In that case the quantity δ no longer plays the role of the penetration depth, but this quantity goes over into the effective dielectric constant for low frequencies and determines the magnetic susceptibility. Since the connection between $\mathbf{j}$ and $\mathbf{A}$ is local, ϵ and χ, expressed in terms of δ, have the usual, London form:

$$\varepsilon = -c^2/\omega^2\delta^2, \tag{10}$$

$$\chi = \frac{1}{4\pi}\left(1 - \frac{2\delta}{d}\tanh\frac{d}{2\delta}\right) \approx -\frac{1}{12\pi}\left(\frac{d}{2\delta}\right)^2. \tag{11}$$

In conclusion the authors express their gratitude to Academician L. D. Landau for discussions of this paper.

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PARTICLE ENERGY LOSSES DUE TO THE EXCITATION OF LONGITUDINAL WAVES

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IT has been pointed out[1-3] that longitudinal plane waves are excited if a medium exhibits spatial dispersion; Ginzburg has noted[1] that a very convenient means for the excitation of these new waves may be the Cerenkov effect.

In this connection it is of interest to develop the theory of the Cerenkov effect as applied to the case in which longitudinal electromagnetic waves are excited by an electron. We consider an isotropic medium. We start by noting that one cannot use the usual Lagrangian $L = (\epsilon E^2 - H^2)/8\pi$ because the energy density of the field, given by the expression $(\epsilon E^2 + H^2)/8\pi$, vanishes for longitudinal waves (both $\mathbf{H}$ and ϵ vanish).

In order to avoid this difficulty we start with the Fock-Podolsky Lagrangian,[4] making the necessary extension to the case of an isotropic madium. We have

$$L = \left\{(n\mathbf{E})^2 - \mathbf{H}^2 - \left(\operatorname{div}\mathbf{A} + \frac{n^2}{c}\frac{\partial\varphi}{\partial t}\right)^2\right\}, \tag{1}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the intensities of the electric and magnetic fields respectively, $\mathbf{A}$ is the vector potential, φ is the scalar potential and n is the index of refraction (n is considered an operator, cf. reference 5).

Introducing the existence condition for a longitudinal field, $\operatorname{curl}\mathbf{A} = 0$, and the fact that there are no free charges, $(\varphi = 0)$, using the analysis in reference 1 we find the following expressions for $\mathbf{A}$ and the energy density respectively:

$$\{\Delta - (n^2\partial^2/c^2\partial t^2)\}\,\mathbf{A} = 0,$$
$$T_{44} = \{(\operatorname{div}\mathbf{A})^2 + (\partial n\mathbf{A}/c\partial t)^2\}/8\pi. \tag{2}$$

In order for Eq. (2) (vector potential) to be consistent with the field equation in the medium, the condition $\epsilon\mathbf{E} = n^2\{\mathbf{E} - \kappa_0(\mathbf{E}\cdot\kappa_0)\}$ must be satisfied; this follows from the analysis in reference 1.

Now, writing the solution of Eq. (2) in the form

$$A = L^{-3}\sum_{\varkappa}\sqrt{2\pi c\hbar/\varkappa n}\left\{\mathbf{a}\exp\left(-ic\frac{\varkappa}{n}t + i\varkappa\cdot\mathbf{r}\right) + \mathbf{a}^+\exp\left(ic\frac{\varkappa}{n}t - i\varkappa\cdot\mathbf{r}\right)\right\},$$

we find the energy of the longitudinal field:

$$H = \sum_{\varkappa}(c\hbar\varkappa/n)\,(\mathbf{a}^+\mathbf{a}), \tag{3}$$

where $\hbar\kappa$ and $c\hbar\kappa/n$ are the momentum and energy of the longitudinal photon. To satisfy the existence condition for longitudinal waves explicitly we write

$$\mathbf{a} = g\boldsymbol{\varkappa}_0, \quad \mathbf{a}^+ = g^+\boldsymbol{\varkappa}_0, \quad \boldsymbol{\varkappa}_0 = \boldsymbol{\varkappa}/\varkappa, \tag{4}$$

then, using Eq. (3) in the usual way (cf. reference 6), we find that the operators g and g^+ satisfy the Bose commutation relations. The phenomenological quantum electrodynamics developed above for the longitudinal field agrees in form with that given in reference 6 — the only differences lie in the meanings of the operators $\mathbf{a}$ and $\mathbf{a}^+$. Hence, in applying the analysis to the Cerenkov effect we can use the results obtained by Sokolov.[7,8] Substituting Eq. (4) in Eq. (8) of the paper by Sokolov and Loskutov[8] and assuming that there are no longitudinal photons at $t = 0$, after some elementary transformations we find the following expression for the energy radiated per unit length of path in the form of longitudinal waves:

$$W^{(1/2)} = \frac{e^2}{2c^2}\int_{\omega_1}^{\omega_2}\omega\left[2\cos^2\theta - \frac{\hbar\omega}{cp}(n\cos\theta + 1/\beta)\right]d\omega \qquad (5)$$

for a particle with half-integral spin and

$$W^{(0)} = \frac{e^2}{c^2}\int_{\omega_1}^{\omega_2}\omega\left(\cos^2\theta - \frac{\hbar\omega n}{cp}\cos\theta\right)d\omega \qquad (6)$$

for a particle of zero spin. Here

$$\cos\theta = 1/\beta n + (n\omega\hbar/2pc)(1 - n^{-2}) \qquad (7)$$

(**p** is the initial momentum of the electron, θ is the angle between **p** and the direction of photon emission). The integrations in Eqs. (5) and (6) are taken over the frequency regions for which the inequality $\cos\theta \leq 1$ is satisfied.

The following remarks may be made concerning the derivation of Eqs. (5) and (6): first, the quantum correction is proportional to $\hbar$ as in the case of the transverse field of longitudinally polarized electrons;[9] second, in the case of the longitudinal field there is no specific quantum correction proportional to $\hbar^2$ due to the electron spin in the transverse field. The latter situation leads one to believe that the indicated correction is due to the transverse field rather than the spin of the electron. There is at least one important difference from the case of the transverse field; in the classical analysis of Cerenkov radiation of transverse waves there is no radiation threshold; in the case of longitudinal waves, however, the radiation remains finite at the threshold, even in the classical approximation. Thus, neglecting terms of order $\hbar$ and higher in Eq. (5) we have

$$W^{(1/2)} = W^{(0)} = (e^2/2c^2)(\omega_2^2 - \omega_1^2). \qquad (8)$$

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COULOMB EXCITATION OF NEON

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IN studying Coulomb excitation of nuclei one can observe various excitations in the target nucleus. It is also possible to observe excitations in the bombarding particle if the latter exhibits levels for which the Coulomb excitation cross section is large. In most of the work on Coulomb excitation the bombarding particles have been protons or α particles. Inasmuch as H^1 and He^4 do not have suitable levels the effect noted above has not been observed. In certain cases, however, the use of heavy ions as bombarding particles makes it possible to observe the excitation of nuclear excitations in these particles.

The present authors have investigated Coulomb excitation in Ne^{20} and Ne^{22}. The first excited levels are at 1.63 and 1.275 Mev respectively. Coulomb excitation of these levels has still not been studied because the intensity of the γ rays produced when neon is bombarded by protons or α particles is low unless the latter have high energies, i.e., energies comparable with the potential barrier. For this reason the measurements are complicated by background effects. In the usual method, when the element being investigated serves as the target (if a thick gas target is used), it is difficult to measure the beam current. It is probably for this reason that in the work reported in references 1 and 2, concerning an investigation of Coulomb excitation of krypton and xenon, the authors were able to determine only the relative values of the γ yields.