RELATIVISTIC CORRECTIONS TO THE PHENOMENOLOGICAL THEORY OF LEVELS OF LIGHT NUCLEI

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Relativistic corrections to the phenomenologically prescribed interaction between a pair of nucleons in a nucleus is computed on the basis of the expression obtained by Yu. M. Shirokov for relativistic corrections to the nonrelativistic two-body Hamiltonian. It is found that the relativistic corrections depend strongly on the shape of the potential and are of the order of 0.02 — 0.2 Mev for a pair of nucleons.

1. The question of the role of relativistic effects in the theory of levels of light nuclei was hardly investigated until recently. Blatt and Weisskopf (reference 1, p. 162) express an opinion that these effects can give a contribution on the order of 10 to 20%. At the present state of meson theory we cannot investigate this problem with any degree of reliability. A phenomenological examination, based on general group-theoretical properties of the relativistic invariance of quantum theory, is therefore of interest. The general method for the investigation of problems of this kind is presented in reference 2.

2. Neglecting relativistic effects, the nucleus is described by the non-relativistic Hamiltonian

$$H = \sum_n T_n + \sum_{m>n} H_{mn}, \quad (1)$$

where T_n is the kinetic energy of the n-th nucleon

$$T_n = \mathbf{p}_n^2/2M, \quad (2)$$

and H_{mn} is Hamiltonian of the paired interaction between the nucleons m and n. In nuclear physics the Hamiltonian, as a rule, is chosen phenomenologically to satisfy the necessary properties of invariance and to agree with the basic experimental data on the nucleon-interaction. In particular, the Hamiltonian H_{mn} should be galilean-invariant, that is, independent of the total momentum of the interacting nucleons.

It is shown in reference 2 that if relativistic corrections are taken into account with accuracy to $(v/c)^2$, the Hamiltonian (1) becomes

$$H = \sum_n T_n + \sum_{m>n} H_{mn} + \sum_n T'_n + \sum_{m>n} H'_{mn}, \quad (3)$$

where T'_n is the correction to the kinetic energy of the n-th nucleon,

$$T'_n = -\mathbf{p}_n^4/8M^3, \quad (4)$$

and H'_{mn} is the relativistic correction to the interaction Hamiltonian, which equals

$$H'_{mn} = (1/8M^2)\{-H_{mn}\mathbf{P}^2 + i(\mathbf{P}\cdot\partial H_{mn}/\partial\mathbf{x})(\mathbf{P}\cdot\partial/\partial\mathbf{p}) + (\sigma_m - \sigma_n)[\mathbf{P}\times\partial H_{mn}/\partial\mathbf{x}] - i(\sigma_m - \sigma_n)H_{mn}[\mathbf{p}\times\mathbf{P}] + iH_{mn}(\sigma_m - \sigma_n)[\mathbf{p}\times\mathbf{P}] - (\mathbf{P}\cdot\partial H_{mn}/\partial\mathbf{p})(\mathbf{P}\cdot\mathbf{p}) + iP_iP_j\partial^2H_{mn}/\partial x_i\partial p_j\}. \quad (5)$$

Here

$$\mathbf{P} = \mathbf{p}_m + \mathbf{p}_n, \quad \mathbf{p} = (\mathbf{p}_m - \mathbf{p}_n)/2, \quad (6)$$

and $\mathbf{x}$ is the operator of the difference of nucleon coordinates.

$$\mathbf{x} = \mathbf{x}_m - \mathbf{x}_n; \quad (7)$$

σ_m and σ_n are the spin matrices of the m-th and n-th nucleons, respectively. The nonrelativistic Hamiltonian is assumed specified in the form of a function of the coordinate and momentum operators of the nucleons. In virtue of conservation of the momentum and of the nonrelativistic center of mass, H_{mn} is independent of $\partial/\partial\mathbf{P}$ and $\mathbf{P}$. The Hamiltonian H_{mn} can naturally depend on the ordinary and isotopic spins.

The correction term H'_{mn} does not satisfy the law of conservation of the nonrelativistic center of mass, since it depends explicitly on the total momentum P. H'_{mn} vanishes in the center-of-mass system of the two nucleons, so that an interaction Hamiltonian H_{mn} phenomenologically constructed from on nucleon-nucleon scattering data need not include special allowance for relativistic effects. However, if a Hamiltonian so constructed is applied to the many-body problem, it becomes necessary to take into account the correction term (5), for in this case it is impossible to go over to a system in which the centers of mass of all pairs of particles are at rest. We note that in expression (5) the additions of order $\mathbf{P}^2/M^2$ and $\mathbf{P}\cdot\mathbf{p}/M^2$ are taken into account exactly, and that the relativistic corrections of order p^2/M^2 are not considered at all. However, the latter do not contradict the nonrelativistic law of conservation of the center of mass. and therefore, when constructing a Hamiltonian from scattering data, they simply enter into the nonrelativistic Hamiltonian H_{mn}.

3. To estimate the influence of relativistic effects on the positions of the levels of light nuclei, the following calculations were performed:

Equation (5) was used to calculate the level shifts for two particles in the states

$$|0s^2_{1/2}01\rangle,\quad |0s^2_{1/2}10\rangle,\quad |1p^2_{3/2}01\rangle,\quad |1p^2_{1/2}01\rangle$$

for a Gaussian potential, a Yukawa potential, and a rectangular well. Oscillator wave functions were used throughout, with $r_0 = 1.65 \times 10^{-13}$ cm (i.e., $h\omega = 15$ Mev). The potentials were chosen in the form

$$V = V_0(0.317 + 0.500P + 0.183PQ)\, f(r/a)$$

with the following constants: for the Gaussian potential:[3]

$$V_0 = -51.9\ \text{Mev},\quad a = 1.73\cdot10^{-13}\ \text{cm},\quad f(x) = e^{-x^2},$$

for the Yukawa potential (reference 1, p. 50)

$$V_0 = -68\ \text{Mev},\quad a = 1.17\cdot10^{-13}\ \text{cm},\quad f(x) = e^{-x}/x,$$

and for the potential well (reference 1, p. 50)

$$V_0 = -33.6\ \text{Mev},\quad a = 2.1\cdot10^{-13}\ \text{cm};$$

The potentials are independent of the velocity. P and Q are the permutation operators of the spatial and spin variables respectively. The following results were obtained (in kev):

Potential	State					
	$s^2_{1/2}\,01$	$s^2_{1/2}\,10$	$p^2_{3/2}\,01$	$p^2_{1/2}\,01$	$s_{1/2}\,p_{1/2}\,00$	$s_{1/2}\,p_{1/2}\,01$
Gaussian	95	150	52	11	125	—22
Yukawa	96	142	205	51	120	—21
Rectangular well	115	182	62	8	150	—14

4. From the foregoing we can draw the following conclusions as to which relativistic effects on the structure of the nuclear levels are significant for a phenomenological statement of the problem.

The relativistic corrections depend greatly on the form of the potential and are of order 0.02 — 0.2 Mev for individual pairs of nucleons in a nucleus. As a consequence of saturation of the nuclear forces, the correction to the ground state can be considered to be proportional to the number of nucleons in the nucleus, i.e., of the order 0.2 — 2 Mev for light nuclei and 2 — 20 Mev for heavy ones. Corrections to the relative placements of the nuclear levels are of the same order as for a pair of nucleons, i.e., 0.02 — 2 Mev.

Relativistic corrections are thus considerably smaller than given reference 1, and can be disregarded in the less accurate present-day methods. On the other hand, these corrections exceed the errors in the experimental measurements of the nuclear levels, and must therefore be taken into account when developing computational methods of accuracy comparable with that of the experimental measurements.

[1] J. Blatt and V. Weisskopf, Theoretical Nuclear Physics, (Russ. Transl. IIL, 1954), Wiley, N. Y., 1952.

[2] Yu. M. Shirokov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 474 (1959), Soviet Phys. JETP, this issue, p. 330.

[3] R. A. Ferrell and W. M. Visscher, Phys. Rev. **102**, 450 (1956).

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