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RELATIVE PROBABILITIES FOR THE PHOTOEFFECT IN SHELLS AND SUBSHELLS OF AN ATOM

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The relative intensities of the K, $L_I + L_{II}$, L_{III} and M + N photoelectric lines produced in various targets by γ rays from several radioactive isotopes were measured by a γ ray spectrometer of resolving power 0.4%. The results are compared with theoretical calculations.

1. INTRODUCTION

In spite of the considerable amount of work on the photoeffect, there is up to now no formula for accurate calculation of the probability of this process for arbitrary energy of the x-rays or γ rays. The process of emission of an electron from the K shell has been considered in greatest detail. For the nonrelativistic case, Heitler[1] gave a formula for the photoelectric cross section in Born approximation. Stobbe[2] and Hall[3] obtained more accurate results. Using relativistic wave functions, Sauter[4] and Hulme[5] obtained formulae for the photoeffect in Born approximation. A simple formula was derived by Hall[6] for the case $h\nu \gg mc^2$. The most accurate calculations of τ_K have been carried out by Hulme et al.[7] for three elements and two γ-ray energies. Formulae for the coefficients of photoabsorption in the L_I, $L_{II} + L_{III}$ and M shells have been given by Stobbe and Hall.[2,3]

2. SURVEY OF EXPERIMENTAL DATA

In so far as experimental results for the relative probabilities of the photoeffect will be given in this article, we first consider similar investigations. We know of three projects carried out with the view of checking theoretical predictions about the relative probabilities of the photoeffect, as well as a series of studies in which relative probabilities of the photoeffect in the K and L shells of the atom were obtained from the study of γ-ray spectra of radioactive elements. A general feature of these works is insufficient resolving power of the apparatus and rather thick targets,

(a) Marty[8] measured the ratio of absorption coefficients τ_K/τ_L for energies of 140 and 411 kev, and drew conclusions concerning the agreement of experimental results with those calculated from Hall's formula for the relativistic case. Our own[9] investigation of the influence of target thickness makes us believe that the results for τ_K/τ_L obtained there are too low.

(b) Davidson and Latyshev[10] determined the ratio τ_K/τ_L for 2614-kev γ rays. This value was 4.8 for Pb and 5.3 for Ta. Later[11,12] Latyshev gave slightly different values, 4.9 and 5.4 respectively. Since the error in the experiments, according to our estimate, constitutes ~ 20%, this difference is not meaningful.

(c) Bazin[13] measured the photoabsorption coefficients for the K_{α_1}-line of molybdenum, $h\nu = 17.5$ kev, in several targets.

The results of this investigation were as follows:

$$\tau_K/(\tau_L+\tau_M) = 11.3 \text{ for sulphur}$$
$$\tau_K/(\tau_L+\tau_M) = 9.4 \text{ for chromium}$$
$$\tau_L/(\tau_M+\tau_N) = 1.8 \text{ for selenium}$$
$$\tau_L/\tau_M = 2.6,\ \tau_L/(\tau_M+\tau_N) = 2.1 \text{ for silver.}$$

Unfortunately, the probable errors in these numbers are not given, but clearly they are appreciable.

(d) Of the works in which the main object was not the study of the photoeffect, the article of Navakov, Hultberg and Anderson[14] should be noted. Here the γ rays coming from the decay of Bi^{206} were studied. A uranium target of thickness 3 mg/cm^2 was employed. For 516 and 880 kev γ rays the ratios τ_K/τ_L were the same and equal to 5.5.

It should be noted that in the works considered, comparison of the results obtained with theoretical ones was somewhat loosely carried out. This was because the influence of the angular distributions of photoelectrons from the various shells and subshells of the atom was not taken into account. This remark applies, to some extent, to our work as well. A complete answer to the problem of the probabili-

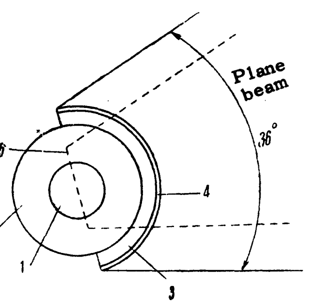

FIG. 1. Arrangement of the source of photoelectrons: 1 – radioactive material ($0.3 \leq \phi \leq 0.5$), 2 – cylindrical ampoule ($1.0 \leq \phi \leq 3.0$), 3 – backing of the target ($d = 0.2$), 4 – target, 5 – usual position of the source of conversion electrons. The height of the source was 35 mm, the width 1 mm.

ties of the photoeffect in various shells of the atom would be provided by measuring the intensities of the photoelectron lines at various angles, and averaging the results, after which they could be compared with theoretical predictions.

3. EXPERIMENTAL CONDITIONS

The results of the study of the photoeffect given below were obtained by us in continuing during the past five years work on the study of the decay of various isotopes. The measurements were carried out using a β spectrometer with a relative half-width of 0.4% for the spread coming from the apparatus.[15]

In our experiments, the photoelectron lines arising as a result of absorption of monochromatic γ rays in the K, $L_{I,II,III}$ and M + N shells of various elements were studied. The half-width of the photoelectron lines was measured to within 0.4 – 0.8%, depending upon the energy of the photoelectrons. In our experiments, a source with axial symmetry[16,17] was employed (see Fig. 1.) and a thin target.

From comparison of our relative γ-ray intensities with the data obtained by other methods, we conclude that, within a wide energy interval (0.1 – 2.0 Mev), the angular distribution of the photoelectrons — which changes with the γ-ray energy — does not affect the results to more than 15%. From this it follows that if there is any difference in the angular distribution of photoelectrons, for example, from the L_{II} and L_{III} subshells in comparison with L_I one, then it does not show up in our experiments. We note that the theory indicates that the angular distribution of photoelectrons from the K and L_I shells will be similar, and that those from the L_{II} and L_{III} ones will be more isotropic.[3]

In the study of the photoeffect from the L shell of medium atoms ($Z \sim 50$), and from M and N shells of heavy atoms ($Z \sim 80$), difficulties arise in the behavior of the background behind the lines, since it is influenced by the photoelectrons emitted from the K shells of the atoms of the material used either as backing of the target or in the ampule containing the radioactive source. Thus, in our use of a brass cylindrical ampule with wall-thickness 0.5 mm and superimposed target backing of 0.2 mm thick aluminum, a noticeable jump occurred in the background in the places where the K peaks of the photoelectrons of copper and zinc should occur.

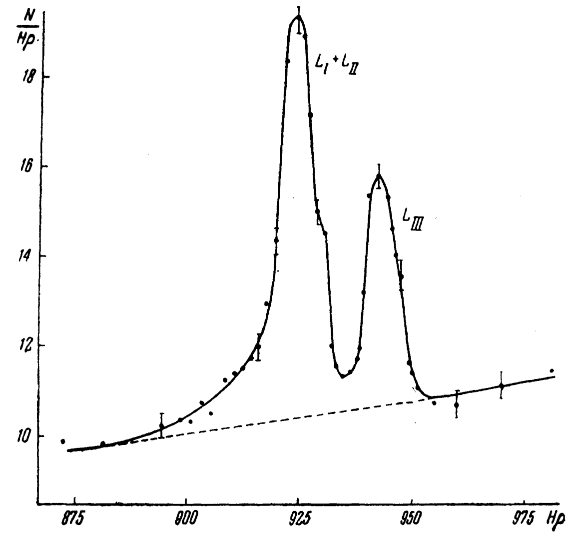

FIG. 2. $L_I + L_{II}$ and L_{III} photoelectron lines from the 86.6-kev Tb^{160} γ rays. The target was 0.25 mg/cm² Bi.

In our experiments, we usually measured the background behind the peaks of the photoelectrons, and we used aluminum both in the ampule and backing. The thickness of the targets was varied from 0.03 to 13 mg/cm², to avoid distortion in the relative intensities: Targets with surface thickness < 0.3, < 1, and 3 – 13 mg/cm² were used in the region of photoelectron energies 30 – 100 kev, up to to 200 kev, and above 500 kev, respectively.

4. THE RATIO $(\tau_{L_I} + \tau_{L_{II}})/\tau_{L_{III}}$

Up till now, none of the experimental data in the literature can be compared with the theoretically predicted relative probabilities of the photoeffect from the L_I, L_{II} and L_{III} subshells of the atom for γ rays of energy greater than 20 kev. The high resolving power of the spectrometer and the use of thin targets made it possible to differentiate either partially or completely, L_I, L_{II} and L_{III} lines in measurement with the bismuth target. Since the L_{II} line of Bi is energetically nearer to the L_I than L_{III} line, we can make a very reliable determination of the ratio $(\tau_{L_I} + \tau_{L_{II}})/\tau_{L_{III}}$. We choose this ratio to compare with the formula of Stobbe,[2] considering the ratio of photoabsorption coefficients $\tau_{L_{II}}$ and $\tau_{L_{III}}$ to be 0.45 : 1 corresponding to the number of electrons in p states in the L_{II} and L_{III} subshells of the atom, and following the theoretical work of Phillips,[18] which

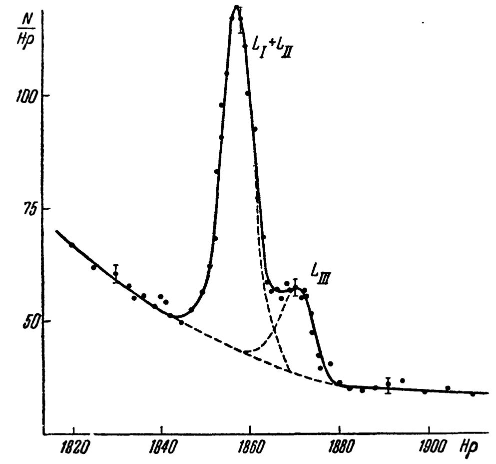

FIG. 3. $L_I + L_{II}$ and L_{III} photoelectron lines from the 265 kev Se^{75} γ rays. The target was 0.1 mg/cm² Bi.

determined this ratio as a function of Z, using relativistic wave functions. The validity of this formula was checked by Patten[19] for energies near to the threshold for absorption by the L subshells and for large Z of order 79 — 83. Examples of measured $L_I + L_{II}$ and L_{III} photoelectron lines are given in Figs. 2 and 3. From the formulae of Stobbe it follows that

$$\frac{\tau_{L_I}}{\tau_{L_{II}}+\tau_{L_{III}}}=\frac{h\nu}{h\nu_2}\,\frac{1+3\,(h\nu_2/h\nu)}{3+8\,(h\nu_2/h\nu)},$$

where $h\nu$ is the γ-ray energy, $h\nu_2 = \frac{1}{4}(Z - s_2)^2\,\mathrm{Ry}$, s_2 is the Slater screening constant for 2s or 2p electrons, equal to 4.15 for all Z larger than 10, Ry is the Rydberg constant, (13.61 ev), and Z is the charge of the element in which the photoeffect takes place. With $\tau_{L_{II}}/\tau_{L_{III}} = 0.45$, we obtain

$$(\tau_{L_I}+\tau_{L_{II}})/\tau_{L_{III}} = 0.45+1.45\tau_{L_I}/(\tau_{L_{II}}+\tau_{L_{III}}).$$

In Table I we give calculated and experimental ratios for four cases.

TABLE I. Experimental and theoretical values of $(\tau_{L_I} + \tau_{L_{II}})/\tau_{L_{III}}$ for Z = 83 (0.1 mg/cm² Bi target, Se^{75} and Tb^{160} γ-ray sources)

hν	Theory	Experiment
86.6	2.51	2.6±0.5
121.2	3.33	3.8±0.8
136.2	3.66	3.9±0.3
265.0	6.28	5.0±0.5

Comparison of these numbers shows that the experimental values coincide with the theoretical ones for small energies, but already at an energy of 265 kev, they diverge more than the limits of experimental error.

5. THE RATIO τ_L/τ_M

We did not succeed in establishing any regularity in the ratio τ_L/τ_M. The mean value from experiments carried out with γ rays of various energies and with various targets was equal to $\tau_L/\tau_M = 3.5 \pm 0.5$. It was obtained from very careful measurements of the photoelectron lines from 411-kev γ-rays from Au^{198} on a thorium target, of 364-kev γ-rays from J^{131} on a bismuth target, etc. The ratios τ_L/τ_M, obtained with large errors, did not go outside the limits of the magnitude indicated, which is in agreement with the value $\tau_L/\tau_M = 4$ used in other work. The value $\tau_L/\tau_M = 3.5$ was used by us where it was not possible to separate the M line from the L line.

6. THE RATIO τ_K/τ_L

Our experimental data on the ratio τ_K/τ_L for various targets and $h\nu$ are collected in Table II.

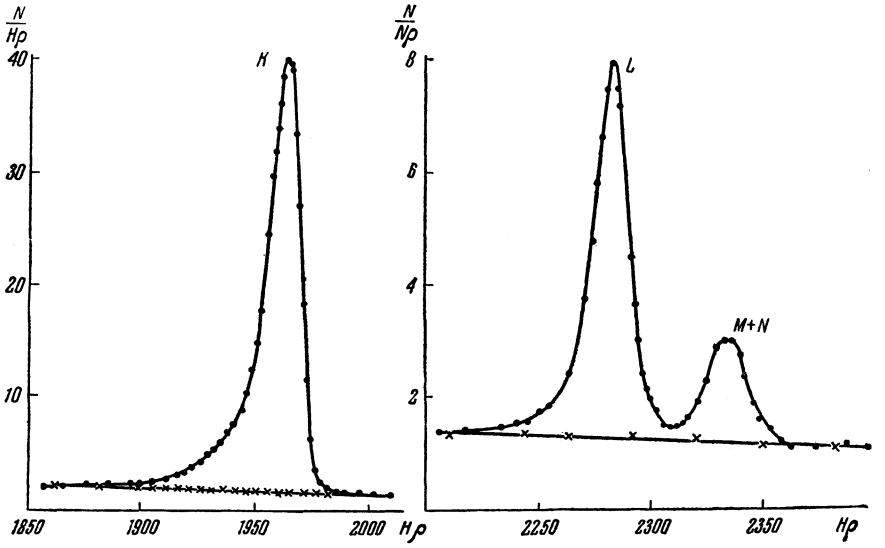

FIG. 4. K, L and M + N photoelectron peaks from the 364 kev J^{131} γ rays. The ampoule was of aluminum. A 3 mg/cm² Bi target was used.

TABLE II. Experimental values of τ_K/τ_L and of the proportion of the absorption coefficient coming from the K-shell to the total photoabsorption coefficient as a function of target material and γ-ray energy

Target	Target thickness, mg/cm²	$h\nu$, kev	τ_K/τ_L	τ_K/τ, %**
Ag	0.25	121	8.3±1.0	86.5±2.2
Ag	0.25	136	7.0±0.8	84.6±2.0
Ag	1.0	121	7.8±0.8	86,0±1.9
Ag	1.0	136	7.1±0.6	84,4±1.9
Ag	1.0	265	9.0±0.9	87.4±1.8
Ag	1.0	401	8.8±1.0	87.2±2.1
Ag	3.0	280	10.2±1.0	90.7±1.3
Sb	1.0	280	9.3±0.3	88.5±0.9
Pt	3.0	603	5.8±0.5	81.8±2.0
Au	7.7	298	5.6±1.0	81.2±4.0
Au	7.7	878	5.7±1.0	81.5±4.0
Au	7.7	967	5.9±1.0	82.1±4.0
Au	7.7	1180	5,1 ±1.0	79.8±4.0
Pb	13	549	4.7 ±0.2*	82.5±0,8
Pb	13	1696	5.1 ±0.3*	83.8±1.0
Bi	0.1	121	4.7 ±0.5	78.3±3.0
Bi	0.1	136	5.0 ±0.4	79.6±2.0
Bi	0.1	265	4.9 ±0.5	79.1±2.6
Bi	3	364	5.9 ±0.2	82.2±0.4
Bi	3	637	6.0 ±0.2	82.4±0.7
Th	3	411	5.2 ±0.6	80.2±2.5
Th	3	513	4.85±0,1*	83.0±0,4
Th	3	603	6.9 ±1.2	84.4±3.1

* – More accurate values of the ratio $\tau_K/\tau_L + \tau_M$ are given.

** – Here τ_L/τ_M has been taken as 3.5

Figure 4 shows the results.

From the experimental results, the dependence of τ_K/τ_L on Z was constructed (Fig. 5). The mean value of τ_K/τ_L for given Z and for energies greater than 200 kev is plotted along the ordinate. The growth in this ratio with decreasing Z shows up very clearly. For comparison, a curve for τ_K/τ_{L_I} obtained from Hall's relativistic formula[6] is given. For the ratio $\tau_K/\tau_{L_I} + \tau_{L_{II}} + \tau_{L_{III}}$ this curve would be somewhat lower because the contribution of the L_{II} and L_{III} subshells to the photoeffect is not very large. The experimental points begin to deviate strongly from the curve in the region Z ~ 50. In Fig. 6 we show the dependence of τ_K/τ_L on γ-ray energy for a group of heavy elements of nearly equal Z, and a comparison with values calculated from both nonrelativistic and relativistic formulae is given. The latter formula predicts the ratio τ_K/τ_{L_I} to be independent of energy. In the region of low γ-ray energy, the calculated ratio $\tau_K/\tau_{L_I+L_{II}+L_{III}}$ should decrease somewhat on account of the contribution of the L_{II} and L_{III} subshells, which grows with decreasing energy. The experimental points lie between the two calculated curves, and not along either. A similar picture is obtained for silver (Z = 47) and antimony (Z = 51) (see Fig. 7). In both this case and the former one, the ratio τ_K/τ_L is noticeably lower in the energy region 100 — 200 kev; this effect cannot be explained by experimental error. Thus, it is established that the experimental ratio τ_K/τ_L disagrees with the theoretical one both in

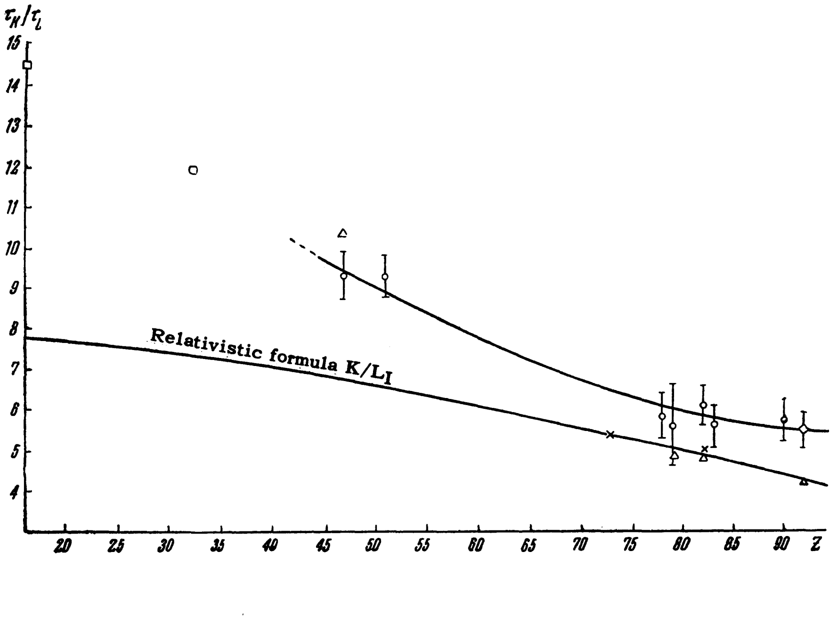

FIG. 5. τ_K/τ_L as a function of Z. The points represent mean values for γ rays of energies greater than 200 kev; o – our results, × – results of Latyshev[10] with $h\nu$ = 2614 kev, □ – results of Bazin[13] with $h\nu$ = 17.5 kev, Δ – results of Marty[8] with $h\nu$ = 411 kev, ◊ – results of Navakov et al.[14] with $h\nu$ = 516 and 880 kev. In the results of Marty[8] and Bazin[13] corrections have been introduced for the M shell. The values of τ_K/τ_L in the work of Bazin may change at higher energies.

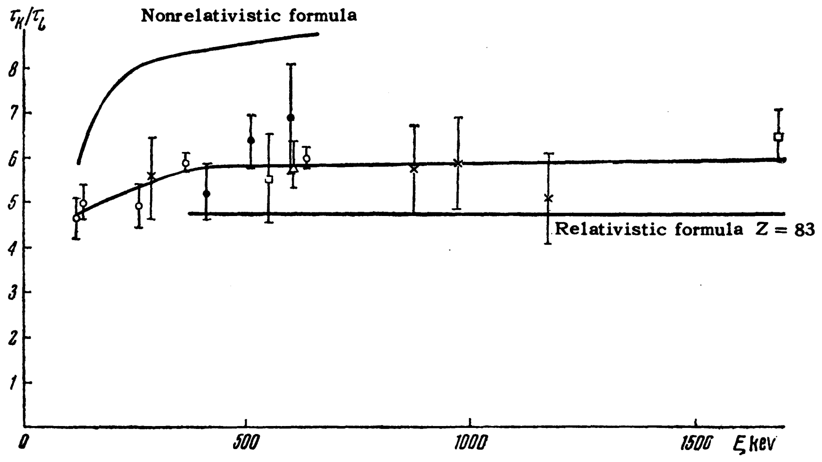

FIG. 6. τ_K/τ_L as a function of γ-ray energy for targets made out of heavy elements: ● – 3 mg/cm^2 Th, ○ – 0.5 mg/cm^2 Bi, □ – 13 mg/cm^2 Pb, × – 7.7 mg/cm^2 Au, Δ – 3 mg/cm^2 Pt.

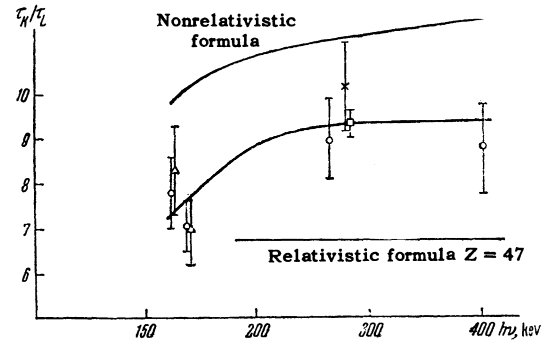

FIG. 7. τ_K/τ_L as a function of γ ray energy for silver ($Z = 47$) and antimony ($Z = 51$) targets. Δ – 0.25 mg/cm^2 Ag, ○ – 1 mg/cm^2 Ag, × – 3 mg/cm^2 Ag, □ – 0.3 mg/cm^2 Sb.

the case of relativistic and in the case of nonrelativistic calculations.

7. THE SHARE OF τ_K IN THE TOTAL ABSORPTION COEFFICIENT τ

From the experimental data given in Table II, and use of the ratio $\tau_L/\tau_M = 3.5$ and the curve given in Fig. 5, we can calculate the share of the K-shell absorption coefficient τ_K in the total absorption coefficient. This ratio is shown in Fig. 8 as a function of Z, for energies greater than 200 kev. The points on the graph are mean values for a given Z. The smooth curve drawn through the experimental points is close to the curve calculated from Allen's formula[20]

$$\frac{\tau_K}{\tau} = \frac{100}{1.13-[(Z-1)/81]\cdot 0.07}$$

in the region of high Z, but diverges noticeably from it in the region $Z \sim 50$.

The results obtained in this work may be useful in the study of relative γ-ray intensities using photoelectrons. This method is used increasingly widely in nuclear spectroscopy.

In conclusion, the authors would like to express their gratitude to those colleagues who took part in different stages of the work. The authors are sincerely grateful to Prof. B. S. Dzhelepov for valuable remarks, made in discussions of the present articles. The authors thank N. A. Bonch-Osmolovskiĭ for the possibility of seeing his survey paper on the photoeffect before publication.

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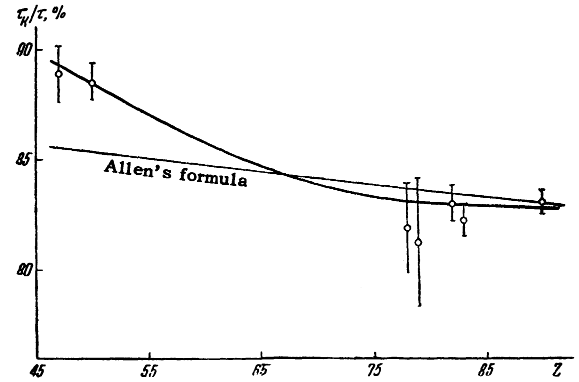

FIG. 8. Absorption coefficients for the K shell τ_K in percent of the total absorption coefficient τ as a function of Z.

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