POLARIZATION CORRELATION IN COULOMB SCATTERING OF ELECTRONS AND MUONS BY LIGHT NUCLEI INCLUDING RADIATIVE CORRECTIONS

I. G. IVANTER

Scientific Information Institute, Academy of Sciences, U.S.S.R.

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THE general expression for the Coulomb scattering cross section of electrons with prescribed directions for the spin and momentum before and after scattering, and with neglect of radiative corrections, was given by Tolhoek.[1] However that work contains an error. Some results were also obtained by Passatore.[2]

We make use of the projection operators method developed by Tolhoek, de Groot, Fano, and Michel.[1]

If lowest order radiative corrections are taken into account one must include the second Born approximation in the calculations. The second Born approximation for this problem was first considered by Mott[3] in connection with the question of asymmetry in double scattering. Dalitz[4] has given the matrix element for the second Born approximation in the Feynman form.

The matrix element for the transition, including the second Born approximation and radiative corrections, has the form

$$M = A\gamma_4 - iB\beta\hat{q} + C. \tag{1}$$

A, B, and C contain no Dirac matrices; $\hat{q} \equiv \hat{p}_2 - \hat{p}_1$, where p_1 and p_2 are the four-momenta of the particle before and after scattering. Further $A = A_0 + A_1$ where $A_0 = 2\pi i Ze^2/|\mathbf{q}|^2$. Terms bilinear in the small quantities A_1, B, and C may be neglected in the matrix element squared.[5]

For the cross section for pure elastic scattering we find

$$d\sigma_s = \frac{1}{2\pi v}\rho_f\frac{do}{4\varepsilon^2}\frac{1}{4}\mathrm{Sp}\,\{(A\gamma_4 - iB\gamma_4\hat{q} + C)(1 + i\hat{S}_1\gamma_5)(\hat{p}_1 + m) \times (A^*\gamma_4 + iB^*\hat{q}\gamma_4 + C^*)(1 + i\hat{S}_2)(\hat{p}_2 + m)\}. \tag{2}$$

where v is the velocity and ϵ the energy of the particle, and S_1 and S_2 are Stokes' four-vectors; the indices 1 and 2 refer to before and after scattering respectively. Expression (2) can be written as the sum of four terms

$$d\sigma_s = {}^1\!/_2\overline{d\sigma_s} + {}^1\!/_2 d\sigma_s(1) + d\sigma_s(2) + d\sigma_s(1,\ 2). \tag{3}$$

Here $\overline{d\sigma_S}$ is the cross section averaged over the polarizations of the incident particle and summed over the polarizations of the scattered particle. The ratio $d\sigma_S(1)/\overline{d\sigma_S}$ gives the asymmetry in the scattering of polarized particles, the ratio $2d\sigma_S(2)/\overline{d\sigma_S}$ gives the degree of polarization in the scattering of unpolarized particles. The ratio $2d\sigma_S(1,2)/\overline{d\sigma_S}$ corresponds to the "degree of polarization correlation" of the scattered and incident particles, or in other words it describes the spin rotation. Mott[3] obtained expressions for $d\sigma_S(1)$ and $d\sigma_S(2)$. Lowest order radiative corrections do not contribute to $d\sigma_S(1)$ and $d\sigma_S(2)$ (i.e., they do not affect the asymmetry in the scattering of polarized particles, nor the polarization in the scattering of unpolarized particles).

For pure elastic scattering we find for $d\sigma_S(1,2)$:*

$$d\sigma_s(1,\ 2) = \frac{1}{2\pi v}\frac{\rho_f do}{4\varepsilon^2}\{[\,|A_0|^2 + 2\mathrm{Re}\,(A_0A_1^*) + 4\mathrm{Im}\,(A_0B^*)\,m] \times (-1)F_1 + [2\,\mathrm{Re}\,(A_0C^*) + 4\,\mathrm{Im}\,(A_0B^*)\varepsilon]F_2\}. \tag{4}$$

Here

$$F_1 \equiv -(\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_2\cdot\mathbf{J}_2) + [(\varepsilon - m)/(\varepsilon + m)][(\mathbf{p}_2\cdot\mathbf{J}_2)(\mathbf{p}_2\cdot\mathbf{J}_1) + (\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_1\cdot\mathbf{J}_2)] + (\mathbf{J}_1\cdot\mathbf{J}_2)(-m^2 - \varepsilon^2) - (\mathbf{J}_1\cdot\mathbf{J}_2)(\mathbf{p}_1\cdot\mathbf{p}_2) + (\mathbf{p}_2\cdot\mathbf{J}_1)(\mathbf{p}_1\cdot\mathbf{J}_2) - [2/(\varepsilon + m)^2](\mathbf{p}_2\cdot\mathbf{J}_2)(\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_1\cdot\mathbf{p}_2);$$

$$F_2 \equiv [(\varepsilon - m)/(\varepsilon + m)][(\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_1\cdot\mathbf{J}_2) + (\mathbf{p}_2\cdot\mathbf{J}_2)(\mathbf{p}_2\cdot\mathbf{J}_1)] + 2\varepsilon m(\mathbf{J}_1\cdot\mathbf{J}_2) - [2/(\varepsilon + m)^2](\mathbf{p}_2\cdot\mathbf{J}_2)(\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_1\cdot\mathbf{p}_2). \tag{4a}$$

$\mathbf{J}_1$, $\mathbf{J}_2$ are the spin directions of the particles in their rest system before and after scattering. The total differential cross section, which includes the emission of long wave length photons accompanying scattering with an energy change $\Delta\epsilon$, is of the same form as (4) with A_1 replaced by some A_1'. The expressions for A, B, and C are given in reference 5.

The expression for F_1 assumes a simple form if longitudinal polarization is considered:

$$F_{1\,\mathrm{long}} \equiv F_{1a} \sim [(\varepsilon^4 + m^4)\cos\theta + \varepsilon^4 + m^4 - 2\varepsilon^2m^2]/(\varepsilon^2 - m^2).$$

It follows that in scattering through an angle $\theta_0 = \cos^{-1}[(\epsilon^2 - m^2)^2/(\epsilon^4 + m^4)$, F_{1a} vanishes but F_{2a} does not. Thus it may be expected that the measurement of residual longitudinal polarization after scattering through an angle θ_0 will provide the best experiment for the study of radiative corrections and the second Born approximation. It should be noted that the contribution to F_{2a} from the second Born approximation decreases with increasing energy. Thus, at $\epsilon = 4m$ the ratio of the contributions from the second Born approximation and the radiative corrections is already Z/10.6.

At low energies, on the other hand, the contribution from the second Born approximation is dominant.

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*The error in reference 1 consists of having the sum $[(\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_2\cdot\mathbf{J}_2) + (\mathbf{p}_1\cdot\mathbf{J}_2)(\mathbf{p}_2\cdot\mathbf{J}_1)]\, \mathbf{p}_2\cdot\mathbf{p}_1$ instead of $2(\mathbf{p}_2\cdot\mathbf{J}_2) \times (\mathbf{p}_1\cdot\mathbf{J}_1)(\mathbf{p}_2\cdot\mathbf{p}_1)$. It is easy to see that if $\hat{S}_1$, $\hat{S}_2$, $\hat{p}_1$ and $\hat{p}_2$ enter the square of the matrix element as factors only once it is impossible to obtain the second term of the sum.

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K-LEVEL X-RAYS FROM FISSION FRAGMENTS AND DISTRIBUTION OF FRAGMENTS BY CHARGES

V. V. SKLYAREVSKII, E. P. STEPANOV, and B. A. MEDVEDEV

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WE have observed earlier[1] that the 30-kev line appearing in the gamma-ray spectrum of U^{235} fission is not monochromatic; it has been supposed that this line represents K-level x-rays from heavy fission fragments with different Z, arising apparently in the internal conversion of the harder gamma rays due to fission. We report here the results of new measurements of the spectrum of soft gamma rays from the fission of U^{235}; these data have been used to estimate the widths of the distributions of the light and heavy fragments by charges.

The experimental setup consisted of an ionization chamber with a layer of U^{235} placed in the thermal-neutron beam of the RFT reactor of the U.S.S.R. Academy of Sciences, and a xenon proportional counter for the gamma rays. The spectra of the gamma-ray pulses coincident with the fission-fragment pulses were measured with an ELA-2 amplitude analyzer.[2]

The diagram shows the gamma-ray spectrum of U^{235} fission in the 10 — 50 kev range. Curve A is the spectrum of all the γ-f coincidences, curve C the random-coincidence spectrum, and curve B the spectrum of true coincidences from hard gammas and fission neutrons. This curve was obtained by measuring the spectrum of the γ-f coincidences with an 180-μ lead absorber placed between the chamber and counter. Curve D was obtained by subtracting B and C from A.

Calibration was with the 32.2-kev barium K line, emitted from a Cs^{137} source, and the Np 17.7-kev L_β line emitted from an Am^{241} source. The Cs^{137} and Am^{241} sources were thin layers of the substance, deposited on aluminum disks. During the measurements these disks were placed in the same position relative to the counter as the U^{235} layer. The half-widths of the calibration 17.7 and 32.2 kev lines were 20 and 14% respectively.

As seen from the diagram, the spectrum contains two intense non-monochromatic lines, whose maxima correspond to 16 ± 1 and 31 ± 1.5 kev, and whose half-widths are 35 and 22%, respectively. There is no doubt that these lines represent the K-level x-rays from the fragments of the light and heavy groups.

It is obvious that the energy distribution of the K-level x-rays of the fragments, W(E), is connected with the charge distribution of the fragments, W(Z). It is therefore possible to estimate the width of W(Z) from the results obtained. For such an estimate we can assume that the distributions W(E) and W(Z) are Gaussian with total widths at half height δ_E and δ_Z. Then $\delta_Z = \delta_E dZ/dE$ and, since $Z \approx 10\,E^{1/2}(\text{kev}) + 1$, where E is the energy of the K-level x-rays from an atom with