HYSTERESIS AND NON-STATIONARY EFFECTS IN THE ELECTRON TEMPERATURE IN PLASMA IN INERT GASES

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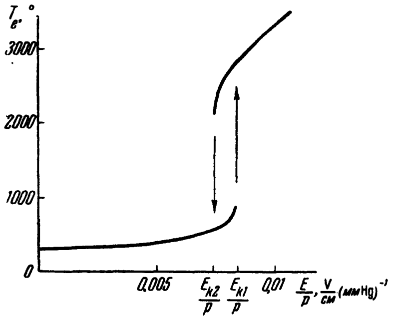
In an earlier paper[1] we have indicated certain features characteristic of the heating of an electron gas in a highly ionized plasma which result from the fact that the frequency of collisions between electrons and ions falls off sharply with increasing electron velocity.* It was found that in a fixed electric field the electron gas is in a stationary state with respect to the ions only at small values of the field ($E < E_k$); when $E \geq E_k$ this state becomes unstable. A similar instability is found in a low-frequency alternating electric field. In this case, however, there is a second stable state characterized by a high electron temperature. The transition between the first and second states takes place at various field amplitudes and leads to hysteresis in the dependence of electron temperature on field.

Similar effects can occur in a weakly ionized plasma; all that is necessary is that the frequency of electron collisions be a sufficiently rapidly decreasing function of velocity: $\nu \sim v^{-\alpha}$, where $\alpha > 1$. In general this condition is not satisfied because the frequency of collisions between electrons and molecules increases rapidly with increasing v. However, an inverse relationship is possible. An inverse relation can obtain, for example, in heavy inert gases such as argon, krypton and xenon at small electron velocities $v \lesssim 5 \times 10^7$ cm/sec (Ramsauer effect).

The calculation of the stationary electron temperature T_e is carried out for a weakly ionized plasma in the same way as for the highly ionized plasma considered in reference 1.† The figure shows the dependence of T_e on the electric field intensity in krypton at a gas temperature of 27°C (p is the pressure in mm Hg). (In this calculation we have used the data of Ramsauer and Kollath[2] and Holtsmark.[3]) It is apparent from the figure that the weakly heated state of the electron gas (the lower curve) becomes unstable at some value of the field intensity in the same way as in a highly ionized plasma. In contrast with the latter, however, in the case being considered here there is a second stable state at a high electron temperature: this state arises because of the increased frequency of collisions between the electrons and krypton ions at high velocities ($v > 5 \times 10^7$ cm/sec). The critical field values associated with the transition from the first state to the second state vary slightly ($E_{k1} = 8.4 \times 10^{-3}$p, $E_{k2} = 7.4 \times 10^{-3}$p (v/cm): here p is the gas pressure in mm Hg). As a consequence the hysteresis loop is small. The electron temperature changes by approximately a factor of 3 in the transition. There is a corresponding change in electronic conductivity: this conductivity increases by a factor of 2.4 in the transition from the first state to the second state but is reduced by a factor of 3 for the reverse transition. The time required for the transition is $10^{-3} p^{-1}$ sec. In an alternating electric field the effect is suppressed at higher frequencies ω and vanishes completely when $\omega > 1.5 \times 10^8$p sec^{-1}.



It should be pointed out that the present calculation applies only when the electron velocity distribution is Maxwellian. As is well-known, this condition is not satisfied in the case of a slightly ionized plasma ($N_e/N_m \lesssim 10^{-10}$), in which case the frequency of collisions with molecules is more than a factor of 10^5 greater than the frequency of collisions between electrons. In the latter case, as is clear, for example, from reference 4, the mean electron temperature is always uniquely related to the electric field intensity so that the effect being discussed here is not obtained.

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*By a highly ionized plasma we mean one in which collisions between electrons and ions predominate; by a weakly ionized plasma we mean one in which collisions between molecules (or atoms) predominate.

†In computing the complex conductivity of a plasma one introduces the coefficients K_σ and K_ϵ which have been considered earlier in references 1 and 4. It should be noted that if the dependence of ν on v is given by a power law the coefficients K_σ and K_ϵ depend only on the parameter ω/ν_{eff}. In the general case, however, they also depend on T_e. This dependence is found to be important in the exam-

ple considered here. We also note that the assumption that the heavy-particle temperature is stationary, necessary for the validity of the entire analysis (cf. reference 1), is always satisfied in a weakly ionized plasma.

[1] A. V. Gurevich, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 392 (1958), Soviet Phys. JETP **8**, 271 (1959).
[2] C. Ramsauer and R. Kollath, Ann. Phys. **5**, 536 (1929).
[3] J. Holtsmark, Z. Physik **66**, 49 (1930).
[4] A. V. Gurevich, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1112 (1956), Soviet Phys. JETP **3**, 895 (1956).

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ON SOME SYMMETRY PROPERTIES OF THE EIGENFUNCTIONS OF THE SCHRÖDINGER EQUATION

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In the present note we wish to call attention to two facts which have not, so far as we know, been noted in the literature: the fact that the symmetry groups of the eigenfunctions of the Schrödinger equation are subgroups of the symmetry group G_H of the corresponding Hamiltonian $\hat{H}$, and the fact that the converse statement is not valid, i.e., the existence of subgroups of the group G_H that are not symmetry groups of the eigenfunctions of the given Schrödinger equation.

The first statement has been made by Melvin,[1] who introduced the concept of the cokernel K^{ij} of the i-th row of the j-th irreducible representation of the group G_H, to which there correspond in the j-th irreducible representation matrices with all the elements in the i-th row equal to zero except for the diagonal element, which is unity. It is easy to see that the symmetry transformations occurring in the cokernel K^{ij} leave the i-th function in the list of eigenfunctions $\psi_1, \psi_2, \ldots, \psi_l$ invariant, and that they form a subgroup of the group G_H.[1] Contrary to Melvin's statement, however, this still does not mean that the cokernel K^{ij} is identical with the symmetry group of the functions ψ_i, since it remains unproved that an eigenfunction of the Hamiltonian $\hat{H}$ with the symmetry group G_H cannot be invariant with respect to some symmetry operator s which does not belong to the group G_H.

We shall prove this last assertion on the assumption that the set (L) of the nodal points of the eigenfunctions of the equation

$$\hat{H}\psi = (\hat{T} + \hat{V})\psi = E\psi, \qquad (1)$$

has no internal points and that the value of the potential at any point ζ of the configuration space can be represented as the limit of the values of the potential at a sequence of points ζ_n that converges to ζ, i.e.,

$$V(\zeta) = \lim V(\zeta_n) \quad \text{for } \zeta_n \to \zeta.$$

Suppose that s does not belong to G_H. We shall show that no eigenfunction of the operator $\hat{H}$, which satisfies our assumptions, can be invariant with respect to the symmetry operation s. Let us assume the opposite, i.e., that there exists a function (whose set of nodal points has no internal points) for which $s\psi = \psi$. Then $s\hat{H}\psi = s(\hat{T} + \hat{V})\psi = \hat{T}s\psi + s\hat{V}\psi = E\psi$ and, on the other hand, $\hat{T}s\psi + \hat{V}s\psi = Es\psi$. Consequently $s\hat{V}\psi = \hat{V}s\psi = \hat{V}\psi$, so that $sV = V$ at points where $\psi \neq 0$.*

On our assumption about the set L of the nodal points, for any point ζ in L we can always find a set of points ζ_n not belonging to L that converge to ζ. Obviously $\psi(\zeta_n) \neq 0$. Therefore from $sV(\zeta_n)\psi(\zeta_n) = V(\zeta_n)\psi(\zeta_n)$ it follows that $sV(\zeta_n) = V(\zeta_n)$. Going to the limit, we get $sV(\zeta) = V(\zeta)$, and consequently the equation $sV = V$ is valid for every point of the configuration space, which contradicts the hypothesis of our argument.

Thus on the assumptions indicated it has been shown that the symmetry groups of the eigenfunctions of the Schrödinger equation are subgroups of the symmetry group G_H of the Hamiltonian $\hat{H}$, namely they are the corresponding cokernels.

We shall show the incorrectness of the converse statement for the example of a Hamiltonian with the symmetry group C_{6v}. The group C_{6v} has as one of its subgroups the group $(E, C_3^{\pm})$. We shall show that this subgroup cannot be a cokernel of the group C_{6v}. From the table of characters[2] of the group C_{6v} it can be seen that the only subgroups that are cokernels corresponding to one-dimensional representations are: for $A_1(C_{6v})$, for $A_2(E, C_2, C_3^{\pm}, C_6^{\pm})$, for $B_1(E, C_3^{\pm}, \sigma_{d_1}, \sigma_{d_2}, \sigma_{d_3})$, and for $B(E, C_3^{\pm}, \sigma_{v_1}, \sigma_{v_2}, \sigma_{v_3})$. In the two-dimensional representation E_1 there corre-