{const}$)

$$\delta b = -\frac{b}{2a}\delta a, \quad \delta\varepsilon = \frac{3}{2}\,\frac{1-\varepsilon^2}{a\varepsilon}\delta a.$$

The total change in energy in the case of such a nuclear deformation is equal to

$$\delta\mathscr{E} = \delta\mathscr{E}_s + \delta\mathscr{E}_q + \delta\mathscr{E}_{rot}, \qquad (5)$$

where $\delta\mathscr{E}_s$ and $\delta\mathscr{E}_q$ are the changes in the surface and in the Coulomb nuclear energies and $\delta\mathscr{E}_{rot}$ is the change in rotational energy due to a change in the moment of inertia of the nucleus as a result of deformation at a given angular momentum.

$$\delta\mathscr{E}_s = \pi O b\left\{\sqrt{1-\varepsilon^2} + \frac{1}{\varepsilon}\arcsin\varepsilon + 3\,\frac{1-\varepsilon^2}{\varepsilon^2}\left[\sqrt{1-\varepsilon^2} - \frac{1}{\varepsilon}\arcsin\varepsilon\right]\right\}\delta a,$$

$$\delta\mathscr{E}_q = \frac{9}{10}\,\frac{(Ze)^2}{a^2\varepsilon^2}\left\{1 - \frac{3-\varepsilon^2}{6\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}\right\}\delta a,$$

$$\delta\mathscr{E}_{rot} = -\frac{L^2}{Ia}\left\{1 + 3\,\frac{1-\varepsilon^2}{\varepsilon^2} + \frac{3}{2}\,\frac{1-\varepsilon^2}{2-\varepsilon^2}\right\}\delta a. \qquad (6)$$

Evidently the nucleus will be stable with respect to deformations of the form indicated above provided $\delta\mathscr{E} > 0$. However, if $\delta\mathscr{E} \le 0$ then the nucleus will be absolutely unstable, i.e., an arbitrarily small deformation will lead to nuclear fission. On substituting into the inequality $\delta\mathscr{E} \le 0$ the expressions (6), and on solving this inequality with respect to the square of the angular momentum L^2, the condition for the fission of a rotating nucleus may be written in the form

$$L^2 \geqslant L_m^2, \qquad (7)$$

where the critical value of the square of the angular momentum is determined by the following expression

$$L_m^2 = Ia\Big\{\pi O b\Big[\sqrt{1-\varepsilon^2} + \frac{1}{\varepsilon}\arcsin\varepsilon + 3\,\frac{1-\varepsilon^2}{\varepsilon^2}\Big(\sqrt{1-\varepsilon^2} - \frac{1}{\varepsilon}\arcsin\varepsilon\Big)\Big] + \frac{9}{10}\,\frac{(Ze)^2}{a^2\varepsilon^2}\Big[1 - \frac{3-\varepsilon^2}{6\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}\Big]\Big\}\{1 + 3(1-\varepsilon^2)/\varepsilon^2 + 3(1-\varepsilon^2)/2(2+\varepsilon^2)\}^{-1}. \qquad (8)$$

From condition (7) it follows that a nonspherical nucleus cannot have an angular momentum greater than the critical value L_m. If a nonspherical nucleus is given an angular momentum exceeding the critical value L_m, then this will immediately lead to the fission of the nucleus.

3. A nonspherical nucleus can acquire an amount of angular momentum which leads to fission by absorbing a fast nucleon with sufficiently large impact parameter with respect to the center of the nucleus. Evidently the maximum value of the impact parameter of the absorbed nucleus is equal to a; in this case the angular momentum communicated to the nucleus is equal to $L = a\sqrt{2mE}$, where m is the mass and E is the kinetic energy of the nucleon. By utilizing (8) one can find the critical value for the energy of the incident nucleon

$$E_m = \frac{1}{8}\,\frac{I}{mr_0^2}\sqrt{1-\varepsilon^2}\,U_s\Big\{\sqrt{1-\varepsilon^2} + \frac{1}{\varepsilon}\arcsin\varepsilon + 3\,\frac{1-\varepsilon^2}{\varepsilon^2}\times\Big(\sqrt{1-\varepsilon^2} - \frac{1}{\varepsilon}\arcsin\varepsilon\Big) + 12\,\frac{\sqrt{1-\varepsilon^2}}{\varepsilon^2}\Big(1 - \frac{3-\varepsilon^2}{6\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}\Big)x\Big\} \times \{1 + 3(1-\varepsilon^2)/\varepsilon^2 + 3(1-\varepsilon^2)/2(2-\varepsilon^2)\}^{-1}. \qquad (9)$$

Here

$$ab^2 = Ar_0^3,\quad U_s = 4\pi r_0^2 O,\quad (Z^2/A)_0 = (40\pi/3)(r_0^3 O/e^2),$$

$$x = (Z^2/A)/(Z^2/A)_0,$$

A is the mass number.

If the energy of the incident nucleon exceeds the critical value $E \ge E_m$, then the absorption of the nucleon may be accompanied by direct fission of the nonspherical nucleus. However, if $E < E_m$, then fission is possible only with the formation of a compound nucleus.

By setting $r_0 = 1.4 \times 1.0^{-13}$ cm, $U_s = 15$ Mev, $I = 2 \times 10^{-47}$ g-cm^2, $a/b = 1.5$ and $x = 0.7$ (the nonspherical nucleus Hf^{180}) we obtain for the critical energy in the case of neutrons the value E_m = 20 Mev. (The value for E_m obtained above is correct only in order of magnitude, since in the course of finding E_m the expression (4) for the moment of inertia of the nucleus has been utilized. If the nucleus is regarded as a solid then we shall obtain for E_m a value which is several times larger.)

We note that the criterion $E \geq E_m$ for the possibility of fission of a nonspherical nucleus under the action of particles of energy E holds only in the case when $E_m > \hbar^2/I$. Indeed, if $E_m < \hbar^2/I$, then even though the inequality $E > E_m$ is fulfilled, the condition $E > \hbar^2/I$ may not be fulfilled, which means that there is a possibility of the incident nucleon transferring energy to the rotational degree of freedom of the nucleus. Therefore the whole discussion is valid only for sufficiently large values of moments of inertia of nuclei.

4. Let us find the cross section for the direct fission of nonspherical nuclei under the action of fast nucleons. For the sake of simplicity we shall consider the nucleus to be perfectly absorbing.

We choose a system of coordinates in such a manner that the momentum of the incident nucleon **p** will be directed along the Y axis, while the axis of symmetry of the nucleus lies in the XY plane. Then the extent of the shadow of the nucleus on the plane perpendicular to the momentum will be defined by the ellipsoid with the semi-axes $\zeta(\theta) = a\sqrt{1-\epsilon^2\cos^2\theta}$ and b, where θ is the angle between the momentum of the incident nucleon and the axis of symmetry of the nucleus.

The angular momentum of the nucleon with respect to the axis perpendicular to the momentum of the nucleon and to the axis of symmetry will be given in the chosen system of coordinates by $L = xp$. When the nucleon is absorbed by the nucleus this angular momentum will be transferred to the nucleus. If $L \geq L_m$, i.e., $x \geq L_m/p$, then the absorption of the nucleon will be accompanied by direct fission of the nucleus. The cross section for such a process will evidently be equal to the area of the segments cut off from the shadow by the straight line $x = +L_m/p$:

$$\sigma(E,\theta) = 2b\zeta\left(\arccos\frac{x_m}{\zeta} - \frac{x_m}{\zeta}\sqrt{1-\frac{x_m^2}{\zeta^2}}\right),\qquad x_m = \frac{L_m}{p}. \tag{10}$$

The cross section (10) corresponds to a definite orientation of the axis of symmetry of the nonspherical nucleus. In the case of nonoriented nonspherical nuclei the expression (10) should be averaged over all the possible orientations of the nuclear axis of symmetry. As a result of such averaging we obtain

$$\sigma(E) = \frac{2ab}{\varepsilon}\int_0^{\eta}\sqrt{1-y^2}\left\{\arccos\frac{\Lambda}{\sqrt{1-y^2}} - \Lambda\frac{\sqrt{1-y^2-\Lambda^2}}{1-y^2}\right\}dy, \tag{11}$$

where $\Lambda = L_m/ap = \sqrt{E_m/E}$, while the upper limit for the integration η should be taken equal to

$$\eta = \begin{cases}\varepsilon, & \varepsilon < \sqrt{1-\Lambda^2},\\ \sqrt{1-\Lambda^2}, & \varepsilon > \sqrt{1-\Lambda^2}.\end{cases} \tag{11a}$$

Expression (11) may be integrated in two limiting cases. Near the threshold the cross section for direct fission is equal to

$$\sigma(E) = (ab/4\varepsilon)(1-E_m/E)^2,\quad E - E_m \ll E. \tag{12}$$

In the case of high energies of incident nucleons ($E \gg E_m$) the cross section of direct fission coincides with the total cross section for the absorption of nucleons by a nonspherical nucleus

$$\sigma = \frac{\pi}{2}ab\left(\sqrt{1-\varepsilon^2} + \frac{1}{\varepsilon}\arcsin\varepsilon\right),\ E \gg E_m \tag{13}$$

Formula (11) determines the maximum possible value of the cross section. Actually the cross section for direct fission will be less than (11) as a result of competition of other processes both direct ones and also those with the formation of a compound nucleus. Qualitatively this may be taken into account by introducing into formula (11) a factor smaller than unity which describes the probability of direct interaction between the incident particle and the nucleus accompanied by transfer of a large amount of angular momentum to the whole nucleus.

5. The study of direct fission of nonspherical nuclei in the case of deuteron stripping may be of particular interest. As a result of the law of conservation of angular momentum the fission products of a nonspherical nucleus will fly apart in the case of a direct fission process in the plane containing the direction of motion of the absorbed nucleon and the axis of symmetry of the nucleus. The other nucleon from the deuteron liberated as a result of stripping will also come out in the same plane. Thus, the direction of emission of the nucleon liberated during stripping and the direction of separation of the fission products of a nonspherical nucleus will be correlated. The establishing of such a correlation may be utilized for the separation of the direct process from fission processes with the formation of a compound nucleus.

Let us determine the cross section for the stripping reaction accompanied by direct fission of a nonspherical nucleus on the assumption that the deuteron radius is considerably smaller than the nuclear dimensions ($R_d \ll a$ and b). Under this assumption the total cross section for the stripping reaction is equal to the perimeter of the area of the nuclear shadow multiplied by one-quarter of the deuteron radius R_d. The cross section for the stripping reaction accompanied by direct fission will be proportional to the length of the arcs cut off by the straight lines $x = + L_m/p$ from the ellipsoid which forms the boundary of the region of the nuclear shadow on the plane perpendicular to the momentum of the incident deuteron. By noting that the length of these arcs is equal to

$$l = 4\zeta \left\{ E\left(\frac{\pi}{2}, \sqrt{1-\frac{b^2}{\zeta^2}}\right) - E\left(\tan^{-1}\frac{x_m}{b\sqrt{1-x_m^2/\zeta^2}}, \sqrt{1-\frac{b^2}{\zeta^2}}\right)\right\}$$

(where $E(\varphi, k)$ is the elliptic integral of the second kind), and on averaging over different orientations of the nuclear axis of symmetry we shall obtain the following expression for the cross section for the stripping reaction accompanied by direct fission of a nonspherical nucleus

$$\sigma(E) = \frac{aR_d}{\varepsilon}\int_0^{\eta}\sqrt{1-y^2}\left\{E\left(\frac{\pi}{2}, \sqrt{1-\frac{1-\varepsilon^2}{1-y^2}}\right) - E\left(\tan^{-1}\frac{\Lambda}{[(1-\varepsilon^2)(1-y^2-\Lambda^2)/(1-y^2)]^{1/2}}, \sqrt{1-\frac{1-\varepsilon^2}{1-y^2}}\right)\right\}dy, \quad (14)$$

where E is half the energy of the incident deuteron, while η is determined by (11a).

Near the threshold of the stripping reaction accompanied by fission the cross section is equal to

$$\sigma(E) = \frac{\pi}{4}aR_d\frac{1-\varepsilon^2}{\varepsilon}\left(1-\frac{E_m}{E}\right), \quad E - E_m \ll E. \quad (15)$$

In the limiting case of high energies $E \gg E_m$ the cross section of the process under consideration coincides with the total cross section of the stripping reaction in the case of a nonspherical nucleus

$$\sigma = \frac{\pi}{4}aR_d\left\{1 + \frac{1-\varepsilon^2}{2\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}\right\}, \quad E \gg E_m. \quad (16)$$

Both in formulas (11) and (14) one should introduce a factor describing the probability of direct excitation of the fission degree of freedom of the nucleus.

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