

RANGE-ENERGY RELATION FOR 660-Mev PROTONS

V. P. ZRELOV and G. D. STOLETOV

Joint Institute for Nuclear Research

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The mean range of protons in copper was determined. The proton energy was computed from the measured angle of Vavilov-Cerenkov radiation emitted by protons in Plexiglas. The corrected range of (658 ± 2) -Mev protons in copper was found to be 257.6 ± 1.2 g/cm². Assuming the ionization potential to be independent of the velocity, the calculated value of I_{Cu} is 305 ± 10 ev. The stopping power relative to copper was also measured for H, Be, C, Fe, Cd and W.

INTRODUCTION

BETHE and Livingston's expression¹ for the average ionization losses of charged particles contains the average ionization potential I of the substance traversed by the particles. In the theory of ionization losses it is assumed that the average ionization potential I depends only on the atomic properties of the medium and is in principle independent of the incident particle's velocity. However, considerable experimental evidence (Table I)² indicates that the average ionization potential tends to decrease with increasing particle velocity, at least for heavy elements (Z > 13).

Lindhard and Scharff⁷ used the Thomas-Fermi statistical model to show that for different energies

the energy losses of charged particles in substances with large Z is a function of only the parameter x and is given by

$$-\frac{dE}{dx} = \frac{4\pi N (ze^2)^2}{mv^2} L(x), \quad x = \frac{v^2}{Zv_0^2}, \quad v_0 = \frac{e^2}{\hbar}. \quad (1)$$

As in the Bethe-Bloch theory, L(x) is a logarithmic relation for x > 100, in which region the specific form of L(x) proposed by Lindhard and Scharff provided a sufficiently good description of the experimental data available at the time and to some extent confirmed Sachs and Richardson's idea⁸ that I might be dependent on energy.

Caldwell³ subsequently analyzed all available experimental data on the determination of ionization potentials of different elements for different

TABLE I. Summary of ionization potentials in ev

Element	Z	From Caldwell ³ for 18 Mev*	From Bloembergen and van Heerden ⁴ for 60 Mev	From Thomson for 270 Mev	From Mather and Segre ⁶ for 340 Mev	From Bakker and Segre ⁵ for 340 Mev
H	1			18.0	15.6	
Li	3				34.0	
Be	4				60.4	59.0
C	6			69.7	76.4	74.4
N	7			75.9		
O	8			87.6		
Al	13	163±3	162±5		150	147.9±3
Cl	17			151.9		
Fe	26				243	
Ni	28	363±19				
Cu	29	377±8	370		279	309.9±3
Rh	45	656±45				
Ag	47	659±50			428	
Cd	48	654±41				
Sn	50	708±59			479	
Ta	73	962±54				
W	74				697	
Au	79	1136±100				
Pb	82		970		758	810.7±12
U	92				881	

*Based on the experimental work of Sachs and Richardson.⁸

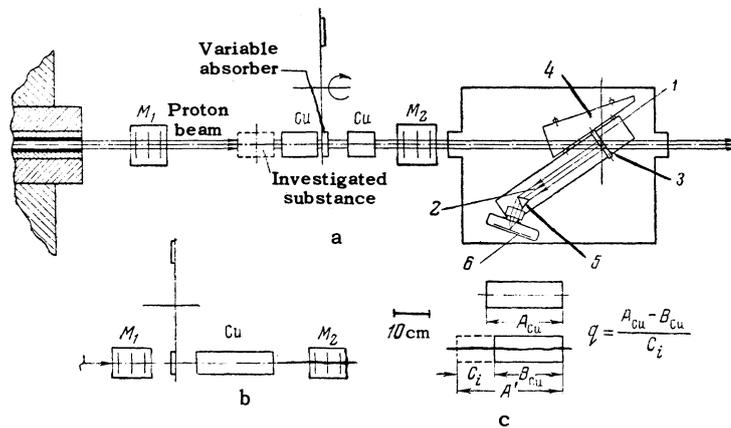


FIG. 1. a - Plan of experiment: 1 - radiator, 2 - radiation, 3 - mirror surface, 4 - angle template, 5 - achromatic prism, 6 - camera; b - arrangement of ionization chambers M_1 and M_2 and copper absorber for total range measurement; c - arrangement for measurements of relative stopping powers.

incident energies, making all essential corrections. The result was strong modification of the experimental curve for $x < 100$. Caldwell concluded that there is no basis at present for assuming that I depends on velocity; he was of the opinion that for high energies we must use higher ionization potentials than those given by Bakker and Segrè⁵ and Mather and Segrè,⁶ which are approximately 13Z ev.

In the present paper we shall present measurements of the range-energy relation for 660-Mev protons in copper.

EXPERIMENT

The mean energy of our external synchrocyclotron beam was 660 Mev, with a spread of ± 4 Mev. The beam was formed by a collimator of 20 mm diameter. The instruments for precise measurement of proton energy and range were placed in line on a special truss behind a four-meter reinforced concrete shield 17 m from the exit window of the accelerator vacuum chamber. The experimental arrangement is shown in Fig. 1.

MEASUREMENT OF MEAN PROTON ENERGY

The mean proton energy was determined by Mather's method which was described in reference 9. In the present paper we shall give the experimental results with only a brief description of the procedure. The method is based on the sharply defined directivity of Vavilov-Cerenkov radiation,¹⁰ with the semiangle of the radiation cone given by $\cos \theta(\lambda) = 1/n(\lambda)\beta$, where $n(\lambda)$ is the refractive index of the medium for a given wavelength λ and $\beta = v/c$ is the particle velocity.

Because of the continuous spectrum of Vavilov-Cerenkov radiation the form of the function $n(\lambda)$ must be taken into account in determining velocities from $\cos \theta(\lambda) = 1/n(\lambda)\beta$. Mather⁹ developed an ingenious scheme for measuring the emission

angle of a given wavelength. In virtue of $n(\lambda)$ the blue portion of the radiation spectrum is located on the outside of the radiation cone while the red portion is located inside. When a portion of the cone of divergent light from the radiator passes through an achromatic prism we obtain an almost parallel light beam which can be focused by a lens into a narrow band of white light with its center of gravity around the position of a given wavelength. The vertex angle α of the achromatic prism is determined from the extinction equation of first order dispersion $d\psi/d\lambda = 0$ written for the case of perpendicular emission from the radiator and minimum prism angle of deviation, where $\psi(\lambda)$ is the direction in which the radiation is viewed after emerging from the prism. We now give this equation, which appears with a misprint in Mather's paper:

$$\frac{d\psi}{d\lambda} = \frac{d\psi}{dn} \frac{dn}{d\lambda}$$

$$= \left\{ \frac{1}{(n^2\beta^2 - 1)^{1/2}} - \frac{2 \sin(\alpha/2)}{(1 - n^2 \sin^2(\alpha/2))^{1/2}} \right\} \frac{dn}{d\lambda} = 0. \quad (2)$$

Hence

$$\sin \frac{\alpha}{2} = \{n_0^2 + 4(n_0^2\beta_0^2 - 1)\}^{-1/2}, \quad (3)$$

where α is the vertex angle of the prism, n_0 is the refractive index for the wavelength λ_0 satisfying the condition $d\psi/d\lambda = 0$, and β_0 is the approximate mean velocity. Equation (3) can be used when the radiator and prism are made of the same material.

Proton velocities were determined by precise measurement of the emission angle of Vavilov-Cerenkov radiation from protons passing through Plexiglas. The choice of radiator material and thickness was dictated by the experimental conditions. Plexiglas was chosen as a substance with low atomic number, low density and relatively low dispersion, thus reducing multiple scattering and slowing down.

Preliminary energy measurements¹¹ were per-

formed with a proton beam of low density (10^6 protons \cdot cm $^{-2}$ sec $^{-1}$) and, consequently, a relatively thick radiator. Radiation was registered by a camera with a 1:1.5 Yupiter-3 objective on "Negative-A" cine film of 50-unit GOST (All-Union State Standard) sensitivity. With a radiator thickness of 2.9 g/cm 2 in the beam direction exposures lasted 3 minutes.

The present measurements were obtained after the system of proton beam extraction was changed, 12 increasing the intensity more than 100 times. The exposures used with a beam of 4×10^7 protons \cdot cm 2 sec $^{-1}$ ranges from 3 to 5 seconds.

MEASUREMENT OF REFRACTIVE INDEX

An IRF-3 refractometer was used to measure the refractive index of a few samples taken from the same piece of material as the radiator. The absolute refractive index for $\lambda = 5461$ A was $1.4926 \pm 5 \times 10^{-4}$. This value of n was confirmed after the experiment by measurements performed on samples cut from the radiator in the region traversed by the proton beam.

EXPERIMENTAL RESULTS

The emission angle of Vavilov-Cerenkov radiation in Plexiglas was found to be $\theta = (34^\circ 0.5') \pm 3'$ for $\lambda = 5461$ A. The proton energy was calculated from

$$E = E_0 \left\{ \frac{n \cos \theta}{(n^2 \cos^2 \theta - 1)^{1/2}} - 1 \right\}, \quad (4)$$

where E_0 is the proton rest mass, 938.2 Mev, 13 and n is the refractive index of Plexiglas for $\lambda = 5461$ A.

According to the measured values of θ and n the proton energy at the center of the radiator was 654.9 Mev. When account is taken of slowing down up to the middle of the radiator the mean proton energy becomes 658.4 Mev ($\Delta E = 3.5$ Mev for $t = 1.45$ g/cm 2 and $-dE/dx = 2.40$ Mev/g \cdot cm $^{-2}$). This was the average of three energy measurements: $E_1 = 658.0$ Mev, $E_2 = 657.6$ Mev, and $E_3 = 659.6$ Mev.

The error in the energy values includes errors in measuring θ and n . $\Delta E_1/\Delta \theta$ and $\Delta E_2/\Delta n$ were determined by means of the relations

$$\frac{\Delta E_1}{\Delta \theta} = E_0 \frac{(n^2 \beta^2 - 1)^{1/2}}{\beta (\beta^2 - 1)^{1/2}}, \quad \frac{\Delta E_2}{\Delta n} = E_0 \frac{\beta^2}{n (1 - \beta^2)^{1/2}}. \quad (5)$$

In our case $\Delta E_1 = \Delta \theta \times 0.59$ Mev when $\Delta \theta$ is given in minutes, and $\Delta E_2 = 2 \times 10^3 \Delta n$ Mev. For $\Delta \theta = \pm 3.0$, $\Delta n = \pm 5 \times 10^{-4}$, $n = 1.493$ and $\beta \approx 0.81$ we obtain $\Delta E_1 = \pm 1.8$ Mev and $\Delta E_2 = \pm 1$ Mev.

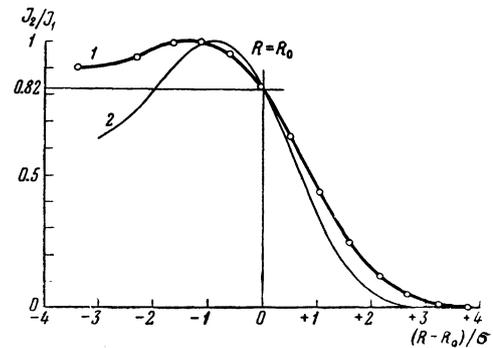


FIG. 2. Ratio of ionization currents in chambers M_1 and M_2 as a function of $(R - R_0)/\sigma$: 1 - experimental curve, 2 - curve computed by Mather and Segrè 6 ($\sigma = 2.68$ g/cm 2).

The total energy error, which is the root mean square of the given errors, is $\Delta E = \pm 2.1$ Mev.

MEASUREMENT OF TOTAL RANGE IN COPPER

While measuring the mean proton energy we also measured the total range in copper (Fig. 1b). A collimated proton beam of 20 mm diameter traversed ionization chambers M_1 and M_2 , which were separated by copper blocks. The ionization ratio J_2/J_1 in the chambers was measured as a function of copper thickness.

Ionization chamber M_1 was filled with helium to 0.5 atmos and M_2 was filled with argon to 1 atmos. Measurements at the end of the absorption curve were performed in steps of 1.5 g/cm 2 . A portion of a Bragg curve is shown in Fig. 2. Comparison of the experimental curve 1 for J_2/J_1 with the theoretical curve 2 of Mather and Segrè 6 yields the energy spread of the proton beam.

The calculation of the theoretical proton range spread due to energy loss fluctuations took a relativistic factor into account: 7

$$(\Delta R_1)^2 = 4\pi z^2 e^4 N Z \int_0^E \left\{ \frac{dE'}{dR'} \right\}^{-2} \frac{1 - \beta^2/2}{1 - \beta^2} dE', \quad (6)$$

where $\beta = \beta(E')$ is the particle velocity. For a copper absorber and proton energy $E = 658$ Mev we have $\Delta R_1 = 2.55$ g/cm 2 .

Besides the indicated spread we took into account range fluctuations due to multiple scattering fluctuations: 14

$$\frac{\bar{s}^2 - \bar{s}^2}{t^2} = \frac{E_s^4 E_0^2}{2a_{ef}^2 (E_0 + \mu)^4} \sum_{n=0}^{\infty} \frac{n+1}{(n+2)^2} \left(\frac{E_0 - \mu}{E_0 + \mu} \right)^{2n}, \quad (7)$$

where

$$a_{ef} = \pi m L_i(E_1) / 2ZxL_r, \quad (8)$$

$L_i(E_1)$ and L_r being given below [see Eq. (9)]. A calculation using (7) and (8) gives

$$\Delta R_2 = \{(\bar{s}^2 - \bar{s}^2)/t^2\}^{1/2} = 0.83 \text{ g/cm}^2.$$

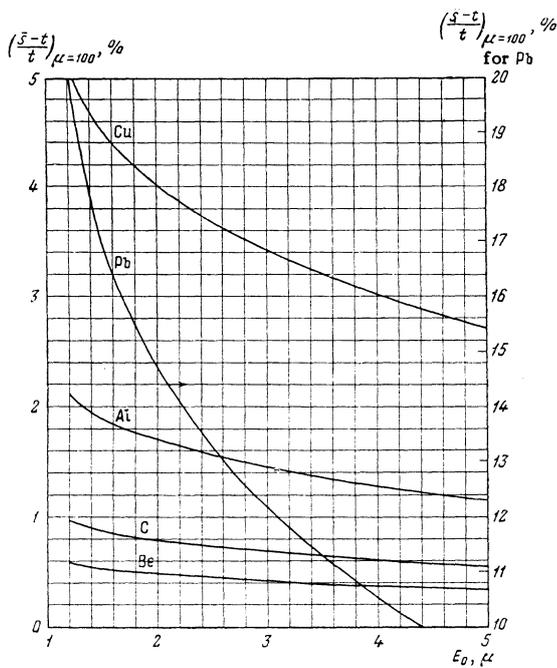


FIG. 3. Multiple-scattering corrections of experimental range as a function of total energy E_0 in units of μ , for different elements and incident particles with rest mass $\mu = 100$.

The total spread $\sigma = \{(\Delta R_1)^2 + (\Delta R_2)^2\}^{1/2}$ was calculated to be 2.68 g/cm^2 whereas the experimental value was $\Delta R_e = 1.30 \sigma = 3.48 \text{ g/cm}^2$. The difference, given by $\{(\Delta R_e)^2 - \sigma^2\}^{1/2} = 2.2 \text{ g/cm}^2$, yields $\pm 3.8 \text{ Mev}$ as the energy spread of the proton beam. It was shown by Mather and Segrè that the mean range corresponds to the ordinate on the Bragg curve for which $J_2/J_1 = 0.82 (J_2/J_1)_{\text{max}}$. The effective range corresponding to this ordinate is 255.9 g/cm^2 . When we take into account the thin copper foils enclosing ionization chambers M_1 and M_2 , and the internal electrodes, with a total thickness of 0.50 g/cm^2 , the total range becomes 256.4 g/cm^2 , which was corrected for multiple scattering of protons in the absorbers. The measured total thickness of an absorber is the effective range, i.e., the range measured in a straight line, whereas the actual range of a particle is somewhat longer because its trajectory is distorted through multiple Coulomb scattering. The total length of the twisted trajectory is the true range which should be used in determining the energy from the range-energy relation.

The difference between the true range \bar{s} and the effective range t was calculated by Pomeranchuk.¹⁴ $(\bar{s} - t)/t$ is given by

$$\frac{\bar{s} - t}{t} \approx \frac{Z\alpha E_s^2 E_0}{\pi m (E_0 - \mu)^2} \frac{L_r}{L_i(E_1)} \ln \frac{(E_0 + \mu)^2}{4E_0\mu}, \quad (9)$$

where Z is the charge of absorber atoms, E_0 is the total particle energy, m is the electron rest

mass in Mev, $E_s = m(4\pi 137)^{1/2} = 21 \text{ Mev}$, μ is the incident particle rest mass, L_r is the "radiation logarithm" and $L_i(E_1)$ is the "ionization logarithm" for an intermediate energy E_1 . E_1 is given by*

$$E_1 = \frac{1}{2} \left[E_0 - \mu - 2\mu \ln \frac{E_0 + \mu}{2\mu} + K\mu b^2 \right] \times \left[\ln \frac{(E_0 + \mu)^2}{4E_0\mu} \right]^{-1}, \quad (10)$$

where

$$K = \int_0^E (1 - b^2 e^{-2t})^{-1} (1 - b e^{-t})^{-2} e^{-2t} t dt, \quad (11)$$

$$b = (E_0 - \mu)/(E_0 + \mu).$$

The values of E_1 (in μ) for E_0 from 1.2 to 5μ are as follows:

$E_0 = 1.2$	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$E_1 = 1.11$	1.16	1.20	1.26	1.31	1.35	1.40	1.44
$E_0 = 2$	2.2	2.5	3	3.5	4	4.5	5
$E_1 = 1.49$	1.58	1.70	1.90	2.07	2.27	2.63	2.63

Figure 3 gives the values of $(\bar{s} - t)/t$ for C, Al, Be, Cu and Pb. The ordinates give $(\bar{s} - t)/t$ as a percentage for a particle with the rest mass $\mu = 100 \text{ Mev}$; for other masses we use the relation $\frac{\bar{s} - t}{t} = \frac{100}{\mu} \left(\frac{\bar{s} - t}{t} \right)_{\mu=100}$. The abscissas give the initial total energy in units of μ . The accuracy of $(\bar{s} - t)/t$ based on (9), (10), and (11) is $\sim 1\%$.

For 658-Mev protons ($E_0 = 1.7\mu$) the correction for multiple scattering of protons in copper is 0.46%; the mean range is then 257.6 g/cm^2 .

Accuracy of the range value is determined by the accuracy in measuring the absorber thickness and by the uncertainty resulting from variation of mean proton energy during the course of the experiment. Several checks of the range during the experiment showed that the deviation did not exceed $\pm 0.2 \text{ g/cm}^2$, while the total error including that involved in measuring absorber thickness amounted to $\pm 0.3 \text{ g/cm}^2$.

DETERMINATION OF IONIZATION POTENTIAL FOR COPPER

Precise measurements of proton energy and range permit determination of the ionization potential for copper by means of the relation

$$R = \int_0^E \left(\frac{dE}{dx} \right)^{-1} dE, \quad (12)$$

where R is the total range (in grams per cm^2) of protons with initial energy E_0 (in Mev) in a

*In Pomeranchuk's paper the expressions for E_1 and K contain errors which have been corrected here.

given substance, and dE/dx is the ionization loss in units of $\text{Mev/g} \cdot \text{cm}^{-2}$, which is determined from Bethe's formula in the form¹⁵

$$-\frac{dE}{dx} = \frac{2\pi z^2 e^4 n}{\rho m c^2 \beta^2} \left\{ \ln \frac{2mv^2 T}{I^2 (1-\beta^2)} - 2\beta^2 - \frac{2}{Z} \sum_{K,L} C_i - \delta \right\}, \quad (13)$$

where e and m are the electron charge and mass, ez is the charge of the incident particle, n is the number of electrons per cm^3 of stopping material with density ρ , $\beta = v/c$ is the particle velocity, I is the mean ionization potential of stopping atoms, $\sum C_i$ represents corrections for particle velocities comparable to electron velocities in K, L, \dots shells, δ is the Fermi density correction,¹⁶ for copper $2\pi n e^4 / \rho m v^2 = 0.0701 \text{ Mev/g} \cdot \text{cm}^{-2}$, and T — the maximum energy transferred by an incident particle to an atomic electron — is given by

$$T = (E^2 - \mu^2 c^4) / \mu c^2 (\mu / 2m + m / 2\mu + E / \mu c^2), \quad (14)$$

where E is the energy of the incident particle with rest mass μ . For heavy particles with $\mu \gg m$ we have $T \cong 2mv^2 / (1 - \beta^2)$.

The corrections C_i which take into account electron binding in K and L shells have been calculated by Walske.^{17,18} The sum of these corrections reduces the calculated energy loss of particles in matter with $Z = 30$ by $\sim 4\%$ for 5-Mev protons, by 0.16% for $E_p = 250 \text{ Mev}$ and by 0.05% for $E_p = 660 \text{ Mev}$. In our case the total correction reduces I_{Cu} by 3 ev.

The most accurate calculation of the density effect in different substances is that of Sternheimer.¹⁹ The empirical expression for δ is

$$\begin{aligned} \delta &= 4.606 X + C + a(X_1 - X)^m & \text{for } X_0 < X < X_1, \\ \delta &= 4.606 X + C & \text{for } X > X_1, \end{aligned} \quad (15)$$

where $X = \log(pc/\mu)$, and pc and μ are the particle momentum and rest mass, X_1 is the value of X above which δ is linearly dependent on X , X_0 is the value of X below which $\delta = 0$ and is

determined from the velocity $\beta_0 = \left\{ 1 + \sum_i f_i / \nu_i^2 \right\}^{1/2}$

(where f_i is the oscillator strength of a given transition, which is equal to the number of electrons on the given level divided by the total number of electrons in the atom), and $\nu_i = E_i/\hbar$ is the i -th transition frequency. For copper we have $C = -4.13$, $10a = 0.99$, $m = 3.40$, $X_1 = 3$, and $X_0 = 0.00$.* The

* C , a , m , X_1 and X_2 were calculated by Sternheimer in connection with a formula containing the value of the copper ionization potential $I_{\text{Cu}} = 279 \text{ ev}$ which was obtained by Bakker and Segrè.⁵ The δ correction to I_{Cu} is so small that the difference between $I_{\text{Cu}} = 279 \text{ ev}$ and the value $I_{\text{Cu}} = 307 \text{ ev}$ used in the present paper has been disregarded.

value of δ is $\sim 0.12\%$ for 400-Mev protons and $\sim 0.25\%$ for 600-Mev protons. In our case the density effect reduces I_{Cu} by 2 ev.

I_{Cu} was computed as follows. Equation (12) was integrated numerically to determine R_1 and R_2 for two different ionization potentials close to the experimental value, such as $I_1 = 375 \text{ ev}$ and $I_2 = 300 \text{ ev}$. We thus determined $\Delta R / \Delta I = (R_2 - R_1) / (I_2 - I_1) = 0.126 \text{ g/cm}^2 \text{ ev}$, and the ionization potential satisfying initial energy $E_0 = 658 \text{ Mev}$ and range $R_0 = 257.6 \text{ g/cm}^2$ was $I_{\text{Cu}} = 305 \text{ ev}$. It should be noted that the range was calculated assuming velocity independence of the ionization potential I_{Cu} . The computed proton range for $I_{\text{Cu}} = 377 \text{ ev}$ is 8.84 g/cm^2 greater than the experimental range.

The error in determining the ionization potential by the foregoing method is a combination of the errors in determining the initial proton energy and the range. The rms error in the range is $\pm 1.2 \text{ g/cm}^2$, which leads to the uncertainty $\Delta I_{\text{Cu}} = \pm 10 \text{ ev}$ in the ionization potential. The value $I_{\text{Cu}} = 305 \pm 10 \text{ ev}$ obtained in the present experiment agrees well within the limits of error with Mather and Segrè's value of 310 ev for 340-Mev protons.

RELATIVE STOPPING POWERS OF DIFFERENT SUBSTANCES FOR 635-Mev PROTONS

The experimental arrangement is shown in Fig. 1. The thicknesses of the specimens were such that the ionization losses of protons were $\sim 50 \text{ Mev}$, so that the measured relative stopping powers refer to protons with the energy 635 Mev at the center of a specimen. The experiment determined the amount of copper which would slow down protons of 635 Mev mean energy to the same extent as 1 g/cm^2 of a given substance. For this purpose we measured $q = (A_{\text{Cu}} - B_{\text{Cu}}) / C_i$, where A_{Cu} is the total proton range in the copper absorber, B_{Cu} is the thickness of copper absorbers remaining after a fraction of the copper had been replaced by the test substance, and C_i is the thickness of the test specimen. Range values were obtained from the mean range; the results are given in Table II.

TABLE II

Element	q	$I, \text{ ev}$	Bloch constant $I_0 = I/Z, \text{ ev}$
H	3.016	15	15
Be	1.167	61±6	15.3
C	1.268	85±8	14.2
Fe	1.034	273±22	10.5
Cu	1.000	305±10	10.5
Cd	0.887	468±35	9.8
W	0.794	680±50	9.2

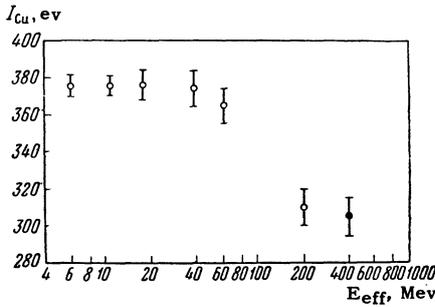


FIG. 4. Ionization potential of copper as a function of proton effective energy. ● — present data (E_{eff} — proton effective energy).

Ionization potentials were calculated assuming that $I_{\text{Cu}} = 305$ ev for $E_0 = 635$ Mev. Corrections were included for the binding of K and L electrons as well as for the density effect. The final expression for ionization potentials based on measurements of relative stopping powers is

$$\ln I = 13.788 - \frac{A}{Z} 3.670q - \frac{1}{Z} \sum_{K,L} C_i - \frac{\delta}{2}, \quad (16)$$

where A is the atomic weight.

Relative stopping powers were measured with an accuracy of about $\pm 1\%$; therefore the accuracy of the ionization potentials is $\pm 10\%$, since $\Delta I/I \approx (\Delta q/q) \ln(2mv^2/I)$. The stopping power of hydrogen* was determined from the $\text{CH}_2 - \text{C}$ difference.

DISCUSSION OF RESULTS

The paper of Sachs and Richardson⁸ contains a graph showing values of the ionization potential for copper that were determined by different methods for different proton energies, as a function of the effective energy, which when I is determined for the entire range equals 0.6 of the initial energy E if we assume $I = I_0 - a \ln E$. The data of the present work for 400 Mev are included in the same graph for comparison (Fig. 4). Up to 60 Mev the ionization potential of copper is ~ 380 ev, while at higher energies it is reduced to ~ 310 ev.

As indicated above, in the present paper I_{Cu} was calculated assuming it to be independent of proton velocity. I_{Cu} was also calculated with the experimental data for low energies taken into account. For proton energies from 0 to 125 Mev we assumed the constant value $I_{\text{Cu}} = 377$ ev; from 125 to 660 Mev we assumed 305 ev. The theoretical and experimental ranges could be brought into agreement only by the value 301 ev for I_{Cu} in the 125 — 660 Mev range. No other assumptions were made regarding the energy dependence of I_{Cu} ; the

*For hydrogen the accuracy of q is $\sim 5\%$ and that of I_{H} is $\sim 50\%$.

assumptions already adopted yield 300 ev as the ionization potential of copper for a mean proton energy of about 400 Mev. The observed drop of 70 — 80 ev in this proton energy range is outside the limits of experimental error and requires an explanation.

If we assume that an ionization potential cannot depend on the velocity of an incident particle the decrease of the former as the particle energy rises can either indicate some additional form of loss at high energies which is not included in Bethe's theory, or that some effect reduces the loss at low energies.

The variation of I with energy may possibly be associated with the effective density at low energies. Indeed, the calculations of ionization potentials from absolute measurements of dE/dx at low energies have usually employed an expression which neglected the Fermi density effect, although this effect does not disappear at low energies. Sternheimer¹⁹ has shown that the reduction of ionization loss $\Delta(dE/dx)$ at low energies due to the density effect is given by

$$\Delta \frac{dE}{dx} = \frac{2\pi n (ze^2)^2}{\rho m v^2} \sum_i f_i \ln \left(\frac{l_i}{v_i} \right)^2 = \frac{2\pi n (ze^2)^2}{\rho m v^2} \delta, \quad (17)$$

where $l_i = (v_i^2 + f_i)^{1/2}$ with all other notation as in (13). In first approximation, i.e., using transition frequencies ν_i taken from Sommerfeld's tables,²⁰ we have $\delta(\text{Cu}) = 0.26$. $\Delta(dE/dx)$ for protons with ~ 20 Mev in copper may be a few percent.

In calculating the reduction of loss at high proton energies we must use the formula¹⁹

$$\Delta \frac{dE}{dx} = \frac{2\pi n (ze^2)^2}{\rho m v^2} \left[\sum_i f_i \ln \frac{v_i^2 + l^2}{v_i^2} - l^2 (1 - \beta^2) \right], \quad (18)$$

where l is defined by

$$\beta^{-2} - 1 = \sum_i f_i / (v_i^2 + l^2).$$

In this case for 600-Mev protons with the same values of the frequencies ν_i , $\Delta(dE/dx)$ for copper is only 0.1% and increases to 3% at 2 Bev.

The reduction of dE/dx by a few percent at low energies is equivalent to reduction of the copper ionization potential by 20 — 30%, i.e., to ~ 300 ev. It should be noted that the value of the sum

$\delta = \sum_i f_i \ln(l_i/v_i)$ is uncertain because the transition frequencies ν_i are uncertain. If a more exact calculation eventually shows that the correction factor δ actually has the given value, there will hardly be any reason for regarding I as velocity dependent. I could then be given a constant

value at 10–11 Z ev and the density effect would be taken into account in calculations of energy losses over the entire energy range. In the event of the contrary result the reduction of I at higher energies must be accounted for in some other manner.

In conclusion we wish to thank Yu. D. Prokoshkin and I. M. Vasilevskii for discussions and for acquainting us with preliminary data on absolute losses for 650-Mev protons.

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Translated by I. Emin

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ERRATA TO VOLUME 9

Page	Reads	Should read
115, Col. 2, line 18 from top	R. Gatto and M. A. Ruderman, [Nuovo cimento 8, 775, (1958)]	T. Goto, Nuovo cimento 8, 625 (1958)
294, Col. 2, line 4 from bottom	$N = N_{\text{exp}}(p, \theta) F(p, \theta)$	$N = N_{\text{exp}}(p, \theta) 1 + F(p, \theta)$
462, Col. 1, line 8 from top	which are approximately 13Z ev	and approximately equal to 13Z ev
646, Col. 1, line 3 from top	$\langle j_1' t_1' \alpha R^{J_2} j_1 t_1 \alpha_1 \rangle$	$\langle j_1' t_1' \alpha R^{J_1} j_1 t_1 \alpha_1 \rangle$
661, Col. 1, line 6 from top	$\lambda = 2.14 \times 10^{-13}$	$\lambda = 1.04 \times 10^{-13}$