SUPERCONDUCTING ALLOYS AT FINITE TEMPERATURES

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Submitted to JETP editor July 16, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 319-320 (January, 1959)

We have recently[1] formulated the electrodynamics of superconductors that contain small atomic concentrations of impurities at $T = 0$. The method of reference 1 is not applicable at finite temperatures.

The authors of the present paper obtained, together with I. E. Dzyaloshinskii, a generalization of the method applied at $T = 0$. It was basically founded upon the formulation of the thermodynamic theory proposed by Matsubara.[2] An exposition of this method will be given in a separate paper. We note here only that the evaluation of the basic functions at $T \neq 0$ is formally very similar to the calculations at $T = 0$. In the case of equilibrium practically the only change involved is the replacement of the integrals over the frequency by sums over the discrete variable

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega f(\omega) \rightarrow \frac{iT}{\hbar}\sum_{n=-\infty}^{\infty} f(i\omega_n), \tag{1}$$

where $\omega_n = (\pi T/\hbar)(2n + 1)$, and T is the temperature in energy units.

This method was applied by us to study the equilibrium properties of superconducting alloys at finite temperatures. All calculations are exactly the same as the evaluations performed[1] for $T = 0$ apart from the transformation (1).

The evaluation of the functions $G(x, x')$ and $F(x, x')$ leads to the conclusion that in alloys, as at $T = 0$, these functions are simply multiplied by an exponential factor

$$\begin{aligned} G(x\ x') &= G_0(x, x')\exp\{-|\mathbf{x}-\mathbf{x}'|/2l\}, \\ F(x, x') &= F_0(x, x')\exp\{-|\mathbf{x}-\mathbf{x}'|/2l\}, \end{aligned} \tag{2}$$

where l is the mean free path in the normal state.

To find the thermodynamic functions it is sufficient to determine the particle density $N(\mu, T)$ as a function of the chemical potential and temperature:

$$N = \langle\psi^+(x)\psi(x)\rangle = -i[G(x, x')]_{x=x';\, t=t'-0}. \tag{3}$$

One sees easily that the function $N(\mu, T)$ is the same as in the case of a pure superconductor. In the model under consideration, where one assumes that the impurities do not influence the interaction between the electrons, but only scatter them, the occurrence of impurities will therefore not change the thermodynamic functions. In particular, the critical temperature T_c is not shifted. We emphasize once again that this result is, of course, valid only if the impurity concentration is supposed to be small.

We have considered the behavior of alloys in a constant magnetic field. The connection between the current and the vector potential is local in the London case. At finite temperatures it is of the form

$$\mathbf{j} = -Q\mathbf{A}, \tag{4}$$

$$Q = \frac{Ne^2}{mc} 2\pi T\Delta^2 \sum_0^\infty 1/[(\hbar\omega_n)^2 + \Delta^2]\left(\sqrt{(\hbar\omega_n)^2 + \Delta^2} + \hbar/2\tau_{tr}\right) \tag{5}$$

where $\omega_n = (\pi T/\hbar)(2n+1)$; $\Delta(T)$ the temperature dependent "gap" occurring in the paper by Bardeen, Cooper, and Schrieffer;[3] $\tau_{tr} = [\int w(\theta)(1-\cos\theta)\, d\Omega]^{-1}$ is the transport collision time for the metal in its normal state, and N is the electron density.

From Eq. (4) together with the Maxwell equation one obtains the connection between Q and the penetration depth:

$$\delta = \sqrt{c/4\pi Q}. \tag{6}$$

In the limiting case of large free paths, $\Delta\tau_{tr}/\hbar \gg 1$, this formula goes over into the usual result for a pure London superconductor, i.e., a superconductor with $\delta \gg \hbar v/\Delta$:

$$\delta = \sqrt{mc^2/4\pi N_s e^2}, \tag{7}$$

where N_s is the number of superconducting electrons (see Ref. 3),

$$N_s/N_0 = 2\pi T\Delta^2 \sum_n [(\hbar\omega_n)^2 + \Delta^2]^{-3/2} = \frac{1}{4T}\int_\Delta^\infty \frac{\varepsilon\, d\varepsilon}{\cosh^2\frac{\varepsilon}{2T}\sqrt{\varepsilon^2 - \Delta^2}}. \tag{8}$$

In the opposite limiting case, $\Delta\tau_{tr}/\hbar \ll 1$,

$$\delta = (c/2\pi)[\hbar/\Delta\sigma \tanh(\Delta/2T)]^{1/2}, \tag{9}$$

where $\sigma = Ne^2\tau_{tr}/m$ is the conductivity of the alloy in the normal state.

It is well known that superconductors fall into two classes. For London superconductors $\delta > \hbar v/\Delta$. In that case the connection between Δ and $\hbar/\tau_{tr}$ can be arbitrary. In the case of the Pippard superconductors, for which $\delta < \hbar v/\Delta$, the condition $\delta \gg l$ leads to $\Delta\tau_{tr}/\hbar \ll 1$. Hence, only Eq. (9) refers to Pippard superconductors.

As was noted in reference 1, all formulae obtained for δ can by analogy with this be applied for the characteristics of films of thickness $d \ll \delta$.

In that case the quantity δ no longer plays the role of the penetration depth, but this quantity goes over into the effective dielectric constant for low frequencies and determines the magnetic susceptibility. Since the connection between $\mathbf{j}$ and $\mathbf{A}$ is local, ϵ and χ, expressed in terms of δ, have the usual, London form:

$$\varepsilon = -c^2/\omega^2\delta^2, \tag{10}$$

$$\chi = \frac{1}{4\pi}\left(1 - \frac{2\delta}{d}\tanh\frac{d}{2\delta}\right) \approx -\frac{1}{12\pi}\left(\frac{d}{2\delta}\right)^2. \tag{11}$$

In conclusion the authors express their gratitude to Academician L. D. Landau for discussions of this paper.

[1] A. A. Abrikosov and L. P. Gor'kov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1558 (1958), Soviet Phys. JETP **8**, 1090 (1959).

[2] T. Matsubara, Progr. Theoret. Phys. **14**, 351 (1955).

[3] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

Translated by D. ter Haar
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PARTICLE ENERGY LOSSES DUE TO THE EXCITATION OF LONGITUDINAL WAVES

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Submitted to JETP editor September 3, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 321-322 (January, 1959)

IT has been pointed out[1-3] that longitudinal plane waves are excited if a medium exhibits spatial dispersion; Ginzburg has noted[1] that a very convenient means for the excitation of these new waves may be the Cerenkov effect.

In this connection it is of interest to develop the theory of the Cerenkov effect as applied to the case in which longitudinal electromagnetic waves are excited by an electron. We consider an isotropic medium. We start by noting that one cannot use the usual Lagrangian $L = (\epsilon E^2 - H^2)/8\pi$ because the energy density of the field, given by the expression $(\epsilon E^2 + H^2)/8\pi$, vanishes for longitudinal waves (both $\mathbf{H}$ and ϵ vanish).

In order to avoid this difficulty we start with the Fock-Podolsky Lagrangian,[4] making the necessary extension to the case of an isotropic madium. We have

$$L = \left\{(n\mathbf{E})^2 - \mathbf{H}^2 - \left(\operatorname{div}\mathbf{A} + \frac{n^2}{c}\frac{\partial\varphi}{\partial t}\right)^2\right\}, \tag{1}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the intensities of the electric and magnetic fields respectively, $\mathbf{A}$ is the vector potential, φ is the scalar potential and n is the index of refraction (n is considered an operator, cf. reference 5).

Introducing the existence condition for a longitudinal field, $\operatorname{curl}\mathbf{A} = 0$, and the fact that there are no free charges, ($\varphi = 0$), using the analysis in reference 1 we find the following expressions for $\mathbf{A}$ and the energy density respectively:

$$\{\Delta - (n^2\partial^2/c^2\partial t^2)\}\,\mathbf{A} = 0,$$
$$T_{44} = \{(\operatorname{div}\mathbf{A})^2 + (\partial n\mathbf{A}/c\partial t)^2\}/8\pi. \tag{2}$$

In order for Eq. (2) (vector potential) to be consistent with the field equation in the medium, the condition $\epsilon\mathbf{E} = n^2\{\mathbf{E} - \kappa_0(\mathbf{E}\cdot\kappa_0)\}$ must be satisfied; this follows from the analysis in reference 1.

Now, writing the solution of Eq. (2) in the form

$$A = L^{-3}\sum_{\varkappa}\sqrt{2\pi c\hbar/\varkappa n}\,\left\{\mathrm{a}\exp\left(-ic\frac{\varkappa}{n}t + i\varkappa\cdot\mathbf{r}\right) + \mathrm{a}^+\exp\left(ic\frac{\varkappa}{n}t - i\varkappa\cdot\mathbf{r}\right)\right\},$$

we find the energy of the longitudinal field:

$$H = \sum_{\varkappa}(c\hbar\varkappa/n)\,(\mathrm{a}^+\mathrm{a}), \tag{3}$$

where $\hbar\kappa$ and $c\hbar\kappa/n$ are the momentum and energy of the longitudinal photon. To satisfy the existence condition for longitudinal waves explicitly we write

$$\mathbf{a} = g\boldsymbol{\varkappa}_0, \quad \mathbf{a}^+ = g^+\boldsymbol{\varkappa}_0, \quad \boldsymbol{\varkappa}_0 = \boldsymbol{\varkappa}/\varkappa, \tag{4}$$

then, using Eq. (3) in the usual way (cf. reference 6), we find that the operators g and g^+ satisfy the Bose commutation relations. The phenomenological quantum electrodynamics developed above for the longitudinal field agrees in form with that given in reference 6 — the only differences lie in the meanings of the operators $\mathbf{a}$ and $\mathbf{a}^+$. Hence, in applying the analysis to the Cerenkov effect we can use the results obtained by Sokolov.[7,8] Substituting Eq. (4) in Eq. (8) of the paper by Sokolov and Loskutov[8] and assuming that there are no longitudinal photons at $t = 0$, after some elementary transformations we find the following expression for the energy radiated per unit length of path in the form of longitudinal waves: