GENERALIZED FORM OF THE DEPENDENCE OF THE PION CROSS SECTION IN COMPLEX NUCLEI ON THE NUMBER OF NUCLEONS

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Submitted to JETP editor July 10, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 244-248 (January, 1959)

A generalized expression is derived for the dependence of the cross section of photoproduction of π mesons in complex nuclei on the number of nucleons. In addition to the ordinary reabsorption the reabsorption by nucleon pairs at the instant of meson creation has been taken into account. The following two calculations are presented: (1) a calculation for uniform density of the nucleons in the nucleus assuming a particular wave function for the nucleon pair, and (2) a phenomenological calculation without a wave function but for a Fermi distribution of the nucleon density in the nucleus.

1. INTRODUCTION

It is well known that the dependence of the photoproduction cross section of π mesons in nuclei as a function of the atomic number is determined to a considerable extent by the secondary process of meson reabsorption.

The effect of these processes was analyzed by Brueckner, Serber, and Watson.[1] They gave the following expression for the dependence of the π^0 production cross section as a function of A (or, respectively for π^+ on Z and π^- on N):

$$(\sigma_{\pi^0})_A = (\sigma_{\pi^0})^0_{p,n} A\eta\chi. \quad (1)$$

The factor χ for the reabsorption has the form

$$\chi = 3\left[{}^1/_2 x^{-1} - x^{-3} + x^{-3}(1+x)e^{-x}\right], \quad x = 2R/\lambda_a,$$

where R — nuclear radius and λ_a — the mean distance for meson absorption in nuclei; $\eta \leq 1$ is a coefficient due to the replacement of free nucleons by nucleons bound in the nucleus.

The secondary meson reabsorption processes can be divided in a rough fashion into single-nucleon and two-nucleon interactions of the meson. At small meson energies the two-nucleon absorption predominates and plays the essential role.

However, as has been pointed out by Wilson[2] and Butler,[3] in the case of meson photoproduction there exists an important specific circumstance which is not accounted for by (1). To be absorbed by a two-nucleon process on its way through the nucleus there have to be present in a certain volume three particles — the meson and two nucleons. Now, at the instant of the production of the meson two particles — the meson and a nucleon — are already contained in this volume and for the two-nucleon absorption process the presence of only a second particle in the same volume is necessary. The circumstance that a quantitative treatment of this effect was not undertaken is the reason for the confrontation "volume production" as described by (1) vs. the so called "surface production." The latter leads to the dependence $\sigma_{\pi^0} \sim A^{2/3}$ independently of the value of λ_a. The necessity of combining these pictures follows from the experimental data on neutral-pion photoproduction in nuclei. In a series of experiments on neutral-pion photoproduction in complex nuclei including our measurements[4-6] it was shown that the dependence of σ_{π^0} on A for nuclei between carbon and lead is close to $A^{2/3}$. Furthermore, the measurements in reference 6 were performed with two counter telescopes in coincidence located 180° apart. They thus counted only π^0 mesons of a limited energy interval (1 to 15 Mev). The mean free path of such mesons in nuclear matter is on the average $\lambda_a \approx 17r_0 = 17 \times 1.4 \times 10^{-13}$ cm. Nevertheless the dependence of σ_{π^0} on A turned out to be close to $A^{2/3}$. This is clearly in contradiction with the usual optical model formula (1) which for these conditions leads to a dependence close to $\sim A$. On the other hand, the results of reference 6 differ perceptibly from the dependence $\sim A^{2/3}$ of "pure surface production."

In the present paper the influence of the two-nucleon absorption of mesons at the instant of their creation on the dependence of σ_{π^0} on A is estimated in the simplest possible manner.

2. DEPENDENCE OF σ_{π^0} ON A FOR A PARTICULAR WAVE FUNCTION AND DENSITY DISTRIBUTION IN THE NUCLEUS

We follow the method of Wilson,[7] which he applied to the analysis of the photodisintegration of the deuteron. We thus assume that it is necessary and sufficient for a meson to be reabsorbed by a two-nucleon group at the instant of its creation that the nucleons at this time be at a separation $r \leq l = \hbar/\mu c$. We assume for the nucleon pairs the Chew-Goldberger wave function

$$\psi(p) = \sqrt{\alpha\hbar}/\pi(\alpha^2\hbar^2 + p^2), \tag{2}$$

where $\hbar^2\alpha^2/2m = 18$ Mev (m = nucleon mass). In the coordinate representation this wave function has the form

$$\psi(r) = \sqrt{\alpha/2\pi}\, e^{-\alpha r}/r. \tag{3}$$

Obviously the probability for the meson not to be absorbed by the quasi-deuteron at the time of its creation equals

$$f_0 = 1 - \int_0^l |\psi(r)|^2 4\pi r^2 dr = e^{-2\alpha l}. \tag{4}$$

The quantity f_0 characterizes the probability for a meson to be produced in the central parts of the nucleus which are further removed from the surface than the distance l. For the given $\psi(r)$ it has a value 0.075. In the vicinity of the surface this probability increases and equals

$$f_s(y) = \frac{1}{2}\{e^{-2\alpha(R-y)} + e^{-2\alpha l}\}$$
$$- \frac{\alpha(R^2-y^2)}{2y}\{-\mathrm{Ei}[-2\alpha(R-y)] + \mathrm{Ei}[-2\alpha l]\} \tag{5}$$
$$+ \frac{1}{8\alpha y}\{[1+2\alpha(R-y)]e^{-2\alpha(R-y)} - [1+2\alpha l]e^{-2\alpha l}\},$$

where y is distance from the center of the nucleus $[(R-l) \leq y \leq R]$. The overall (i.e., averaged over the nucleus) probability for the meson not to be reabsorbed in a two-nucleon process at the instant of its creation is given by

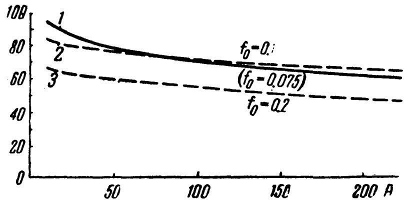

FIG. 2

$$f_q = f_0(1 - A^{-1/3})^3 + \frac{3}{A}\int_{A^{1/3}-1}^{A^{1/3}} z^2 f_s(z)\,dz, \tag{6}$$

where $z = y/l$.

Curve 1 of Fig. 1 shows the function $f_q A$. The values are plotted with respect to carbon (A = 12). Obviously the curve falls in between the straight line $\sim A$ and the curve $\sim A^{2/3}$. It thus denotes a mixture of "volume" and "surface" production which are characterized respectively by the first and second term of (6).

The dependence of the contribution of "surface production" of π mesons on A (i.e., the production in a surface layer of thickness l) is plotted as curve 1 in Fig. 2.

In addition to the reabsorption of mesons at the instant of their creation one obviously has to take into account the usual reabsorption in the nucleus. The fraction of the "volume-produced" mesons that leave the nucleus will be given approximately by Eq. (1) multiplied by $\exp(-l/\lambda_a)$ for $x = 2(R-l)/\lambda_a$. The fraction of the "surface-produced" mesons that leave the nucleus will be given approximately by

$$\chi_s = \frac{1}{2}\left\{1 + \frac{1-e^{-u}}{u}\right\}, \tag{7}$$

where

$$u = 2(R - {}^1/_2\, l)/\lambda_a.$$

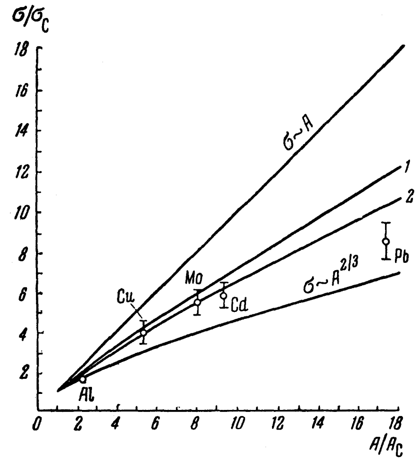

FIG. 1

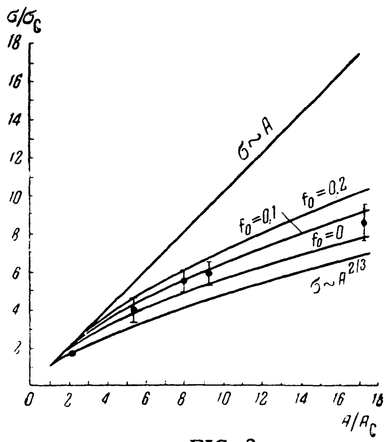

FIG. 3

Curve 2 of Fig. 1 shows the ratio of the photoproduction cross section of slow mesons in different nuclei to that in carbon for $\lambda_a/r_0 = 17$ (this corresponds, for example, to π^0 mesons with an energy of approximately 8 Mev). A comparison with the experimental points taken from reference 6 favors the generalized formula as compared with the usual expression (1) or with the simple dependence $\sim A^{2/3}$.

3. PHENOMENOLOGICAL CALCULATION OF THE DEPENDENCE OF σ_{π^0} ON A FOR A FERMI DENSITY DISTRIBUTION OF THE NUCLEONS IN THE NUCLEUS

The above calculation of the two-nucleon reabsorption of π mesons at the instant of their creation was performed for the case of uniform nucleon density in the nucleus. A consideration of different nucleon density distributions results in considerable complication of all computations. We therefore tried to account for the density distribution purely phenomenologically without assuming any particular form for the two-nucleon wave function. By giving the approximate character of the density distribution of the nucleons in the nucleus and the approximate dependence of the reabsorption probability at the instant of production on the density, we can obtain the dependence of σ_{π^0} on A in terms of the parameter f_0 which describes the probability of a meson to be created in the center of the nucleus without being reabsorbed in a two-nucleon process. Comparing the obtained curves with the experimental results we can establish which value of f_0 gives the closest agreement with the observed A-dependence. The calculation was done for a density distribution which is in agreement with the electron scattering data,[8] i.e., for the Fermi distribution

$$\rho(r) = \rho_0 / [e^{(r-c)/d} + 1], \tag{8}$$

where ρ_0 approximately equals the density at $r = 0$ (since $e^{-c/d} \ll 1$), c is the radius at which the density has one half its central value, $d = t/4.4$ where t is the surface thickness (the density changes in this layer from $0.9\rho_0$ to $0.1\rho_0$).

The probability of photoproduction of mesons in a nucleus without reabsorption actually depends on the density ρ of nuclear matter. The photoproduction probability averaged over the nuclear volume is given by

$$f_q^* = \frac{4\pi}{A}\int_0^\infty r^2 \rho(r) f(\rho)\, dr. \tag{9}$$

We shall assume that the dependence of f on ρ has the form

$$f(\rho) = 1 - (1 - f_0)\rho/\rho_0. \tag{10}$$

Then

$$f_q^* = {}^4/_3 (\pi c^3/A)\rho_0 \{f_0 [1 + \pi^2 (d/c)^2] + (1 - f_0)[3d/c + \pi^2 (d/c)^3]\}. \tag{11}$$

The first term in (11) gives the contribution of "volume production" and yields the proportionality $f_q^* A \sim c^3$. The second term, the contribution of "surface production," contains a factor 1/c and thus yields $f_q^* A \sim c^2$. The magnitude of the second term is proportional to the volume of the surface layer of the nucleus.

The obtained expression still has to be corrected for the usual reabsorption of the mesons, similarly as in the case treated in Sec. 2.

In Fig. 3 the results of the computation are plotted for $\lambda_a = 17 r_0$ and for the three values $f_0 = 0$, 0.1, and 0.2.

As one sees from a comparison with the experimental points from reference 6, which are also plotted in Fig. 3, the value of f_0 and the dependence $\sigma_{\pi^0} = f(A)$ agrees with the above obtained results. The relative magnitude of the contribution of "surface production" to the meson production as given by (11) is plotted in Fig. 2 as the dotted curves 2 and 3.

We shall not discuss here different possible reasons for changes in the obtained dependence $\sigma_{\pi^0}(A)$ (e.g., the influence of the binding energy and the momentum distribution of the nucleons, the charge exchange scattering of π^+ mesons etc.). We only wish to point out the as yet unfeasible direct establishment of the facts of the two-nucleon absorption of slow mesons not at the instant of creation but on the subsequent passage through the nucleus. When a fast (~70 Mev) π^+ meson is absorbed by a quasideuteron or a fast π^0 meson is absorbed by a pair of protons two protons are emitted with approximately equal energy into roughly opposite directions while the emission is roughly isotropic with respect to the direction of the incoming γ-ray beam.

The cross section for the production of such pairs which are due to the reabsorption of π^+ mesons must have the form

$$\sigma = (\sigma_{\pi^+})_p^0 \eta Z \left\{ f_0 (1-\chi)(1 - A^{-1/3})^3 + \frac{3}{A}(1-\chi_s)\int_{A^{1/3}-1}^{A^{1/3}} z^2 f_s(z)\, dz \right\} \frac{\lambda_a}{\lambda_2}\Gamma f_{pp}, \tag{12}$$

where Γ is a factor that describes the two-nucleon correlation in the nucleus ($\Gamma \approx 6$), λ_a and λ_2 are the meson mean free paths with respect to the total and the two-nucleon absorption respectively, and f_{pp} is the probability that both protons leave the nucleus (equal, for example 0.6 for carbon, 0.4 for iron and 0.25 for lead).

The possibility of actually observing such processes is indicated by the work of Rosengren and Dudley[9] who report the existence of an isotropic component in the angular distribution of photoprotons with an energy close to 70 Mev which is due to the absorption of π mesons by pairs of nucleons.

The authors thank E. M. Leĭkin for his participation in an evaluation of this work.

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Translated by M. Danos
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