ESTIMATE OF THE CONTRIBUTION OF NUCLEON-ANTINUCLEON INTERACTION IN THE DISPERSION RELATION FOR NUCLEON-NUCLEON SCATTERING

L. I. LAPIDUS

Joint Institute for Nuclear Research

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The relation between the scattering lengths and effective radii in the s and p states, as predicted by the dispersion relations for N-N scattering, are discussed. Estimates are derived for the contribution of the N-N interaction to the dispersion relation for N-N scattering from the experimental data on n-p and p-p scattering at low energies. The N-N interaction contribution has been found to be small. Its magnitude depends pronouncedly on the sign of the scattering path in the s states.

1. INTRODUCTION

AN analysis of dispersion relations for N-N scattering and their use for the processing of experimental data is made quite difficult by the presence of contribution of the antinucleon-nucleon interaction. This pertains both to the energy dependence of the total cross sections of the N-N interaction and, in particular, to the observable region that remains even in the case of forward scattering.

There is reason for believing that the influence of the nucleon-antinucleon interaction is slight in the region of low energies. Using this assumption, an attempt can be made[1,2] to obtain dispersion relations for N-N scattering containing no unobservable region.

A clarification of the role of the nucleon-antinucleon interaction in nucleon-nucleon scattering is of interest both from the point of view of obtaining approximate relations and from the general point of view.

In this paper we attempt an estimate of the contribution of the N-N interaction to the scattering of nucleons by nucleons at low energies. For this purpose we use the approach used previously[3] in the analysis of the dispersion relations for π-N scattering, based on the "theory of the effective radius."[4]

For π-N scattering we obtain[3] relations between the scattering paths and the effective radii in different states, and these admitted of direct verification, since the final relations contained quantities that could be determined directly by experiment. It has been shown that the result of the analysis depends strongly on the small scattering phases in the p state.

The analogous relations for N-N scattering contain unknown quantities that characterize the nucleon-antinucleon interactions. The use of data on N-N scattering at low energies (and of the energy dependences of the total cross sections) makes it possible to estimate the contribution of the nucleon-antinucleon interaction to the dispersion relation for the N-N scattering.

Let us consider the dispersion relation for forward N-N scattering. We write the N-N scattering amplitude in the form

$$M = \frac{m}{\omega_b}\{\alpha + \beta(\sigma_1\cdot\mathbf{n})(\sigma_2\cdot\mathbf{n}) + i\gamma(\sigma_1+\sigma_2)\cdot\mathbf{n} + \delta(\sigma_1\cdot\mathbf{m})(\sigma_2\cdot\mathbf{m}) + \varepsilon(\sigma_1\cdot\mathbf{l})(\sigma_2\cdot\mathbf{l})\}, \quad (1)$$

where $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$ are unit vectors in the directions $\mathbf{k}_b \pm \mathbf{k}'_b$ and $\mathbf{k}_b \times \mathbf{k}'_b$, while $\omega_b = \sqrt{m^2 + k_b^2}$ is the energy of one particle in the c.m. system. We confine ourselves to the dispersion relations for the quantity

$$\alpha(\omega) = (\omega_b/4m)\,\mathrm{Sp}\,M(0^\circ).$$

The dispersion relations for N-N scattering were considered by various authors.[5-8] They were treated most fully and in greatest detail by Goldberger, Nambu, and Oehme.[8] Many interesting ideas are due to Ioffe.[6]

2. SCATTERING OF NEUTRONS BY PROTONS

The forward dispersion relation for the α_{np} is represented in the form

$$\mathrm{Re}\Big[\alpha_{np}(\omega)-\frac{1}{2}(1+\omega/m)\,\alpha_{np}(m)-\frac{1}{2}(1-\omega/m)\,\alpha_{n\bar{p}}(m)\Big]$$
$$=(\omega^2-m^2)\Big\{\frac{f^2}{4\pi}\frac{1}{2(\omega_\mu+\omega)(\omega_\mu^2-m^2)}$$
$$+\Gamma_\alpha(0)\Big(\frac{\omega_d+m}{2m}\Big)^{1/2}\frac{1}{(\omega_d-\omega)(\omega_d^2-m^2)}$$
$$+\frac{1}{8\pi^2}\,\mathrm{P}\int_m^\infty\frac{d\omega'}{k'}\Big[\frac{\sigma_{np}(\omega')}{\omega'-\omega}+\frac{\sigma_{n\bar{p}}(\omega')}{\omega'+\omega}\Big]$$
$$+\frac{1}{\pi}\int_{\omega(2\mu)}^{m}d\omega'\,\frac{\mathrm{Im}\,\alpha_{n\bar{p}}(\omega')}{(\omega+\omega')(\omega'^2-m^2)}\Big\}, \qquad (2)$$

where ω is the total energy of the nucleon in the laboratory system, m and μ are the masses of the nucleon and of the pion, $B=\kappa^2/m$ is the binding energy of the deuteron, and

$$\omega_d=m-2B+B^2/2m,$$
$$\omega_\mu=\mu^2/2m-m,\ \Gamma_\alpha(0)=3(\kappa/m)/(1-\kappa r_{0t}).$$

The effective radius r_{0t} is given by Eq. (4.40) of reference 8, the notation of which is used in this article.

The contribution of the nucleon-antinucleon interaction to (2) will be taken to mean the term proportional to the integral from $\omega(2\mu)$ to m (unobservable region) as well as the terms containing $\alpha_{n\bar{p}}(m)$ and $\sigma_{n\bar{p}}(\omega')$.

Since we are interested in the values of $\mathrm{Re}\,\alpha_{np}(\omega)$ for small $\eta_{2b}^2=(k_b/m)^2$, we write the dependence of the scattering phases on the energy in the following form[4]

$$\eta_b^{2L+1}\cot(^{2S+1}L_J)=\frac{1}{a_L}+\frac{1}{2}r_0^{(L)}\eta_b^2+Q_L\eta_b^4\equiv A(^{2S+1}L_J), \qquad (3)$$

where $a_L\equiv a(^{2S+1}L_J)$ is the scattering length (for $\eta\to 0$) in a state with momentum J, parity $(-1)^L$, and spin s; $k_b=\eta_b m$ is the momentum of the nucleon in the center-of-mass system, with

$$k/k_b=\eta/\eta_b=(2+2\omega/m)^{1/2}\approx 2. \qquad (4)$$

The scattering lengths in the $^3S_1$ and $^1S_0$ states are sometimes denoted $a_t\equiv a(^3S_1)$ and $a_s\equiv a(^1S_0)$. In accordance with definition (3) we have $a_t<0$ and $a_s>0$. The symbol b denotes quantities in the center-of-mass system.

Using the expression for α_{np} in terms of the scattering phases, we readily obtain

$$\alpha_{np}^{(b)}(\omega)=\frac{\omega_b}{m}\frac{1}{4k_b}\sum_{J=0}^{\infty}(2J+1)e^{i\delta_J}\sin\delta_J$$
$$=\frac{\hbar}{mc}\frac{\omega_b}{m}\frac{1}{4\eta_b}\sum_{J,l,S}(2J+1)e^{i\delta_J}\sin\delta_J, \qquad (5)$$

where δ_J denotes the scattering phases in states with total momentum J [the mixing coefficients drop out from (5)]. Since

$$\sigma_{np}=\frac{4\pi}{k_b}\frac{m}{\omega_b}\,\mathrm{Im}\,\alpha_{np}^{(b)}(\omega_b)=\frac{8\pi}{\sqrt{\omega^2-m^2}}\,\mathrm{Im}\,\alpha_{np}(\omega), \qquad (6)$$

$\alpha_{np}(\omega)$ is expressed in the laboratory system in terms of $\alpha_{np}^b(\omega_b)$ by the following relation ($\lambda_c=\hbar/mc=2.1\times10^{-14}$ cm):

$$\alpha_{np}(\omega)=\frac{k}{2k_b}\frac{m}{\omega_b}\,\alpha_{np}^{(b)}(\omega_b)$$
$$=\Big(\frac{1}{2}+\frac{1}{2}\frac{\omega}{m}\Big)^{1/2}\frac{\lambda_c}{4\eta_b}\sum_{JLS}(2J+1)e^{i\delta_J}\sin\delta_J. \qquad (7)$$

From (3) and (7) we have

$$\mathrm{Re}\,\alpha_{np}(m)\equiv D_{np}(m)=\frac{\lambda_c}{4}(3a_t+a_s), \qquad (8)$$

and the expression

$$D_{n\bar{p}}(m)=\frac{\lambda_c}{4}\{3a_{n\bar{p}}(^3S_1)\exp[-2\beta_{n\bar{p}}(^3S_1)]$$
$$+a_{n\bar{p}}(^1S_0)\exp[-2\beta_{n\bar{p}}(^1S_0)]\} \qquad (9)$$

takes into account the inelastic process — annihilation — with the aid of $\beta_{n\bar{p}}(^3S_1)$ and $\beta_{n\bar{p}}(^1S_0)$, the imaginary parts of the phases in the corresponding states of $n\bar{p}$ system.

Using a procedure analogous to that of reference 3, we consider the relation obtained from Eq. (2) when the latter is differentiated with respect to η^2 and η^2 is then set equal to zero. Taking into account the available experimental data, we confine ourselves to a single differentiation. Denoting the derivative of $D_{np}(\omega)$ with respect to η^2 by $D'_{np}(\omega)$, denoting its value at $\omega=m$ by $D'_{np}(m)$, and considering that all the lengths are expressed in terms of λ_c, we get

$$D'_{np}(m)+\frac{1}{4}[D_{n\bar{p}}(m)-D_{np}(m)]$$
$$=\lambda_c\Big\{-\frac{f^2}{4\pi}\Big(\frac{m}{\mu}\Big)^4+\frac{3}{8}\Big(\frac{m}{B}\Big)^{3/2}\frac{1}{1-\kappa r_{0t}}$$
$$+\frac{1}{8\pi^2}\,\mathrm{P}\int_1^\infty\frac{d\omega'}{\eta'}\Big[\frac{\sigma_{np}(\omega')}{\omega'-1}+\frac{\sigma_{n\bar{p}}(\omega')}{\omega'+1}\Big]$$
$$+\frac{1}{\pi}\int_{\omega(2\mu)/m}^{1}d\omega'\,\frac{\mathrm{Im}\,\alpha_{n\bar{p}}(\omega')}{(\omega'+1)(\omega'^2-1)}\Big\} \qquad (10)$$

The contribution of the deuteron state to (2) is calculated with accuracy to terms on the order of B/m, and therefore the terms of order B/m are discarded in the second term of Eq. (10). Terms of the order $(\mu/2m)^2$ have been omitted from the term proportional to f^2.

From Eqs. (3) and (7) we obtain for $D'_{np}(m)$

$$D'_{np}(m) = {}^1/_8 D_{np}(m) - {}^1/_{16}\{a^2({}^1S_0)[a({}^1S_0) + {}^1/_2 r({}^1S_0)] + 3a^2({}^3S_1)[a({}^3S_1) + {}^1/_2 r({}^3S_1)] - a({}^3P_0) - 3[a({}^1P_1) + a({}^3P_1)] - 5a({}^3P_2)\}. \quad (11)$$

According to the experimental data[9,10]

$$a({}^3S_1) = -(0.537 \pm 0.004)\cdot 10^{-12}\text{cm} = -(25.6 \pm 0.19)\lambda_c,$$
$$a({}^1S_0) = (2.373 \pm 0.007)\cdot 10^{-12}\text{cm} = (113 \pm 0.33)\lambda_c,$$
$$r({}^1S_0) \equiv r_{0s} = 2.7\cdot 10^{-13}\text{ cm} = 12.84\lambda_c,$$
$$r({}^3S_1) \equiv r_{0t} = 1.7\cdot 10^{-13}\text{ cm} = 8.1\lambda_c, \quad (12)$$

hence

$$D_{np}(m) = (0.190 \pm 0.0025)\cdot 10^{-12}\text{ cm} = 9.05\lambda_c,$$
$$D'_{np}(m) = -9.20\cdot 10^4\lambda_c. \quad (12')$$

In calculating $D'_{np}(m)$, the principal contribution is that of the ${}^1S_0$ state. The triplet s scattering gives a contribution not exceeding 5% of the singlet contribution, while the inaccuracy in the value of a_s leads to an inaccuracy of approximately 1% in the value of $D'_{np}(m)$. Smaller still is the contribution of scattering in p states. We note that if, for a rough estimate, we use the values yielded by the potential of Gammel, Christian and Thaler,[11] the contribution of the p states do not exceed a value determined by the inaccuracy in the calculation of the deuteron term and comparable with $D_{np}(m)$. An estimate based on the potential of Signell and Marshak,[12] yields

$$a({}^1P_1) = -146.4\lambda_c, \quad a({}^3P_0) = 57.2\lambda_c,$$
$$a({}^3P_1) = -46.5\lambda_c, \quad a({}^3P_2) = 27.8\lambda_c.$$

Estimates of various terms in (10) show that numerically the deuteron state is considerably more important than the single-meson term, which played an important role in π-N scattering. In the low-energy region considered here, the presence of $D_{np}(m)$ in Eq. (10), as can be seen from Eqs. (11) and (12), is hardly significant in practice. The value of $D_{np}(m)$ does not exceed the error in the calculation of the contribution of the deuteron state. The smallness of this term confirms the suggested unimportance of the role of "subtraction" in the nonrelativistic region. Furthermore, this gives grounds for assuming that the contribution of $D_{n\bar{p}}(m)$, which so far cannot be estimated directly, is small.

The contribution of the deuteron state amounts to +(5450 ± 13) (when B = 2.2 Mev, we have in the right half $1 - \kappa r_{0t} = 0.608$). The contribution of the single-meson state is 162 when $f^2/4 = 0.08$ and −184 when $f^2/4 = 0.09$.

The essential difference between (11), (12), and the analogous formulas of reference 3 lies naturally in the entirely different role of the s and p states in the np and pN scattering. The presence of a "resonance" pN interaction in the p state causes $D_{\pi N}(\omega)$ to be an increasing function at low energies, while the "resonance" interaction of nucleons in the s state makes $D_{np}(\omega)$ diminish at low energies. The sign of $a({}^1S_0)$ becomes significant here.

The calculation of the dispersion integral

$$J_{np}(m) = \frac{1}{8\pi^2} P\int_1^\infty \frac{d\omega'\sigma_{np}(\omega')}{\sqrt{\omega'^2 - 1}(\omega' - 1)} = \frac{F_{np}(m)}{8\pi^2} \quad (13)$$

leads to a value

$$J_{np}(m) = -\frac{7.70\cdot 10^6}{8\pi^2} = -9.65\cdot 10^4. \quad (14)$$

The integral $F_{np}(m)$ is understood to mean the limit

$$F_{np}(m) = \lim_{\omega_0\to 1} P\int_1^\infty \frac{\sigma_{np}(\omega')d\omega'}{(\omega' - \omega_0)(\omega'^2 - 1)^{1/2}} = F_s(\omega_0) + 3F_t(\omega_0). \quad (15)$$

The entire interval of integration is broken up into sections, on each of which the function $\sigma_{np}(\omega')$ is approximated by a simple expression. For the region of kinetic energies up to 20 Mev, the formula of Smorodinskiĭ[13] is used

$$\sigma_{np}(E_0) = 1.3\cdot 10^{-24}\left\{\frac{3}{(1.22 - 0.06E_0)^2 + E_0/2} + \frac{1}{(0.27 + 0.0\ E_0)^2 + E_0/2}\right\}\text{cm}^2 \quad (16)$$

(E_0 is the laboratory-system kinetic energy of the neutron in Mev). In other regions, a rough approximation is used. The roughness of the approximation of $s_{np}(\omega)$ at high energies introduces no noticeable error whatever, since the principal role in the calculation of (15) is played, naturally, by the region $\omega' \sim \omega_0$. The contribution of this principal region, when the energy dependence of σ_{np} is given by Eq. (16), is easy to calculate

$$F_1(m) = \lim_{\omega_0\to 1} F_1(\omega_0) = \lim_{\omega_0\to 1}\{3F_{1t}(\omega_0) + F_{1s}(\omega_0)\},$$

where, for example,

$$F_{1s}(\omega_0) = P\int_1^{1+\varkappa^2/2} \frac{d\omega'}{\eta'}\frac{\sigma(\omega')}{\omega' - \omega_0} = \pi a^2({}^1S_0)\left\{-\frac{1}{k_0}\ln\left|\frac{k_0 + \varkappa}{k_0 - \varkappa}\right| + \left(\frac{k_0^2 + k_{2s}^2}{k_{1s}^2 - k_{2s}^2}\right)\frac{2}{k_{1s}}\tan^{-1}\frac{\varkappa}{k_{1s}} - \left(\frac{k_0^2 + k_{1s}^2}{k_{1s}^2 - k_{2s}^2}\right)\frac{2}{k_{2s}}\tan^{-1}\frac{\varkappa}{k_{2s}}\right\} \quad (17)$$

and an analogous expression for the contribution of the triplet state. In Eq. (17) k_{1s}^2 and k_{2s}^2 are the roots of the equation

$$\left(\frac{0.27}{\sqrt{m}} + 0.06\sqrt{m}\frac{k_s^2}{2}\right)^2 + \frac{k_s^2}{4} = 0 \quad (k_1^2 > k_2^2).$$

Using (17) and (15) we obtain directly

$$F_{np}^{(1)}(m) = -\frac{2}{\varkappa}\sigma_{np}(m) - 6\pi a_t^2 \left\{\frac{k_{1t}^2 \tan^{-1}(\varkappa/k_{2t})}{k_{2t}(k_{1t}^2 - k_{2t}^2)} - \frac{k_{2t}^2 \tan^{-1}(\varkappa/k_{1t})}{k_{1t}(k_{1t}^2 - k_{2t}^2)}\right\}$$

$$-2\pi a_s^2 \left\{\frac{k_{1s}^2 \tan^{-1}(\varkappa/k_{2s})}{k_{2s}(k_{1s}^2 - k_{2s}^2)} - \frac{k_{2s}^2 \tan^{-1}(\varkappa/k_{1s})}{k_{1s}(k_{1s}^2 - k_{2s}^2)}\right\} = -7.70\cdot 10^6. \qquad (18)$$

In Eq. (18) k_{1t}^2 and k_{2t}^2 are the roots of the equation

$$\left(\frac{1.22}{\sqrt{m}} - 0.06\sqrt{m}\frac{k_t^2}{2}\right)^2 + \frac{k_t^2}{4} = 0.$$

The contribution of the entire energy region above 20 Mev is not more than 0.3% of the value given by Eq. (18). In the subsidiary region itself, the energy section above 100 Mev yields approximately 12% of the contribution of this region.

Gathering the results and transferring all the known factors in Eq. (10) to one side, we obtain for the contribution of the nucleon-antinucleon interaction

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\frac{\sigma_{n\bar{p}}(\omega')}{\omega'+1} + \frac{1}{\pi}\int_{\omega(2\mu)}^1 d\omega' \frac{\operatorname{Im}\alpha_{n\bar{p}}(\omega')}{(\omega'+1)(\omega'^2-1)}$$

$$-\frac{1}{4}D_{n\bar{p}}(m) = (-9.2 + 9.1)\,10^4 = -1000. \qquad (19)$$

A direct comparison of the dispersion relations with the experimental data on the n-p scattering at low energies shows therefore that the contribution of the nucleon-antinucleon interaction to the scattering of the nucleons is small in this energy region. It follows hence, in particular, that for this energy region the exact dispersion relation (2) can be replaced, without excessive error, by the approximation

$$D_{np}(\omega) - D_{np}(m) = (\omega^2 - m^2)\left\{\left(\frac{\omega_d + \omega}{2m}\right)^{1/2}\frac{\Gamma_\alpha(0)}{(\omega_d - m)(\omega_d^2 - m^2)}\right.$$

$$\left. + \frac{1}{8\pi^2} P\int_1^\infty \frac{d\omega'}{\eta'}\frac{\sigma_{np}(\omega')}{\omega' - m}\right\}. \qquad (20)$$

Equation (76) of reference 8 may be useful to obtain information on the contribution of the unobservable region at various energies.

3. SCATTERING OF PROTONS BY PROTONS

The amplitude α_{np} can be expressed in terms of the amplitude of N-N scattering in states with definite values of the isotopic spin α_0 and α_1, using the relation

$$2\alpha_{np} = \alpha_0 + \alpha_1 = \alpha_0 + \alpha_{pp}. \qquad (21)$$

As a result we obtain for p-p scattering, in lieu of Eq. (10), a relation with a different numerical factor in front of f^2, containing no contribution of the deuteron state. In the unobservable region, naturally, there enter also states of several mesons with T = 1.

In lieu of (8) and (11) we obtain

$$D_{pp}(m) = \frac{1}{2}a(^1S_0) = 56.5\lambda_c,$$

$$D'_{pp}(m) = \frac{1}{8}\left\{a_s - a_s^2\left(a_s + \frac{1}{2}r_{0s}\right)\right.$$

$$\left. + a(^3P_0) + 3a(^3P_1) + 5a(^3P_2)\right\} = -19.00\cdot 10^4\lambda_c. \qquad (22)$$

Making use of the isotopic invariance and calculating for the corresponding integral only the contribution up to 20 Mev, we obtain with the Smorodinskiĭ formula

$$J_{pp}(m) \approx -\frac{a_s^2}{2\pi}\left\{\frac{1}{\varkappa} + \frac{\tan^{-1}(\varkappa/k_{2s})}{k_{2s}}\right\} = -18.75\cdot 10^4. \qquad (23)$$

Equation (19) is replaced by

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\frac{\sigma_{p\bar{p}}(\omega')}{\omega'+1}$$

$$+\frac{1}{\pi}\int_{\omega(2\mu)}^1 d\omega' \frac{\operatorname{Im}\alpha_{p\bar{p}}(\omega')}{(\omega'+1)(\omega'^2-1)} - \frac{1}{4}D_{p\bar{p}}(m) = -2500, \qquad (24)$$

indicating that the corresponding values in p-p and n-p scattering are of the same scale. Analysis shows that this is caused by the fact that the singlet scattering plays the important role in (19) and (23). We can thus use for p-p scattering also an approximate dispersion relation similar to (20) in which $\Gamma_\alpha(0) = 0$.

4. SCATTERING OF NUCLEONS IN STATE WITH T = 0

Using (21), it is possible to obtain a dispersion relation for N-N scattering in states with T = 0. The relation will be similar to (2), except that the right hand will not contain the contribution of the single-meson state, while the deuteron contribution is double. Relation (10) is correspondingly transformed:

$$D_{NN}^{(0)}(m) = \frac{3}{2}a(^3S_1) = -38.4\lambda_c. \qquad (25)$$

Instead of (11) we get

$$D_{NN}^{(0)\prime}(m) = \frac{1}{4}D_{NN}^{(0)}(m)$$

$$-\frac{3}{8}\left\{\frac{a_t}{2} + a_t^2\left(a_t + \frac{1}{2}r_{0t}\right) - a(^1P_1)\right\} \approx 5160\lambda_c. \qquad (26)$$

Here the relative role of the p states is naturally greater than in the cases previously considered, but the role of D(m), as before, is insignificant. If, in calculating the dispersion integral

$$J_{NN}^{(0)} = \frac{1}{8\pi^2} \mathrm{P} \int_1^\infty \frac{d\omega'}{\eta'} \frac{\sigma_{NN}^{(0)}(\omega')}{\omega' - 1}$$

we again confine ourselves to the contribution of the kinetic-energy region below 20 Mev, we get

$$J_{NN}^{(0)}(m) = -\frac{3}{2}\frac{a_t^2}{\pi}\left\{\frac{1}{\varkappa} + \frac{k_{1t}^2 \tan^{-1}(\varkappa/k_{2t})}{k_{2t}(k_{1t}^2 - k_{2t}^2)} - \frac{k_{2t}^2 \tan^{-1}(\varkappa/k_{1t})}{k_{1t}(k_{1t}^2 - k_{2t}^2)}\right\} = -8.35\, a_t^2 = -5450.$$

The contribution of the deuteron state now amounts to 10900, so that, in analogy with (19) and (24),

$$\frac{1}{8\pi^2}\int_1^\infty \frac{d\omega'}{\eta'}\frac{\sigma_{NN}^{(0)}(\omega')}{\omega'+1} + \frac{1}{\pi}\int_{\omega(2\mu)}^{1} d\omega' \frac{\mathrm{Im}\,\alpha_{NN}^{(0)}(\omega')}{(\omega'+1)(\omega'^2-1)} - \frac{1}{4} D_{NN}^{(0)}(m) = +300. \quad (27)$$

The approximate dispersion relation differs from (20) in the numerical factor in front of $\Gamma_\alpha(0)$.

5. DISCUSSIONS

The net result of this article is the derivation of estimates like (19), (24), and (27) and the justification of approximate relations like (20). The estimate (27) is the least reliable; nevertheless, all three relationships show the magnitude of the contribution of the nucleon-antinucleon interaction to the dispersion equation for N-N scattering.

The approximate relations (20) may be useful in the analysis of experimental data on N-N scattering at low energies, particularly after more detailed data are obtained on N-N scattering in the p states. An approximate dispersion relation for nucleon-nucleon scattering was first postulated by Blank and Isaev.[1] In the present paper it is corroborated by direct comparison with experimental data. The principal result of this paper — relations of type (10) — can be used to obtain information on the contribution of the unobservable region in the future, when data on the interaction between antinucleons and nucleons become available for a wide range of energies.

It is interesting to note that the deduction that the nucleon-nucleon interaction contributes little is connected with the signs of a_s and a_p.

The dispersion relations for the pion-nucleon scattering was used in its time by many investigators to establish that $D_{\pi+p}$ and α_{33} are positive below resonance.

In the calculations presented here, the signs of a_s and a_t were assumed to be those that follow from data on the scattering of neutrons in para-hydrogen and ortho-hydrogen (see, for example, reference 10).

Since $D'_{pp}(m)$ is determined essentially by the singlet S scattering, while $D'_{np}(m)$ is essentially determined by this scattering, a change in the sign of a $(^1S_0)$ would lead to the conclusion that nucleon-antinucleon interaction plays a large role at low energies. It is interesting to call attention to the fact that, from the point of view of dispersion relations, the positive sign of a $(^3S_1)$ is determined both by the presence of a real deuteron state and by the fact that follows, from a somewhat different point of view, from the "effective radius theory" (see, for example, reference 14).

The analysis performed can be of interest in the evaluation of data on interactions between nucleons and low-energy antinucleons. The dispersion relation for N-N scattering is obtained from (2) by formally making the substitution $\omega \to -\omega$ and replacing α_{np} by $\alpha_{n\bar{p}}$ in the proper places. The role of single-meson and deuteron states is greatly reduced, but the role of the unobservable region increases noticeably instead. In the presence of necessary data, a relation of type (10) can be obtained to estimate the contribution of the unobservable region in this case.

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