DUAL SCATTERING OF RELATIVISTIC PIONS BY NUCLEONS

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The formalism of the dual scattering theory based on invariance of the interaction Hamiltonian under rotations in isotopic and x space is extended to relativistic energies. The expression for the differential cross section is obtained in a form which is convenient for analysis of the experimental data. To illustrate the application of the formal theory, the problem is considered in the approximation of radiation damping theory.

INTRODUCTION

THE production of pions in low-energy meson-nucleon collisions ("dual scattering" of pions) has been considered in a dissertation by d'Espagnat[1] and in a number of other papers.[2-4] The theory developed in these papers is subject to a number of formal limitations: (1) In deriving the equations for the scattering matrix T it is assumed that dual scattering is much less significant than single-meson scattering.* (2) Nucleon recoil is neglected, thus leading to the use of an approximate density function $\rho(1, 2)$ of the final states which is independent of angular variables, with the result that the angular distribution of scattered mesons is distorted at high energies. (3) The differential cross section is derived[1] on the assumption that only s and p waves are important in the scattering.

It is the purpose of the present paper to examine the formalism of the dual scattering theory at relativistic energies. Racah's procedure enables us to derive a formula for the differential cross section which is valid at any energy and can be used conveniently to analyze the experimental angular and momentum distributions of scattered pions (Sec. 3A). The scattering theory establishes an equation that relates the matrix T to the reaction matrix K. In Sections 1 and 2 this equation is studied for moderate relativistic energies, when effects associated with ternary scattering and the production of "new" particles are still small but the nucleon must be regarded as a relativistic particle.

The formalism that is developed is used for a solution of the problem in the approximation of the radiation damping theory (Sec. 3B). Although the damping theory cannot account for experimental scattering results[5] at low energies of the incident meson in the laboratory system $T_{kin} \ll M$ (nucleon rest mass), the results of this section may be of interest in explaining the role of damping at energies comparable with or greater than M.

*We here exclude the nonrigorous procedure based on the assumption that the dual scattering matrix can be represented by the product of a function depending on the energy of the incident meson and a function of the energy of the scattered mesons.[1]

1. K AND T MATRICES IN DUAL SCATTERING

Lippman and Schwinger[6] and Goldberger[7] have shown that an exact statement of the scattering problem leads to a set of two equations in the matrix T and reaction matrix K. The first is determined on a constant-energy surface while the second is determined both on and outside of a constant-energy surface:

$$K_{\beta a} = H_{\beta a} + P\sum_{\gamma} H_{\beta\gamma} K_{\gamma a} / (E_a - E_\gamma), \quad (1)$$

$$T_{ba} = K_{ba} - i\pi \sum_{c} K_{bc}\delta(E_a - E_c) T_{ca}. \quad (2)$$

Here $H_{\beta a} = (\Phi_\beta, \hat{H}\Phi_a)$ is the matrix element of the interaction Hamiltonian for the transition between states a and β (following reference 7, we use the indices b, c for states with $E_b = E_a$ and β, γ for $E_\beta \neq E_a$) and P in (1) indicates that the principal value of the integral is taken.

The probability of a transition from state a to state b is related to the matrix T_{ba} very simply as follows:[6]

$$w_{ba} = 2\pi |T_{ba}|^2 \rho(E_b), \quad (3)$$

where $\rho(E_b)$ is the density of states.*

Let us now consider dual scattering of a pion by a nucleon. In the center-of-mass system a meson with energy W_i and momentum $\mathbf{k}_i = \kappa_i \mathbf{n}_i$ ($\mathbf{n}_i$ is

*We use a system of units in which $m_\pi = c = \hbar = 1$.

the unit vector of direction), the isotopic state of which will be denoted by α ($\alpha = +, -, 0$), collides with a nucleon (energy E_i, momentum $\mathbf{p}_i$, spin s_i and isotopic spin t_i). In the final state we have a nucleon (E_f, $\mathbf{p}_f$, s_f, t_f) and two mesons (W_1, $\mathbf{k}_1$, β) and (W_2, $\mathbf{k}_2$, γ).

It is known that the solution of (1) is difficult and requires a number of field approximations. We confine ourselves to (2) and use (1) only to establish the general form of the reaction matrix K. The interaction operator of the nucleon and meson fields will be the Hamiltonian of symmetric pseudoscalar theory,

$$\hat{H}(x) = g\overline{\Psi}\gamma_5\boldsymbol{\tau}\Psi\boldsymbol{\varphi} + if\overline{\Psi}\gamma_5\gamma_\nu\boldsymbol{\tau}\Psi\frac{\partial\boldsymbol{\varphi}}{\partial x_\nu}, \quad (4)$$

where Ψ is the nucleon field operator, φ is the meson field operator, τ is the nucleon isotopic spin vector, $\gamma_5 = \beta\gamma^1\gamma^2\gamma^3$ and $\gamma_5 = (\beta\alpha^k, \beta)$. $\hat{H}(x)$ is invariant under rotations and reflections in x space and under rotations in isotopic spin space.

Let Ω_i denote the set of variables that characterize the initial (nucleon-meson) state and let Ω_f denote the variables of the final (nucleon-2meson) state. When isotopic spin is excluded the corresponding sets of variables will be denoted by ω_i and ω_f.

From the definition of the center-of-mass system,

$$\mathbf{p}_i + \mathbf{k}_i = \mathbf{p}_f + \mathbf{k}_1 + \mathbf{k}_2 = 0, \quad (5)$$

the nucleon momenta $\mathbf{p}_i$ and $\mathbf{p}_f$ can be expressed in terms of the meson momenta $\mathbf{k}_i$ and $\mathbf{k}_1$, $\mathbf{k}_2$. We then have

$$\omega_i = \{\varkappa_i, \mathbf{n}_i; s_i\}, \qquad \omega_f = \{\varkappa_1, \mathbf{n}_1; \varkappa_2, \mathbf{n}_2; s_f\}, \quad (6)$$

where κ_i, κ_1, κ_2 are related by energy conservation as follows:

$$E = (M^2 + \varkappa_i^2)^{1/2} + (1 + \varkappa_i^2)^{1/2} \quad (7)$$
$$= (M^2 + \varkappa_1^2 + \varkappa_2^2 + 2\varkappa_1\varkappa_2(\mathbf{n}_1\cdot\mathbf{n}_2))^{1/2} + (1 + \varkappa_1^2)^{1/2} + (1 + \varkappa_2^2)^{1/2}.$$

The matrix K in isotopic and momentum space is determined through its representation by an iteration series [see Eq. (1)]. It is easily shown from a study of the general term of this series that for interaction (4) the matrix element of K on a constant-energy surface is

$$\langle\Omega_f|K|\Omega_i\rangle = \langle t_f|\tau_\gamma^+\tau_\beta^+\tau_\alpha K_1 + \tau_\beta^+\tau_\gamma^+\tau_\alpha K_2 + \tau_\gamma^+\tau_\gamma\tau_\beta^+ K_3$$
$$+ \tau_\beta^+\tau_\alpha\tau_\gamma^+ K_4 + \tau_\alpha\tau_\gamma^+\tau_\beta^+ K_5 + \tau_\alpha\tau_\beta^+\tau_\gamma^+ K_6|t_i\rangle, \quad (8)$$

where τ_α are related to the Cartesian components of τ by

$$\tau_\pm = \mp(\tau_x \pm i\tau_y)/\sqrt{2}, \qquad \tau_0 = \tau_z, \quad (9)$$

and each of the K_j ($j = 1, 2, \ldots 6$) can be represented by a linear combination of the pseudoscalars $(\boldsymbol{\sigma}\cdot\mathbf{n}_1)$, $(\boldsymbol{\sigma}\cdot\mathbf{n}_2)$, $(\boldsymbol{\sigma}\cdot\mathbf{n}_i)$ and $i(\mathbf{n}_1\times\mathbf{n}_2\cdot\mathbf{n}_i)\eta$ (where η is the unit matrix) with coefficients that are invariant functions of $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_i$ and κ_1, κ_2, κ_3. Expressing κ_2 of (7) as a function of κ_1, E and $\cos\theta_{12} = \mathbf{n}_1\cdot\mathbf{n}_2$ and expanding these coefficients in series of the Legendre polynomials $P_{\lambda_1}(\mathbf{n}_1\cdot\mathbf{n}_2)$, $P_{\lambda_2}(\mathbf{n}_2\cdot\mathbf{n}_i)$ and $P_\lambda(\mathbf{n}_1\cdot\mathbf{n}_2)$, we obtain for K_j:

$$\begin{aligned}\langle\omega_f|K_j|\omega_i\rangle = \{&A_0 + B_0P_1(\mathbf{n}_1\cdot\mathbf{n}_i) + C_0P_1(\mathbf{n}_2\cdot\mathbf{n}_i)\\ &+ D_0P_1(\mathbf{n}_1\cdot\mathbf{n}_2) + \ldots\}(\sigma\cdot\mathbf{n}_i)_{s_fs_i}\\ &+ \{A_1 + B_1P_1(\mathbf{n}_1\cdot\mathbf{n}_i) + C_1P_1(\mathbf{n}_2\cdot\mathbf{n}_i)\\ &+ D_1P_1(\mathbf{n}_1\cdot\mathbf{n}_2) + \ldots\}(\sigma\cdot\mathbf{n}_1)_{s_fs_i}\\ &+ \{\text{symm. terms } 1\rightleftarrows 2\} + \{A + \ldots\}\, i([\mathbf{n}_1\times\mathbf{n}_2]\cdot\mathbf{n}_i)(\eta)_{s_fs_i},\end{aligned} \quad (10)$$

where A_0, B_0, ... depend only on κ_i; κ_1.

The complete matrix $\langle\Omega_f|K|\Omega_i\rangle$ must of course be symmetric with respect to the interchange of all variables of the emitted mesons, $1\rightleftarrows 2$ (pions are bosons).

For not too high energies ($T_{kin} < 1.5$ Bev) we can in (2) neglect ternary scattering and the production of "new" particles. For dual scattering T is then determined from a system of equations written schematically as follows:

$$\begin{aligned}\langle 1_f, 2_f|T|1_i\rangle = \langle 1_f, 2_f|K|1_i\rangle\\ - i\pi\sum_{1'}\rho(1')\langle 1_f, 2_f|K|1'\rangle\langle 1'|T|1_i\rangle\\ - i\pi\sum_{2'}\rho(2')\langle 2_f|K|2'\rangle\langle 1_f, 2'|T|1_i\rangle\\ - i\pi\sum_{1'}\rho(1')\langle 1_f|K|1'\rangle\langle 1', 2_f|T|1_i\rangle\\ - i\pi\sum_{1',2'}\rho(1', 2')\langle 1_f, 2_f|K|1', 2'\rangle\langle 1', 2'|T|1_i\rangle;\\ \langle 1_f|T|1_i\rangle = \langle 1_f|K|1_i\rangle - i\pi\sum_{1'}\rho(1')\langle 1_f|K|1'\rangle\langle 1'|T|1_i\rangle\\ - i\pi\sum_{1',2'}\rho(1', 2')\langle 1_f|K|1', 2'\rangle\langle 1', 2'|T|1_i\rangle.\end{aligned} \quad (11)$$

Here $\langle 1_f, 2_f|T|1_f\rangle$ is the matrix of dual scattering and $\langle 1_f|T|1_i\rangle$ is the matrix of single-meson scattering. Then density functions $\rho(1)$ and $\rho(1, 2)$ are obtained from δ functions by integrating over the energy. The term

$$X_1 = \sum_{2'}\rho(2')\langle 2_f|K|2'\rangle\langle 1_f, 2'|T|1_i\rangle,$$

for example, describes rescattering of one of the two scattered mesons (meson "2") in another direction (with observation of the conservation laws). The matrices $\langle 1_f | K | 1_i \rangle$ and $\langle 1_f, 2_f | K | 1', 2' \rangle$ are given similarly by (8) and (10).

2. K AND T MATRICES IN THE TOTAL MOMENTS REPRESENTATION

The state of our system is described by the following quantum numbers: The total isotopic spin I, its projection M_I, the total angular momentum J, its projection μ_J, and parity ξ. In addition, the initial state is characterized by the orbital angular momentum l and the final state by the total L and partial l_1, l_2 orbital moments as well as by the isotopic spin Λ of the two-meson system (the isotopic spin of a meson is 1). Since the interaction Hamiltonian is invariant under rotations and reflections K has the following diagonal form in the representation based on eigenstates of total isotopic spin and total angular momentum:

$$\begin{aligned}(I'M_I'\Lambda;\ J'\mu_J'L(l_1'l_2)\xi'|K|IM_I;\ J\mu_J l_\xi) \\ = K^{I\Lambda}_{JlL(l_1l_2)}\delta_{II'}\delta_{JJ'}\delta_{M_IM_I'}\delta_{\mu_J\mu_J'}\delta_{\xi\xi'},\end{aligned} \tag{12}$$

where $l_1 + l_2 - l$ is odd (because of the pseudoscalar pion wave function). We have the following orthogonal transformation to the total isotopic spin representation:*

$$\begin{aligned}\langle\Omega_f|K|\Omega_i\rangle \\ = \sum_{I=1/2}^{I=3/2}\sum_{\Lambda=I-1/2}^{I+1/2} C^{IM_I}_{\Lambda 1/2;\ Mt_f} C^{\Lambda M}_{11;\ \beta\gamma}(\omega_f|K|\omega_i)^{I\Lambda} C^{IM_I}_{1\,1/2;\ \alpha t_i}.\end{aligned} \tag{13}$$

For the matrix $\langle\Omega_f | K | \Omega_i\rangle$ in the form (8) we have

$$\begin{aligned}(\omega_f|K|\omega_i)^{3/2;\,2} &= -\sqrt{10}\,(\omega_f|K_3+K_4|\omega_i),\\ (\omega_f|K|\omega_i)^{3/2;\,1} &= \sqrt{2}\,(\omega_f|2(K_6-K_5)+K_3-K_4|\omega_i),\\ (\omega_f|K|\omega_i)^{1/2;\,1} &= \sqrt{2}\,(\omega_f|3(K_2-K_1)\\ &\quad +K_3-K_4+K_5-K_6|\omega_i),\\ (\omega_f|K|\omega_i)^{1/2;\,0} &= (\omega_f|3(K_1+K_2+K_5+K_6)-K_3-K_4|\omega_i).\end{aligned} \tag{14}$$

We now expand the matrix $(\omega_f | K | \omega_i)^{I\Lambda}$ for arbitrary I and Λ in eigenfunctions of the total angular momentum:

$$\begin{aligned}(\omega_f|K|\omega_i)^{I\Lambda} = \sum_{J}\ \sum_{l,\ L=J-1/2}^{J+1/2}\ \sum_{|l_1+l_2|=L} K^{I\Lambda}_{JlL(l_1l_2)}(\varkappa_1, \varkappa_i)\\ \times\sum_{\mu_J} G^{J,\,\mu_J}_{L(l_1l_2)}(s_f, \mathbf{n}_1, \mathbf{n}_2)\, g_l^{J,\,\mu_J+}(s_i;\ \mathbf{n}_i).\end{aligned} \tag{15}$$

Here

$$g_l^{J,\,\mu_J}(s_i, \mathbf{n}_i) = \sum_m C^{J\mu_J}_{l\,1/2;\ ms_i} Y_{lm}(\mathbf{n}_i), \tag{16}$$

$$\begin{aligned}G^{J,\,\mu_J}_{L(l_1l_2)}(s_f;\mathbf{n}_1,\mathbf{n}_2) &= \sum_M C^{J\mu_J}_{L\,1/2;\ Ms_f} Y^M_{L(l_1l_2)}(\mathbf{n}_1,\mathbf{n}_2)\\ &= \sum_{m_1,\,m_2,\,M} C^{J\mu_J}_{L\,1/2;\ Ms_f} C^{LM}_{l_1l_2;\ m_1m_2} Y_{l_1m_1}(\mathbf{n}_1)\,Y_{l_2m_2}(\mathbf{n}_2)\end{aligned} \tag{17}$$

are orthogonal and normalized functions of the nucleon-meson system and of the nucleon-2 meson system; respectively; $Y_{lm}(\mathbf{n})$ is the usual spherical function.

When the matrix $(\omega_f | K | \omega_i)^{I\Lambda}$ is given by (10) its matrix elements in the J, μ_J representation, $K^{I\Lambda}_{JlL(l_1l_2)}(\kappa_1, \kappa_i)$, will be linear combinations of A_0, B_0, ... etc. The expansions (13) and (15) also apply to the T matrix.

In order to obtain a set of equations relating T and K that are reduced with respect to total isotopic spin and total angular momentum we represent all matrices in (11) as expansions in functions of the total moments. When expanding the matrices $\langle 2_f | K | 2' \rangle$ that are in the X terms it must be taken into account that because of energy and momentum conservation these matrices will also depend on the momentum of meson 1_f, so that is more convenient to expand them in momentum space "on the left" in functions of the nucleon-2 meson system.

For elastic scattering $\kappa = \kappa_i$, whence the density function of nucleon-meson states becomes

$$\rho(1) = \rho_1(\varkappa_i) = \varkappa_i W_i (E - W_i)/(2\pi)^3 E. \tag{18}$$

From (7) the density function of nucleon-2 meson states is

$$\begin{aligned}\rho(1, 2) &= \rho_2(\varkappa_1, \cos\theta_{12})\\ &= \frac{1}{(2\pi)^6}\frac{\varkappa_1^2\varkappa_2 W_2(E - W_1 - W_2)}{E - W_1 + (W_2/\varkappa_2)\varkappa_1\cos\theta_{12}},\end{aligned} \tag{19}$$

where the angle θ_{12} is related to the emission angles ϑ_1, φ_1 and ϑ_2, φ_2 of the scattered pions by

$$\cos\theta_{12} = \cos\vartheta_1\cos\vartheta_2 + \sin\vartheta_1\sin\vartheta_2\cos(\varphi_1 - \varphi_2). \tag{20}$$

At relativistic energies the dependence of $\rho(1, 2)$ on $\cos\theta_{12}$ becomes significant and complicates the reduction because, when integrating over angles, in terms containing $\rho(1, 2)$ we cannot directly make use of the orthogonality of the functions $Y^M_{L(l_1l_2)}(\mathbf{n}_1 \cdot \mathbf{n}_2)$.

The separation of isotopic, angular and spin variables results in the following system of linear algebraic integral equations:

*Here and hereinafter $C^{c\gamma}_{ab;\ \alpha\beta}$ are Clebsch-Gordan coefficients, W(abcd; ef) are Racah coefficients, Z(abcd; ef) are the coefficients used in reference 8 and $U\begin{pmatrix} a & b & e \\ c & d & e' \\ f & f' & g \end{pmatrix}$ are generalized Racah coefficients.[9]

$$T^{I\Lambda}_{JlL\,(l_1 l_2)}(\varkappa_1, \varkappa_i)$$

$$= K^{I\Lambda}_{JlL\,(l_1 l_2)}(\varkappa_1, \varkappa_i) - i\pi\rho_1(\varkappa_i)\, K^{I\Lambda}_{JlL\,(l_1 l_2)}(\varkappa_1, \varkappa_i)\, T^{I}_{Jl}(\varkappa_i)$$

$$- i\pi \sum_{I'', J'', \ldots} a^{I\Lambda(I'')\Lambda'} \tilde{\rho}_1(\varkappa_1)\, K^{I''}_{J''l''L''(l''_1 l_2)}(\varkappa_1)\, T^{I\Lambda'}_{JlL'\,(l'_1 l'_2)}(\varkappa_1, \varkappa_i)$$

$$-i\pi\{\text{symm. terms } 1 \rightleftarrows 2\} \qquad (21)$$

$$- i\pi \sum_{\Lambda', L', \ldots} \int_0^{\varkappa'_{1\,max}} d\varkappa'_1 \tilde{\rho}_2(\varkappa'_1)\, K^{I\Lambda\Lambda'}_{JL'(l''_1 l''_2)L(l_1 l_2)}(\varkappa_1, \varkappa'_1)\, T^{I\Lambda'}_{JlL'\,(l'_1 l'_2)}(\varkappa'_1, \varkappa_i),$$

$$T^{I}_{Jl}(\varkappa_i) = K^{I}_{Jl}(\varkappa_i) - i\pi\rho_1(\varkappa_i)\, K^{I}_{Jl}(\varkappa_i)\, T^{I}_{Jl}(\varkappa_i)$$

$$- i\pi \sum_{\Lambda', L', \ldots} \int_0^{\varkappa'_{1\,max}} d\varkappa'_1 \tilde{\rho}_2(\varkappa'_1)\, K^{I\Lambda'}_{JL'(l''_1 l''_2)\,l}(\varkappa_i, \varkappa'_1)\, T^{I\Lambda'}_{JlL'(l'_1 l'_2)}(\varkappa'_1, \varkappa_i).$$

The summation in (21) is taken over all primed indices. We have here the coefficient

$$a^{I\Lambda\,(I'')\Lambda'} = (2I''+1)\sqrt{(2\Lambda+1)(2\Lambda'+1)}$$

$$\times W(11{}^1\!/_2 I; \Lambda I'')\, W(11{}^1\!/_2 I; \Lambda' I''). \qquad (22)$$

$\tilde{\rho}_1(\kappa_1)$ and $\tilde{\rho}_2(\kappa'_1)$ are abbreviations for

$$\tilde{\rho}_1(\varkappa_1) = \sum_\lambda A_\lambda \int d\cos\theta_{12}\rho_1(\varkappa_1, \cos\theta_{12'})\, P_\lambda(\cos\theta_{12'}), \qquad (23)$$

$$\tilde{\rho}_2(\varkappa'_1) = \sum_\lambda B_\lambda \int d\cos\theta_{1'2'}\, \rho_2(\varkappa'_1, \cos\theta_{1'2'})\, P_\lambda(\cos\theta_{1'2'}), \qquad (24)$$

where

$$A_\lambda = \sum_r \frac{1}{2\sqrt{4\pi}}(-1)^{L''+L'+L+l'_1+l'_2+l_2+l''+\lambda}\, i^{\,l''_1-l'_1-l_1-\lambda}$$

$$\times (2J''+1)(2\lambda+1)^{1/2} C^{l'_2 0}_{\lambda l''; 00}\, W(LJL''J''; \tfrac{1}{2} r)$$

$$\times W(L'Jl''J''; \tfrac{1}{2} r)\, Z(l_1 L l''_1 L''; l_2 r)\, Z(l'_1 L'\lambda l''; l'_2 r), \qquad (25)$$

$$B_\lambda = \tfrac{1}{2}(-1)^{L''+l''_2+l'_2}\, i^{\,l''_1-l'_1-\lambda}\, C^{\lambda 0}_{l''_2 l'_2; 00}\, Z(l''_1 l''_2 l'_1 l'_2; L'\lambda). \qquad (26)$$

3. DUAL SCATTERING CROSS SECTIONS

A. General Relations

Using (3), (13) and (19), we can represent the differential cross section for dual scattering by

$$d\sigma(t_f; \beta, \gamma \mid t_i, \alpha) = \sum_{I'\Lambda'} \sum_{I, \Lambda} \Delta^{I'\Lambda'; I\Lambda}_{\alpha t_i; \beta\gamma t_f}\, d\sigma^{I'\Lambda'; I\Lambda}, \qquad (27)$$

where the coefficient $\Delta^{I'\Lambda'; I\Lambda}_{\alpha t_i; \beta_\gamma t_f}$ contains the charge character of the process (see reference 10, for example) and

$$d\sigma^{I'\Lambda'; I\Lambda} = \frac{2\pi}{v}\frac{1}{2} \sum_{s_f, s_i} (\omega_f \mid T \mid \omega_i)^{I'\Lambda'+} (\omega_f \mid T \mid \omega_i)^{I\Lambda}$$

$$\times \rho_2(\varkappa_1, \cos\theta_{12})\, d\varkappa_1 d\Omega_1 d\Omega_2. \qquad (28)$$

Here $d\Omega_1 = \sin\vartheta_1 d\vartheta_1 d\varphi_1$ and $d\Omega_2 = \sin\vartheta_2 d\vartheta_2 d\varphi_2$ are the solid angles of pion scattering and v is the velocity of the incident meson. In (28) we assume that $(\omega_f \mid T \mid \omega_i)$ is represented by the expansion (15) with the functions g and G given by (16) and (17).

In the coordinate system where the polar axis has the direction of incident meson momentum,

$$g^{J\mu J}_l(s_i, \mathbf{n}_i) \to \sqrt{(2l+1)/4\pi}\, C^{Js_i}_{l^{1}/_2; 0s_i}.$$

Following Racah,[8,9,11] we can obtain an equation for the differential cross section that is directly comparable with experiment. Integration of $d\sigma^{I'\Lambda'; I\Lambda}$ in (28) over the azimuthal angles φ_1 and φ_2, followed by rather complicated transformations gives*

$$d\sigma^{I'\Lambda'; I\Lambda}(\varkappa_1; \vartheta_1, \vartheta_2) = \frac{1}{32v} \sum_{J'l'L'(l'_1 l'_2)} \sum_{JlL(l_1 l_2)} T^{I'\Lambda'*}_{J'l'L'(l'_1 l'_2)}(\varkappa_1, \varkappa_i)\, T^{I\Lambda}_{JlL(l_1 l_2)}(\varkappa_1\ \varkappa_i)$$

$$\times \sum_{L_1, L_2} \sum_\lambda D_{L_1 L_2; \lambda}\, \rho_\lambda(\varkappa_1)\, P_{L_1}(\cos\vartheta_1)\, P_{L_2}(\cos\vartheta_2)\, d\varkappa_1 d\cos\vartheta_1 d\cos\vartheta_2, \qquad (29)$$

where

$$\rho_\lambda(\varkappa_1) = \int_{-1}^{+1} \rho_2(\varkappa_1, \cos\theta_{12})\, P_\lambda(\cos\theta_{12})\, d\cos\theta_{12}, \qquad (30)$$

$$\times D_{L_1 L_2; \lambda} = (-1)^{l'_1+l'_2+L_1+L_2-L-\lambda}\, i^{\,l+L_1-l'-L_2}(2\lambda+1)\sqrt{(2J'+1)(2L'+1)(2l'_1+1)(2l'_2+1)(2J+1)(2L+1)(2l_1+1)(2l_2+1)}$$

$$\times \sum_{\Lambda_1, \Lambda_2} \sum_R (-1)^R C^{\Lambda_1 0}_{l'_1 l_1; 00}\, C^{\Lambda_2 0}_{l'_2 l_2; 00}\, C^{L_1 0}_{\Lambda_1\lambda; 00}\, C^{L_2 0}_{\Lambda_2\lambda; 00}\, W(LJL'J'; {}^1\!/_2 R)\, Z(lJl'J'; {}^1\!/_2 R)\, Z(L_1\Lambda_1 L_2\Lambda_2; \lambda R)\, U\begin{pmatrix} l'_1 & l'_2 & L' \\ l_1 & l_2 & L \\ \Lambda_1 & \Lambda_2 & R \end{pmatrix}. \qquad (31)$$

*The integral functions $\rho_\lambda = \int_0^{\varkappa\,max} \rho_\lambda(\varkappa)d\varkappa$ diminish as λ increases. For example, when $T_{kin} \approx 1.5$ Bev we have $\rho_0 : \rho_1 : \rho_2 = 1 : (-0.27) : 0.085$. Therefore it is sufficient to use only small values of λ in practical calculations by means of (29). In the non-relativistic limit (for a static nucleon) all $\rho_{\lambda \neq 0} = 0$.

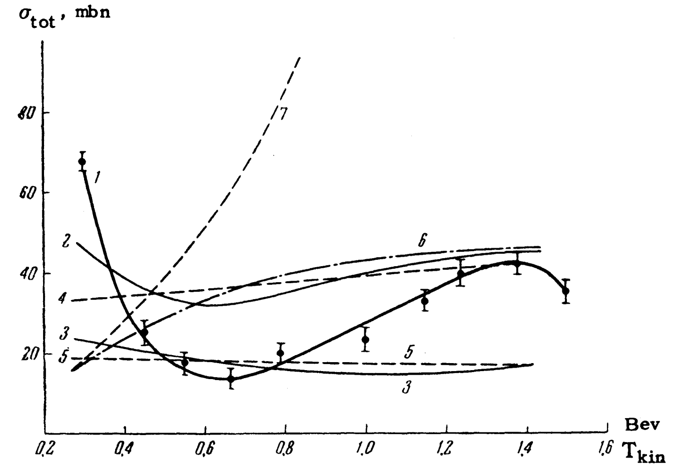

FIG. 1. Energy dependence of the total (elastic + inelastic) cross section for π^+ meson scattering by protons. Curve 1) experimental; curves 2–7) theoretical (including only dual scattering among the inelastic processes); 2–5) for ps(ps) coupling; 2, 3) by damping theory (2 – g^2 = 6; 3 – g^2 = 3); 4, 5) by perturbation theory (4 – g^2 = 3; 5 – g^2 = 2.2); 6, 7) for ps(pv) coupling; 6) by damping theory (f^2 = 1/12); 7) by perturbation theory (f^2 = 1/12)

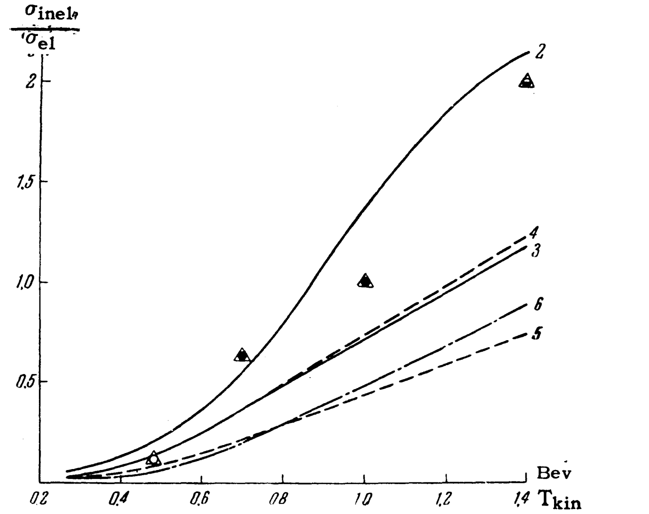

FIG. 2. Ratio of inelastic to elastic scattering cross section for π^+ meson scattering by protons. The notation is the same as in Fig. 1. Experimental data were taken from reference 14 for π^+, p scattering (T_{kin} = 0.5 Bev) and π^-, p scattering (T_{kin} 0.7 Bev, 1 Bev); also π^-, n (T_{kin} = 1.4 Bev) (since for this energy there are no corresponding data for π^+, p scattering).

The momentum distribution is obtained by integrating over $d\cos\vartheta_1$ and $d\cos\vartheta_2$, with the result

$$d\sigma^{I'\Lambda';I\Lambda}(\varkappa_1)=\frac{1}{8v}\sum_{JlL(l_1l_2)}\sum_{l_1'l_2'}T^{I'\Lambda'*}_{JlL(l_1'l_2')}(\varkappa_1;\varkappa_i)\times T^{I\Lambda}_{JlL(l_1l_2)}(\varkappa_1,\varkappa_i)\sum_{\lambda}D_{00;\lambda}\rho_{\lambda}(\varkappa_1)\,d\varkappa_1, \tag{32}$$

where

$$D_{00;\lambda}=(-1)^{l_2'-l_2-L}i^{l_1'-l_1-\lambda}(2J+1)C^{\lambda 0}_{l_2'l_2;00}Z(l_1l_2'l_1l_2;L\lambda). \tag{33}$$

B. Damping Theory

The dual scattering cross section and the related single-meson scattering cross section were calculated according to the theory of radiation damping, where the first Born approximation is taken for the reaction matrix K. The covariant formalism of Fukuda[12] and Pirenne[13] was used; pseudoscalar ps(ps) and pseudovector ps(pv) coupling were considered [see (4)].

When we do not limit ourselves to nonrelativistic energies, $K^{I,\dots}_{J,\dots}(\kappa_1,\dots)$ are complicated functions and the solution of (21) is difficult. We replace each $K^{I,\dots}_{J,\dots}(\kappa_1,\dots)$ by its value for $\kappa_1=\kappa_2$, in which case (21) becomes an ordinary system of linear algebraic equations with its solution given by the ratio of two determinants consisting of equation coefficients. It can be shown by studying the character of the functions $K^{I,\dots}_{J,\dots}(\kappa_1,\dots)$ and of the density function $\rho_2(\kappa_1,\cos\theta_{12})$ that the total dual scattering cross section is not essentially changed.

In the present work values of $K^{I\Lambda}_{JlL(l_1l_2)}(\kappa_1,\kappa_2)$ (for $\kappa_1=\kappa_2$) were calculated for four incident meson energies (in the laboratory system): T_{kin} = 0.275, 0.52, 0.84 and 1.4 Bev, and divided into two groups such that, for a given energy, each member of the second group was not greater in absolute magnitude than ⅓ of any member of the first group.

In order to obtain the matrix elements $T^{I\Lambda}_{JlL(l_1l_2)}\times(\kappa_1,\kappa_2)$ corresponding to the first group we solved a set of approximate equations obtained from (21) by dropping the last term of the first and second equations (two mesons — 1′, 2′ — in the intermediate state). The single-meson scattering matrix elements $T^{I}_{Jl}(\kappa_i)$ were derived from the same equations.

We assumed $T^{I\Lambda}_{JlL(l_1l_2)}=K^{I\Lambda}_{JlL(l_1l_2)}$ for the second group, which in the present case represents the first approximation of damping theory with respect to the coupling constant. For purposes of comparison we also considered perturbation theory ($T^{I\Lambda}_{JlL(l_1l_2)}=K^{I\Lambda}_{JlL(l_1l_2)}$ for all scattering amplitudes). Figures 1 and 2 show the results for π^+, p scattering. A comparison of the curves calculated by perturbation theory and by damping theory for ps(ps) coupling shows that damping somewhat "improves" the energy dependence of the total cross section, although the pronounced resonant character of the experimental curves cannot be obtained with any of the coupling constants. Inclusion of damping enables us to increase the fraction of inelastic processes (dual scattering) for the same total cross section. (The experimental points in Fig. 2 lie within a region bounded by curves calculated by the damping theory but go outside this region in the case of perturbation theory.)

With ps (pv) coupling damping sharply reduces the growth of the total cross section, but the same coupling constant cannot be used to obtain a total cross section and a cross section ratio which are both close to the experimental values.

A calculation of the total cross section for π^-, p scattering with ps (ps) coupling shows that the damping theory cannot account for the second peak in the energy-dependence curve of the total cross section. For any one coupling constant the theoretical curves for π^-, p scattering lie below the curves for π^+, p.

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APPENDIX

ANGULAR POLYNOMIALS FOR DUAL SCATTERING

For some features of the theory (the semi-phenomenological theory and the derivation of dispersion relations) it is useful to know the expansion of matrix T in operators that project the initial (one-meson) eigenstate with given total isotopic spin and angular momentum into the final (two-meson) state.

We represent (15) for the T matrix as follows:

$$(\omega_f \mid T \mid \omega_i)^{I\Lambda} = \frac{1}{(4\pi)^{3/2}} \sum_J \sum_{l,L=J-1/2}^{J+1/2} \sum_{|\mathbf{l}_1+\mathbf{l}_2|=L} T^{I\Lambda}_{JlL(l_1 l_2)} Q^{fi}_{JlL(l_1 l_2)}; \quad \text{(A.1)}$$

$$Q^{fi}_{JlL\,(l_1 l_2)} = (4\pi)^{3/2} \sum_{\mu_J} G^{J,\,\mu_J}_{L(l_1 l_2)} (s_f; \mathbf{n}_1, \mathbf{n}_2)\, g^{J,\,\mu_J^+}_{l} (s_i, \mathbf{n}_i) \quad \text{(A.2)}$$

form a set of orthogonal operators for dual scattering (the so-called "angular polynomials").* If we confine ourselves to $l,\ l_1,\ l_2 \leq 2$ we have the following operators:†

$$Q^{(\pm)}_{1/2;0,1(10)} = -\frac{1}{\sqrt{2}} (\sigma\cdot\mathbf{n}^{(\pm)}_{12}),$$

$$Q^{(\pm)}_{1/2;0,1(21)} = \frac{3}{2} [(\mathbf{n}_2\cdot\mathbf{n}_1) \mp 1/3] (\sigma\cdot\mathbf{n}^{(\pm)}_{12}),$$

$$Q^{(+)}_{3/2;1,2(11)} = -3\sqrt{\frac{3}{10}}\Big[(\mathbf{n}_2\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_1)$$

$$+ (\mathbf{n}_1\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_2) - \frac{2}{3}(\mathbf{n}_2\cdot\mathbf{n}_1)(\sigma\cdot\mathbf{n}_i)\Big],$$

$$Q^{(\pm)}_{3/2;1,2(20)} = -\frac{3}{\sqrt{2}}\Big[(\mathbf{n}_1\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_1) \pm (\mathbf{n}_2\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_2) - \frac{1\pm 1}{3}(\sigma\cdot\mathbf{n}_i)\Big],$$

$$Q^{(+)}_{1/2;1,0(l_1 l_1)} = (-1)^{l_1}\sqrt{2l_1+1}\, P_{l_1}(\cos\theta_{12})(\sigma\cdot\mathbf{n}_i),$$

$$Q^{(\pm)}_{3/2;2,1(10)} = -\frac{3}{\sqrt{2}}\Big[(\mathbf{n}^{(\pm)}_{12}\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_i) - \frac{1}{3}(\sigma\cdot\mathbf{n}^{(\pm)}_{12})\Big],$$

$$Q^{(-)}_{1/2;1,1(11)} = \sqrt{\frac{3}{2}}\{(\mathbf{n}_2\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_1)$$

$$-(\mathbf{n}_1\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_2) - i([\mathbf{n}_2\times\mathbf{n}_1]\cdot\mathbf{n}_i)\,\eta\},$$

$$Q^{(-)}_{3/2;1,1(11)} = -\sqrt{\frac{3}{2}}\{(\mathbf{n}_2\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_1)$$

$$-(\mathbf{n}_1\cdot\mathbf{n}_i)(\sigma\cdot\mathbf{n}_2) + 2i([\mathbf{n}_2\times\mathbf{n}_1]\cdot\mathbf{n}_i)\,\eta\}, \quad \text{(A.3)}$$

where

$$\mathbf{n}^{(\pm)}_{12} = \mathbf{n}_1 \pm \mathbf{n}_2.$$

*Ritus[15] has considered such operators for reactions of the type a + b → c + d (without the production of additional parcles).

†The sign +(−) denotes a symmetric (antisymmetric) state of the system with respect to the exchange $\mathbf{n}_1 \rightleftarrows \mathbf{n}_2$.

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