Letters to the Editor

WAVE SOLUTIONS OF NONLINEAR GENERALIZATIONS OF RELATIVISTIC EQUATIONS

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ACCORDING to the general idea of the nonlinear unified field theory, the entire set of elementary particles is constructed from one fundamental spinor field. Since, however, the spinor equation is taken to be nonlinear, the fields obtained for the other particles are also nonlinear.

The relativistically-invariant equation for the elementary particles can, in the linear theory, be written in the well-known general form[1]

$$(\Gamma_\mu \partial / \partial x_\mu + k_0)\psi = 0, \tag{1}$$

where Γ_μ is a matrix; in particular, if $\Gamma_\mu = \gamma_\mu$, we have the Dirac equation, if $\Gamma_\mu = \beta_\mu$, the Duffin-Kemmer equation, etc.

As was shown in reference 1, there exists a nondegenerate invariant form $y \equiv (\psi^* T \psi)$ (T is a matrix) for all finite-dimensional representations of the full Lorentz group. Hence, if we change k_0 in (1) to $C(y) = k_0 + f(y)$, we again obtain a relativistically invariant equation

$$(\Gamma_\mu \partial / \partial x_\mu + C(y))\psi = 0, \tag{2}$$

which is now nonlinear.

We arrive at an equation of this form, if, for example, all particles are "merged" into one field obeying the nonlinear generalization of the Dirac equation in conformity with group theory:

$$(\gamma_\mu \partial / \partial x_\mu + A(y))\psi = 0. \tag{3}$$

As in the linear theory, Eq. (3) is an irreducible representation $D_{1/2}$ in the space of the basis vectors $\{\psi_k\}$. By forming the product $D_{1/2} \times D_{1/2}$ from two spinor equations of form (3), we obtain then the irreducible representations $D_0 + D_1$ in the space of the basis vectors $\{\psi_i \psi_k\} \equiv \{\psi_{ik}\}$. We write the resulting equation in the general form

$$(\beta_\mu \partial / \partial x_\mu + B(y))\psi = 0. \tag{4}$$

In reference 2 we determined the exact wave solutions of Eq. (3). The method developed there may, however, also be applied to the general equation (2). Indeed, if we require a solution in the form

$$\psi_\alpha = u_\alpha(\varepsilon, \alpha_1 \ldots \alpha_i, k_\mu)\varphi(\sigma), \quad \sigma = k_\mu x_\mu, \quad u^*_\alpha u_\alpha = 1,$$
$$k_\mu = (k_n, i\omega), \quad \omega = \varepsilon E, \quad \varepsilon = \pm 1, \quad c = \hbar = 1, \tag{5}$$

where $\epsilon, \alpha_1, \ldots, \alpha_i$ are parameters specifying the state of the particles, Eq. (2) takes the form

$$(\Gamma_\mu k_\mu d\varphi / d\sigma + C(\rho)\varphi) u(\varepsilon, \alpha_1, \ldots \alpha_i, k_\mu) = 0, \quad \rho = \varphi^*\varphi, \tag{6}$$

We further add the conjugate equation

$$\bar{u}(\varepsilon, \alpha_1 \ldots \alpha_i, k_\mu)(\Gamma_\mu k_\mu d\varphi^* / d\sigma - \bar{C}(\rho)\varphi^*) = 0. \tag{7}$$

If we now subject the functions $\varphi(\sigma)$ and $\varphi^*(\sigma)$ to the system of equations (2.4) of reference 2, we obtain for the amplitude

$$(i\Gamma_\mu k_\mu + \lambda) u = 0, \quad \bar{u}(i\Gamma_\mu k_\mu + \lambda) = 0. \tag{8}$$

$\varphi(\sigma)$ and $\varphi^*(\sigma)$ were determined in reference 2. The solutions of (8) can be taken from the linear theory. In particular, in the case of equation (4), where $\Gamma_\mu = \beta_\mu$, we have $\bar{\psi} = \psi^* \eta_4$, $\eta_4 = 2\beta_4^2 - 1$, $\bar{C}(y) = C^*(y)$, with the solutions of equation (8) known from reference 3.

On the other hand, if $C(y)$ has a form such that the four-dimensional current is conserved,

$$\frac{\partial}{\partial x_\mu}(\bar{\psi}\Gamma_\mu \psi) = (\bar{u}\Gamma_\mu k_\mu u)\frac{d}{d\sigma}(\varphi^*\varphi) = 0, \tag{9}$$

then $\varphi^*\varphi = \text{const}$, or $(\bar{u}\Gamma_\mu k_\mu u) = -\lambda(\bar{u}u) = 0$.

The unique complex solution $\varphi(\sigma)$ $\varphi^*(\sigma)$ is given by (2.13) of reference 2. The real solution $\varphi^* = \varphi \neq \text{const}$ leads to $(\bar{u}\Gamma_\mu k_\mu u) = -\lambda(\bar{u}u) = 0$. These solutions were found in reference 2 with $\Gamma_\mu = \gamma_\mu$ and the normalization $u^*u = 1$.

We further note that, since the equation for $\varphi(\sigma)$ with given $C(y)$ has identical form for all $\Gamma_\mu = \gamma_\mu$, β_μ, etc., the quadratic equation resulting from it also has the same form for all fields (just as the relativistic wave equation in the linear theory).

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