Letters to the Editor

QUANTUM CORRECTIONS TO RADIATION FROM A RIGID ROTATOR

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A series of Soviet and foreign papers, quite complicated mathematically, have been devoted to the calculation of quantum corrections to synchrotron electron radiation (in particular, see reference 1.) The authors begin from the equations of motion for particles in an axially-symmetric magnetic field that assures circular motion and provides weak-focusing forces around the stable orbit.

It is shown in the present note that practically the same results can be obtained by entirely neglecting the excitation due to the betatron oscillations and considering the electron to be an absolutely rigid rotator. The character of the forces that ensures such a rigid focus are not specified. A possible interaction of the electron spin with the focusing fields is not taken into account.

We taken the orbit of radius a to lie in the xy plane, with its center at the origin. Dirac's equation for free azimuthal motion has the form

$$(\hbar c/ia)(\sin\varphi\alpha_x - \cos\varphi\alpha_y)\,\partial\psi/\partial\varphi = (E + \beta mc^2)\psi. \quad (1)$$

We obtain the positive energy eigenvalues

$$E_l = \hbar\omega_l = \{(\hbar c/a)^2 l(l+1) + (mc^2)^2\}^{1/2} \quad (l = 1, 2, \ldots). \quad (2)$$

Two eigenfunctions correspond to each E_l:

$$\Psi_l^{(-)} = \frac{e^{il\varphi}}{\sqrt{2\pi(1+\varkappa_{l-}^2)}}\begin{pmatrix} i \\ 0 \\ 0 \\ \varkappa_{l-}e^{i\varphi}\end{pmatrix};$$

$$\Psi_l^{(+)} = \frac{e^{il\varphi}}{\sqrt{2\pi(1+\varkappa_{l+}^2)}}\begin{pmatrix} 0 \\ -ie^{i\varphi} \\ \varkappa_{l+} \\ 0\end{pmatrix};$$

$$\varkappa_{l-} = \frac{E_l + mc^2}{(\hbar c/a)(l+1)}, \qquad \varkappa_{l+} = \frac{E_l + mc^2}{(\hbar c/a)\,l}. \quad (3)$$

They are also eigenfunctions of the z component of the angular momentum, belonging to the eigenvalue $\hbar(l + \frac{1}{2})$. In the nonrelativistic approximation $\Psi_l^{(-)}$ corresponds to the z components of orbital angular momentum $m = l + 1$ and spin $s = -\frac{1}{2}$, while $\Psi_l^{(+)}$ corresponds to $m = l$ and $s = +\frac{1}{2}$.

We are interested in motion at relativistic velocities in a circle of macroscopic radius a. Such motion is quasi-classical, that is, $l \gg 1$, so $l(l+1) \approx l^2$, therefore, $\kappa_l = \kappa_{l+} = \kappa_{l-}$. We consider the transition of an electron from the state $\Psi_l^{(-)}$ to the state Ψ_{l_1}; furthermore, we shall take $l - l_1 = n \ll l$, so that we can restrict ourselves to the first order in an expansion in powers of n/l. The frequency of the photon emitted in this transition is

$$\omega_n = \omega_l - \omega_{l_1} = \sigma n\frac{c}{a}; \quad \sigma = \beta\left[1 - \frac{1-\beta^2}{2}\frac{n}{l}\right];$$
$$\beta = \sqrt{1-\gamma^{-2}}; \qquad \gamma = \frac{E_l}{mc^2}. \quad (4)$$

It is easy to see that the probability of a transition $\Psi_l^{(\pm)} \to \Psi_l^{(\mp)}$ with a change in the z component of the spin is proportional to $(n/l)^2$; therefore, only transitions with conservation of the spin component play a role in our approximation.

In first-order perturbation theory, the intensity of radiation of frequency ω_n in a solid angle $d\Omega$, given in terms of the angles θ' and φ', is equal to

$$I_n d\Omega = \frac{e^2c}{2\pi a^2}\sigma^2 n^2\left\{[\cot^2\theta' J_n^2(n\sigma\sin\theta') + \sigma^2 J_n'^2(n\sigma\sin\theta')]\right.$$
$$\left. + \frac{n}{l}\left[\frac{\beta^2\sqrt{1-\beta^2}}{\sigma}\frac{1+\cos^2\theta'}{\sin\theta'}J_n(n\sigma\sin\theta')\,J_n'(n\sigma\sin\theta')\right]\right\}d\Omega. \quad (5)$$

In the limit $l \to \infty$ we get precisely the classical formula.[2] Integrating Eq. (5) over the angles, we get the full power for the n-th harmonic:

$$I_n = \frac{e^2cn}{a^2}\left\{\left[2\sigma^3 J_{2n}'(2n\sigma) - \sigma(1-\sigma^2)\int_0^{2n\sigma} J_{2n}(x)\,dx\right]\right.$$
$$\left. + \frac{n}{l}\beta^2\sqrt{1-\beta^2}\left[J_{2n}(2n\beta) - \frac{1}{2n\beta}\int_0^{2n\beta} J_{2n}(x)\,dx\right]\right\}. \quad (6)$$

In the most interesting, ultrarelativistic case, when $\gamma \gg 1$, $\beta \approx 1$, it is possible to calculate the total radiated power.

For this we sum Eq. (6) over n and with the help of an asymptotic formula (see reference 3) we find

$$W = \frac{2e^2c}{3a^2}\gamma^4\left[1 - \left(\frac{385}{32\sqrt{3}} - \frac{3}{2}\right)\frac{\hbar}{mac}\gamma^2\right]. \quad (7)$$

The result obtained has the same appearance as that of motion in a magnetic field,[1] but the numerical coefficients are somewhat different for the quantum correction.

In conclusion, I should like to express my gratitude to Professor M. S. Rabinovich for much valuable advice.

[1] Sokolov, Klepikov, and Ternov, J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 632 (1952) and **24**, 249 (1953). Also J. Schwinger, Proc. Natl. Acad. Sci. (U.S.A.) **40**, 132 (1954).

[2] G. A. Schott, Electromagnetic Radiation, Cambridge, 1912.

[3] J. Schwinger, Phys. Rev. **75**, 1912 (1949).

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ENERGY SPECTRUM OF FRAGMENTS FROM THE PHOTOFISSION OF U^{238}

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A systematic study of the variation of the nature of fission with excitation is possible by comparison of the energy spectra of any one isotope for different excitation energies. This should show a preference for excitation of the nucleus by gamma quanta, since the number of nucleons in the nucleus is not changed by this method. Some data for such a comparison can be gained at present from the behavior of U^{238}, for which the spectrum of spontaneous fission fragments[1,2] and the spectrum of fragments from photofission for a maximum gamma-ray energy of 16.7 Mev[3] have been investigated. The purpose of the present work was to obtain the energy spectrum of fragments from the photofission of U^{238} at a maximum betatron gamma-ray energy of 12.5 Mev.

The measurement of the kinetic energy of the photofission fragments was made in an apparatus consisting of a differential pulse ionization chamber, an amplifying channel, and a photo-recording pulsed oscillograph. The chamber had two parts — working and compensating — with a common collecting electrode and high voltage electrodes of opposite sign. The working part of the chamber was an ordinary pulse chamber with a grid. The compensating part of the chamber gave better compensation in the absence of a grid electrode within it. A layer of U_3O_8, which had been a target for a beam of gamma-rays and had a surface density of 0.4 mg/cm^2, was placed on the negative electrode. No collimation of the direction of the fragments was made.

The angular distribution of fragments in photofission is anisotropic with a maximum in the direction perpendicular to the gamma-ray beam. When the orientation of the chamber axis was parallel to the beam axis an additional distortion of the fragment spectrum could have arisen at the expense of absorption, since a significant part of the fragments could have flown out at very small angles to the plane of the preparation. Therefore in practice the chamber axis was set up at an angle of 15° to the beam axis.

The energy spectrum of fragments from the photofission of U^{238} for a maximum betatron bremsstrahlung energy of 12.5 Mev is shown in Fig. 1. The energy distribution of fragments from the slow neutron fission of U^{235} was obtained with the same apparatus and with the same preparation. This distribution is also shown in Fig. 1.

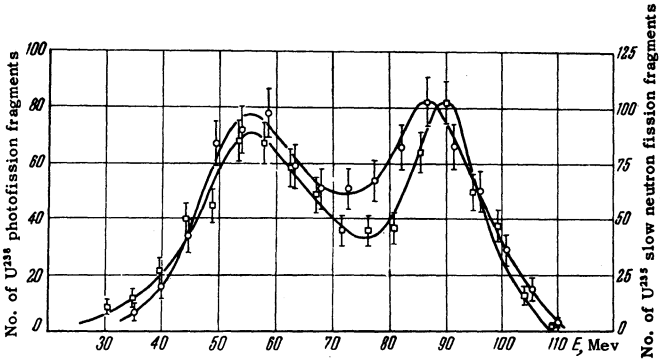

FIG. 1. Fragment energy spectra: ◎ – photofission of U^{238} at a maximum gamma-ray energy of 12.5 Mev, □ – slow neutron fission of U^{235}.

The spectrum of the photofission fragments has the most probable energies of (55.1 ± 1) and (86.9 ± 1) Mev for the groups of heavy and light fragments, respectively. To take the absorption in the preparation layer into account, these values must each be increased by approximately 5 Mev.

In Fig. 2 the U^{238} photofission spectrum is compared with the spontaneous fission spectrum obtained in reference 2. The comparison was made by means of the energy spectra of fragments from