Element	$K(\varepsilon)$	$F(\varepsilon)$
Ca^{48}	$2.1 \cdot 10^3$	$2.0 \cdot 10^3$
Zr^{96}	$4.8 \cdot 10^2$	$4.4 \cdot 10^2$
Te^{130}	$2.7 \cdot 10^2$	$2.4 \cdot 10^2$
Cd^{116}	$3.2 \cdot 10$	$2.8 \cdot 10$
Sn^{124}	3.5	2.8

latest data on the β decay of neutrons[9] evidently show that in β decay the V and A interaction types are realized; the precision of the experiment does not, however, exclude the possibility that there may also be present small amounts of the S and T types. Therefore it is of interest to calculate the probability of double β decay also in this scheme (transitions $0^+ \rightarrow 1^- \rightarrow 0^+$). We have obtained the following expression for the half-value period

$$T_{1/2} \leqslant 2 \cdot 10^{15} (ft)^2 Z^2 / (|I|^2 + |J|^2) F(\varepsilon) \text{ sec}, \qquad (3)$$

where $F(\epsilon)$ is a function that depends on the energy ϵ of the transition (see table). A large uncertainty in the value of $T_{1/2}$ is introduced by the quantity ft, which for the transitions in question can neither be calculated theoretically nor satisfactorily estimated from the experimental data. Making the usual assumption that $ft = 10^7$ for first-forbidden transitions,[10] and supposing that $|C_S/C_A| = 0.1$ (or $|C_T/C_V| = 0.1$), we get for Ca^{48} the value $T_{1/2} = 2 \times 10^{22}$ years.

The results of our calculations show that at the present time the question of the existence of neutrinoless double β decay cannot be regarded as finally settled by the work of Dobrokhov and others[1] and the search for this effect for the purpose of establishing higher values of the lower limit on $T_{1/2}$ is of real interest for the theory.

In conclusion the writer expresses his deep gratitude to I. S. Shapiro for the suggestion of this topic and help in studying it.

***The function $f(\varepsilon)$ used in references 2, 5, and 6 differs from our $K(\varepsilon)$ by the fact that in obtaining it account was taken of the energy dependence of the Coulomb correction factors. In references 5 and 6 the Coulomb factors are regarded as independent of the energy, but the writers use $f(\varepsilon)$ instead of $K(\varepsilon)$.**

†In Eqs. (2) and (3) the β-decay constant is included in ft, so that $0 < |C| \leq 1$.

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PAIR PRODUCTION BY A CIRCULARLY POLARIZED PHOTON

I. M. NADZHAFOV

Moscow State University

Submitted to JETP editor October 14, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 337-339 (January, 1959)

THE theoretical prediction and experimental detection of longitudinally polarized electrons and positrons in the β decay of non-oriented nuclei has heightened interest in the problem of bremsstrahlung and electron-positron pair production, taking into account the polarization properties of the particles participating in these processes. Since the bremsstrahlung of a polarized electron has already been examined in references 1 to 4, we confine our investigation here to the pair production process. The bremsstrahlung and pair production process in the ultrarelativistic case was considered in reference 5.

In the present work, we investigate pair production by a circularly polarized photon in the field of a point nucleus. The treatment is general, and is suitable for all values of angles and energies.

In the Born approximation, the effective cross section for pair production is given by the formula

$$d\sigma_p(\theta_+, \theta_-)\, d\Omega_+ d\Omega_- = \frac{Z^2}{\pi^2} \left(\frac{e^2}{c\hbar}\right)^3 \frac{K_+ K_- k_+ k_- dK_+}{\varkappa \varkappa'^4} (S^+S)_p\, d\Omega_+ d\Omega_-, \qquad (1)$$

where $(S^+S)_p$ is the square of the matrix element for pair production, $E_\pm = c\hbar K_\pm = c\hbar\sqrt{k_0^2 + k_\pm^2}$, $\hbar \mathbf{k}_\pm$ are the total energies and momenta of the positron and the electron, $\epsilon_{ph} = c\hbar\kappa = E_+ + E_-$, $\hbar\boldsymbol{\kappa}$ are the energy and momentum of the photon, $\hbar\boldsymbol{\kappa}' = \hbar\boldsymbol{\kappa} - \hbar\mathbf{k}_+$

$-\hbar\mathbf{k}_-$ is the momentum transferred to the nucleus, and $d\Omega_\pm$ are the solid angles of the emerging positron and electron.

Taking the spins of the photon, positron, and electron into account we obtain the following expression for $(S^+S)_p$:

$$(S^+S)_p = (4K_+K_-\varkappa^2)^{-1}[F_{p0}(\theta_+,\theta_-)$$

$$+ s_+s_-F_{p1}(\theta_+,\theta_-) - ls_+F_{p2}(\theta_+,\theta_-) - ls_-F_{p3}(\theta_+,\theta_-)], \quad (2)$$

where

$$F_{p0}(\theta_+,\theta_-) = -(K_-^2 - \varkappa'^2/4)k_+^2\sin^2\theta_+/\Delta_+^2$$
$$-(K_+^2 - \varkappa'^2/4)k_-^2\sin^2\theta_-/\Delta_-^2 + (\varkappa^2/2 - K_+K_- - \varkappa'^2/4)$$
$$\times 2\delta/\Delta_+\Delta_- + (k_+^2\sin^2\theta_+ + k_-^2\sin^2\theta_-)\varkappa^2/2\Delta_+\Delta_-,$$

$$F_{p1}(\theta_+,\theta_-) = -(2k_+k_-)^{-1}\{[2(k_0^2 - K_+K_-)(K_-^2 - \varkappa'^2/4)$$
$$- k_0^2(3K_-^2 + K_+^2)]k_+^2\sin^2\theta_+/\Delta_+^2$$
$$+ [2(k_0^2 - K_+K_-)(K_+^2 - \varkappa'^2/4)$$
$$- k_0^2(3K_+^2 + K_-^2)]k_-^2\sin^2\theta_-/\Delta_-^2 - 2[2(k_0^2 - K_+K_-)(\varkappa^2/2$$
$$- K_+K_- - \varkappa'^2/4) - k_0^2(K_- - K_+)^2]\delta/\Delta_+\Delta_-$$
$$+ \varkappa k_0^2(K_- - K_+)\delta(\Delta_+^{-2} - \Delta_-^{-2}) + 4k_0^2\varkappa^2K_+K_-/\Delta_+\Delta_-$$
$$+ \varkappa^2K_+K_-(k_+^2\sin^2\theta_+ + k_-^2\sin^2\theta_-)/\Delta_+\Delta_-$$
$$+ \varkappa k_0^2(K_- - K_+)(k_+^2\sin^2\theta_+ - k_-^2\sin^2\theta_-)/\Delta_+\Delta_-$$
$$- \varkappa^2k_0^2(K_-/\Delta_- + K_+/\Delta_+) - \varkappa^2k_0^2(K_-(K_- + \boldsymbol{\varkappa}^0\mathbf{k}_-)/\Delta_+^2$$
$$+ K_+(K_+ + \boldsymbol{\varkappa}^0\cdot\mathbf{k}_+)/\Delta_-^2)\},$$

$$F_{p2}(\theta_+,\theta_-) = -(2k_+)^{-1}$$
$$\times\{(K_+\boldsymbol{\varkappa}\cdot\mathbf{k}_- - K_+\boldsymbol{\varkappa}\cdot\mathbf{k}_+ + K_+\varkappa^2 - \varkappa k_0^2)k_+^2\sin^2\theta_+/\Delta_+^2$$
$$+ (K_+\boldsymbol{\varkappa}\cdot\mathbf{k}_- - K_+\boldsymbol{\varkappa}\cdot\mathbf{k}_+ - K_+\varkappa^2 + \varkappa k_0^2)k_-^2\sin^2\theta_-/\Delta_-^2$$
$$- 2K_+(\boldsymbol{\varkappa}\cdot\mathbf{k}_- - \boldsymbol{\varkappa}\cdot\mathbf{k}_+)\delta/\Delta_+\Delta_- + \varkappa(K_+\varkappa - k_0^2)(k_+^2\sin^2\theta_+$$
$$- k_-^2\sin^2\theta_-)/\Delta_+\Delta_- - \varkappa k_0^2\delta(\Delta_+^{-2} - \Delta_-^{-2}) + \varkappa^2k_0^2(k_-^2\sin^2\theta_-$$
$$- k_+^2\sin^2\theta_+)/\Delta_+^2\Delta_-\},$$

$$F_{p3}(\theta_+,\theta_-) = (2k_-)^{-1}\{(K_-\boldsymbol{\varkappa}\cdot\mathbf{k}_- - K_-\boldsymbol{\varkappa}\cdot\mathbf{k}_+$$
$$+ K_-\varkappa^2 - \varkappa k_0^2)k_+^2\sin^2\theta_+/\Delta_+^2 + (K_-\boldsymbol{\varkappa}\cdot\mathbf{k}_- - K_-\boldsymbol{\varkappa}\cdot\mathbf{k}_+$$
$$- K_-\varkappa^2 + \varkappa k_0^2)k_-^2\sin^2\theta_-/\Delta_-^2 - 2K_-(\boldsymbol{\varkappa}\cdot\mathbf{k}_- - \boldsymbol{\varkappa}\cdot\mathbf{k}_+)\delta/\Delta_+\Delta_-$$
$$- \varkappa k_0^2\delta(\Delta_+^{-2} - \Delta_-^{-2})$$
$$+ \varkappa(K_-\varkappa - k_0^2)(k_+^2\sin^2\theta_+ - k_-^2\sin^2\theta_-)/\Delta_+\Delta_-$$
$$+ \varkappa^2k_0^2(k_-^2\sin^2\theta_- - k_+^2\sin^2\theta_+)/\Delta_+\Delta_-^2\},$$
$$\cos\theta_\pm = \boldsymbol{\varkappa}^0\cdot\mathbf{k}_\pm/k_\pm, \quad \Delta_\pm = K_\pm - k_\pm\cos\theta_\pm,$$
$$\delta = k_+k_-\sin\theta_+\sin\theta_-\cos(\varphi_+ - \varphi_-), \quad \boldsymbol{\varkappa}^0 = \boldsymbol{\varkappa}/\varkappa. \quad (3)$$

The quantities l, s_+, $s_- = \pm 1$ in Eq. (2) determine the projections of the spins of the photon (in units of $\hbar$), the positron and the electron (in units of $\hbar/2$), respectively, in the directions of their motion.

Integrating the effective cross section over the angle of emergence θ_- of the electron, we obtain

$$d\sigma_p(\theta_+)d\Omega_+ = C_p[Q_{p0}(\theta_+) + s_+s_-Q_{p1}(\theta_+)$$
$$- ls_+Q_{p2}(\theta_+) - ls_-Q_{p3}(\theta_+)]d\Omega_+, \quad (4)$$

where

$$Q_{pi}(\theta_+) = \int(F_{pi}(\theta_+,\theta_-)d\Omega_-)/\varkappa'^4, \; i = 0, 1, 2, 3,$$

can be expressed in terms of elementary functions (which we do not present here), and

$$C_p = Z^2(e^2/c\hbar)^3k_+k_-dK_+/4\pi^2\varkappa^3.$$

After integrating Eq. (4) over θ_+, we obtain

$$d\sigma_p = C_p[G_{p0} + s_+s_-G_{p1} - ls_+G_{p2} - ls_-G_{p3}],$$
$$G_{pi} = \int Q_{pi}(\theta_+)d\Omega_+, \; i = 0, 1, 2, 3. \quad (5)$$

for the effective cross section.

In the general case, the expressions for G_{pi} are quite unwieldy. However, in the ultra-relativistic case, they are substantially simplified:

$$G_{p0} = \eta_p(2K_+K_- + 3K_+^2 + 3K_-^2), \quad G_{p1} = -\eta_p(K_+ - K_-)^2,$$
$$G_{p2} = -\eta_p\varkappa(3K_+ - K_-), \quad G_{p3} = -\eta_p\varkappa(3K_- - K_+),$$
$$\eta_p = 2(L_p - 1)\pi^2/3K_+K_-k_0^2, \quad L_p = 2\ln(2E_+E_-/\varepsilon_{ph}mc^2). \quad (6)$$

The result obtained by McVoy and Dyson[5] follows from Eqs. (5) and (6) as a special case.

In the case of complete screening, the quantity $L_p - 1$ in Eq. (6) is replaced by $2\ln(183Z^{-1/3})$ (to within $2/9$).

The degree of polarization of the pair can be determined as follows:

$$P_1(\theta_+) = \frac{d\sigma_p(\theta_+)_{\uparrow\uparrow} - d\sigma_p(\theta_+)_{\downarrow\downarrow}}{d\sigma_p(\theta_+)_{\uparrow\uparrow} + d\sigma_p(\theta_+)_{\downarrow\downarrow}},$$

$$P_2(\theta_+) = \frac{d\sigma_p(\theta_+)_{\uparrow\downarrow} - d\sigma_p(\theta_+)_{\downarrow\uparrow}}{d\sigma_p(\theta_+)_{\uparrow\downarrow} + d\sigma_p(\theta_+)_{\downarrow\uparrow}}, \quad (7)$$

where the first and second of the symbols ↑ and ↓ correspond to the values +1 and −1 of the spins s_+ and s_-. In the general case, Eqs. (4), (5), and (7) determine the angular and energy dependence of the effective cross section and the degree of polarization of the pair.

Recently, Olsen and Maximon investigated the process of bremsstrahlung and pair production in the ultra-relativistic case, taking Coulomb corrections and screening into account[6] (the effective cross section was averaged over the final spin states of the electron). It turned out that these corrections have no significant influence on the degree of polarization.

In conclusion, I would like to thank Prof. A. A. Sokolov for guidance, and B. K. Kerimov for discussion of the results.

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Translated by D. Lieberman
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RADIATION FROM A SPIN-2 PARTICLE MOVING UNIFORMLY IN A MEDIUM

A. I. BEDRITSKIĬ

Vitebsk Pedagogical Institute

Submitted to JETP editor October 13, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 339-341 (January, 1959)

THE energy radiated from a particle with spin 2, moving uniformly in a medium with a velocity higher than the phase velocity of light in that medium (Cerenkov effect), can be determined in analogy to the phenomenological theory of this effect for the electron (see reference 1, §32).

The free field operator for a particle with spin 2 has, according to the general, relativistically-covariant equations of first order,[2] the form

$$D = i\hbar\gamma_4\partial / \partial t - \hbar c\,(\gamma\nabla) - mc^2, \tag{1}$$

where the matrices γ, of dimension thirty, are known from references 3 and 4. The interaction of charged particles with the electromagnetic field is obtained by changing the operator $\partial/\partial x_k$ to the operator $\partial/\partial x_k - (ie/\hbar c) A_k$. The field of virtual photons interacting with the particle moving in a dielectric with a refraction coefficient $n = c/c'$ is obtained from the fundamental formula (1) in analogy to the theory for the electron.[1] The result is

$$w^+(\mathbf{r}, t) = -ieL^{-3/2}\sum_{\mathbf{K}}\sqrt{2\pi c'\hbar / K}\,(\boldsymbol{\gamma}\cdot\mathbf{a}^+) \times \exp(ic'K_0t - i\mathbf{K}\cdot\mathbf{r}), \tag{2}$$

where e is the charge of the particle, K_1, K_2, K_3, and $K_0 = K = \sqrt{K_1^2 + K_2^2 + K_3^2}$ form the four-dimensional wave vector of the photon, $\mathbf{a}^+(a_1^+, a_2^+, a_3^+)$ are the amplitudes of the photon field acting on a function of the number of photons included in the wave function. The a_n^+ satisfy the commutation relations

$$a_{n'}^+ a_n = 0, \quad a_n a_{n'}^+ = \delta_{nn'} - K_n K_{n'} / K^2 \tag{3}$$

and the condition of transversality $(\mathbf{K}\cdot\mathbf{a}^+) = 0$.

The probability for the radiation process can be determined by considerations similar to those of the electron case. The energy W radiated from the particle per unit time is

$$W = e^2c\int_0^{\omega_m}\frac{k_0'K^2}{nk}G\,dK, \quad G = \frac{1}{5Kn}\sum_{s,s'}\frac{b^+(\boldsymbol{\gamma}^+\mathbf{a})\gamma_4^+b'\cdot b'^+\gamma_4(\boldsymbol{\gamma}\mathbf{a}^+)b}{b'^+\gamma_4^+b'\cdot b'^+\gamma_4 b'}, \tag{4}$$

where b and b′ are thirty-component functions of the wave vectors k_e and k_e'; the primes refer to the state of the particle after radiation; ω_m is the maximal radiation frequency, which depends on the momentum of the particle according to the formula

$$\omega_m = \frac{2pc}{n\hbar}\,\frac{1 - 1/n\beta}{1 - n^{-2}}, \tag{5}$$

the direction of the radiation is determined by

$$\cos\theta = 1/n\beta + (K/2k)(1 - n^{-2}), \tag{6}$$

$\beta = v/c$, where v is the velocity of the particle. The conservation laws are fulfilled: $\mathbf{k}' = \mathbf{k} - \mathbf{K}$, $k_0' = k_0 - K/n$. The frequency of the light radiated from the particle is equal to $\omega = cK/n$.

The classification of the wave functions according to the projection of the spin on the momentum of the particle and the normalization with respect to the charge $\psi^*A\gamma_4 = 1$ in the calculation of the quantity G were carried out with the help of the covariant method proposed by Fedorov.[5]

In the general case, as well as in the nonrelativistic approximation, we were faced with exceedingly complex calculations, which we were unable to master. The comparatively simple calculation in the extreme relativistic case leads, with (4), to

$$W = \frac{e^2E^2}{5120\,cE_0^4}\int_0^{\omega_m}\omega\Big\{32\frac{(E - n\hbar\omega\cos\theta)^4}{(E - \hbar\omega)^2} + 32(E - n\hbar\omega\cos\theta)^2 + \frac{(\hbar\omega)^2\sin^2\theta}{E - \hbar\omega}(E - n\hbar\omega\cos\theta) + (E - \hbar\omega)(E - n\hbar\omega\cos\theta) + 4\cos^2\theta\left[\frac{n^6(\hbar\omega)^6\sin^6\theta}{(E - \hbar\omega)^4} + 4\frac{n^4(\hbar\omega)^4\sin^4\theta}{(E - \hbar\omega)^2} + 3n^2(\hbar\omega)^2\sin^2\theta\right] + 8\cos\theta\sin\theta\left[\frac{n^5(\hbar\omega)^5\sin^5\theta}{(E - \hbar\omega)^4}(E - n\hbar\omega\cos\theta) + 8\frac{n^3(\hbar\omega)^3\sin^3\theta}{E - \hbar\omega} + 3\frac{n^3(\hbar\omega)^3\sin^3\theta}{(E - \hbar\omega)^2}(E - n\hbar\omega\cos\theta)\right]\Big\}d\omega, \tag{7}$$