RADIATION FROM A CHARGED PARTICLE TRAVERSING A LAYERED MEDIUM

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An analysis is made of the transition radiation produced when a charged particle passes through a large number of slabs which are separated by vacuum spaces.

THE transition radiation produced when a charged particle moves from one semi-infinite medium into another has been treated in references 1 and 2. Later, in references 3 and 4, an investigation was made of the radiation field which is produced when a charged particle moves through a slab of material. In the present paper we calculate the transition radiation produced by the passage of a charged particle through an arbitrary number of material layers which are separated by vacuum spaces.

The present work is motivated by the following considerations. Ginzburg and Frank[1] have shown that the intensity of the transition radiation increases logarithmically with increasing particle energy. It is tempting to explore this effect as a means of measuring the energy of highly relativistic particles. Because the effect is so small it is impossible to observe the transition radiation due to a single particle. For this reason it is of interest to investigate the possibility of intensifying the transition radiation by allowing a particle to pass through a large number of material slabs.

1. Suppose that a charged particle with velocity **v** moves along the z axis in the positive direction and traverses m slabs of material, which are perpendicular to the direction of motion of the particle. The slabs are of thickness a and are separated by a distance b; all have the same dielectric constant $\epsilon(\omega)$.

The radiation field in the slabs and in the voids between them will consist of waves which move in the positive (E') and negative (E'') z directions. In the region $z < 0$, however, there will exist only reflected waves; in the region $z > ma + (m-1)b$ there will be only waves which move in the positive z direction. We introduce the notation $\lambda^2 = (\omega^2\epsilon/c^2) - \kappa^2$ and $\lambda_0^2 = (\omega^2/c^2) - \kappa^2$, where it is assumed that the real and imaginary parts of λ and λ_0 are positive.* To compute the radiation fields, E_0'' for $z < 0$ and E_m' for $z > ma + (m-1)b$, we proceed in the following manner.* Writing the continuity conditions for the electric vectors at the boundaries of the $(l+1)$-th slab (cf. references 2 and 4), we obtain four equations by means of which it is possible (eliminating the fields in the slab) to obtain a relation between the tangential Fourier components of the electric fields in the l-th and $(l+1)$-th voids. This relation is of the form:

$$F_{l+1} = MF_l, \tag{1}$$

where

$$F_l = \begin{pmatrix} F_l'' \\ F_l' \end{pmatrix},$$

$$F_l'' = E_{l,t}''(\mathbf{k}) \exp\{-i(\lambda_0 + k_z)(a+b)l\} + \alpha'',$$
$$F_l' = E_{l,t}'(\mathbf{k}) \exp\{i(\lambda_0 - k_z)(a+b)l\} + \alpha',$$

$$\alpha'' = \frac{ei\varkappa}{2\pi^2}\frac{\lambda\lambda_0}{4\varepsilon}\frac{A\exp\{-i\lambda_0 b - ik_z(a+b)\} + B\exp\{-ik_z(a+2b)\}}{(k_1-1)(k_2-1)},$$

$$\alpha' = \frac{-ei\varkappa}{2\pi^2}\frac{\lambda\lambda_0}{4\varepsilon}\frac{C\exp\{i\lambda_0 b - ik_z(a+b)\} + D\exp\{-ik_z(a+2b)\}}{(k_1-1)(k_2-1)},$$

$$\left.\begin{matrix} A \\ B \end{matrix}\right\} = \left(\frac{\varepsilon}{\lambda} + \frac{1}{\lambda_0}\right)\alpha e^{\mp i\lambda a}$$
$$+ \left(\frac{\varepsilon}{\lambda} - \frac{1}{\lambda_0}\right)\beta e^{\pm i\lambda a} + \frac{2\varepsilon}{\lambda}\gamma\exp\{\pm ik_z a\},$$

$$\left.\begin{matrix} C \\ D \end{matrix}\right\} = \left(\frac{\varepsilon}{\lambda} - \frac{1}{\lambda_0}\right)\alpha e^{\mp i\lambda a}$$
$$+ \left(\frac{\varepsilon}{\lambda} + \frac{1}{\lambda_0}\right)\beta e^{\pm i\lambda a} + \frac{2\varepsilon}{\lambda}\delta\exp\{\pm ik_z a\},$$

$$k_{1,2} = q_{1,2}\exp\{-ik_z(a+b)\}$$
$$= \frac{C_2 + D_1}{2} \pm \left[\left(\frac{C_2 + D_1}{2}\right)^2 - \exp\{-2ik_z(a+b)\}\right]^{1/2}.$$

*We retain the notation reference 4.

*This method was suggested by I. I. Gol'dman.

and the matrix M is

$$M = \begin{pmatrix} D_1 & C_1 \\ D_2 & C_2 \end{pmatrix},$$

where

$$D_1 = -\frac{\lambda\lambda_0}{4\varepsilon}\exp\{-i\lambda_0 b - ik_z(a+b)\}$$

$$\times\left[\left(\frac{\varepsilon}{\lambda}-\frac{1}{\lambda_0}\right)^2 e^{i\lambda a} - \left(\frac{\varepsilon}{\lambda}+\frac{1}{\lambda_0}\right)^2 e^{-i\lambda a}\right],$$

$$D_2 = \frac{\lambda\lambda_0}{4\varepsilon}\exp\{i\lambda_0 b - ik_z(a+b)\}\left(\frac{\varepsilon^2}{\lambda^2}-\frac{1}{\lambda_0^2}\right)(e^{i\lambda a}-e^{-i\lambda a}),$$

$$C_1 = -\frac{\lambda\lambda_0}{4\varepsilon}\exp\{-i\lambda_0 b - ik_z(a+b)\}$$

$$\times\left(\frac{\varepsilon^2}{\lambda^2}-\frac{1}{\lambda_0^2}\right)(e^{i\lambda a}-e^{-i\lambda a}),$$

$$C_2 = \frac{\lambda\lambda_0}{4\varepsilon}\exp\{i\lambda_0 b - ik_z(a+b)\}$$

$$\times\left[\left(\frac{\varepsilon}{\lambda}+\frac{1}{\lambda_0}\right)^2 e^{i\lambda a} - \left(\frac{\varepsilon}{\lambda}-\frac{1}{\lambda_0}\right)^2 e^{-i\lambda a}\right].$$

The quantities α, β, γ, and δ are given in Eq. (4) of reference 4.

Applying (1) m times we have

$$F_m = M^m F_0, \qquad (2)$$

with

$$F_0 = \begin{pmatrix} E''_{0,t}(\mathbf{k}) + \alpha'' \\ \alpha' \end{pmatrix}$$

$$F_m = \begin{pmatrix} \alpha'' \\ E'_{m,t}(\mathbf{k})\exp\{i(\lambda_0 - k_z(a+b)m\} + \alpha' \end{pmatrix}.$$

In principle the relation in (2) represents a solution of the problem since it yields two equations with two unknowns, $E''_{0,t}(\mathbf{k})$ and $E'_{m,t}(\mathbf{k})$. However, because it is necessary to calculate M^m when this procedure is used the problem remains essentially unsolved. The following method can be used to circumvent the difficulty. We introduce the matrix $S(t)$ defined by the relation

$$S(t) = e^{Mt}, \qquad (3)$$

where t is some variable. It is apparent that

$$M^m = (d^m S/dt^m)_{t=0}. \qquad (4)$$

It follows from Eq. (3) that S satisfies the differential equation

$$dS/dt = MS. \qquad (5)$$

Solving this system of differential equations for the elements of the matrix S and taking account of the fact that $S(0) = 1$, we obtain the following expression:

$$S = \frac{1}{k_1 - k_2}\begin{pmatrix} (D_1 - k_2)e^{k_1 t} + (k_1 - D_1)e^{k_2 t} & C_1(e^{k_1 t} - e^{k_2 t}) \\ D_2(e^{k_1 t} - e^{k_2 t}) & (C_2 - k_2)e^{k_1 t} + (k_1 - C_2)e^{k_2 t} \end{pmatrix}. \qquad (6)$$

Using Eqs. (2), (4), and (6) we obtain a system of two equations; the solution yields:

$$\mathbf{E}''_{0,t}(\mathbf{k}) = \frac{ei\kappa}{2\pi^2 G}\{e^{-i\lambda_0 b}A(q_1^m - q_2^m) \qquad (7)$$

$$+ (e^{-i\lambda_0 b}A + e^{-ik_z b}B)[e^{ik_z(a+b)}(q_1^{m-1} - q_2^{m-1})$$

$$+\ldots+e^{ik_z(a+b)(m-1)}(q_1 - q_2)]\},$$

$$\mathbf{E}'_{m,t}(\mathbf{k}) = -\frac{ei\kappa}{2\pi^2 G}e^{-i(\lambda_0 - k_z)(a+b)m}\{e^{-ik_z b}D(q_1^m - q_2^m)$$

$$+ (e^{i\lambda_0 b}C + e^{-ik_z b}D) \qquad (8)$$

$$\times[e^{-ik_z(a+b)}(q_1^{m-1} - q_2^{m-1}) + \ldots + e^{-ik_z(a+b)(m-1)}(q_1 - q_2)]\},$$

where

$$G = e^{-i\lambda_0 b}\left[\left(\frac{\varepsilon}{\lambda}+\frac{1}{\lambda_0}\right)^2 e^{-i\lambda a} - \left(\frac{\varepsilon}{\lambda}-\frac{1}{\lambda_0}\right)^2 e^{i\lambda a}\right](q_1^m - q_2^m)$$

$$-\frac{4\varepsilon}{\lambda\lambda_0}(q_1^{m-1} - q_2^{m-1}).$$

When m = 1 the last two expressions coincide with the corresponding formulas given in reference 4.

2. It is now necessary to integrate the expressions with respect to $dk_x dk_y = \kappa\, d\kappa\, d\varphi$. The integrals over φ are carried out by means of Bessel functions. We shall be interested in the field at large distances R from the points of entry and exit of the particle in the medium so that the asymptotic expansion for the Bessel functions can be used; the integration over κ is carried out by the method of steepest descent.

We first consider the radiation emitted in the backward direction $(z < 0)$. The expression for $E''_{0\rho}$ is

$$E''_{0\rho}(\mathbf{r}, t) = i\sqrt{\frac{2\pi}{R\sin\theta}}\int E''_{0,t}(\mathbf{k})$$

$$\times(e^{f(\kappa)R - 3\pi i/4} + e^{\varphi(\kappa)R + 3\pi i/4})e^{-i\omega t}\sqrt{\kappa}\,d\kappa\,dk_z,$$

where

$$f(\kappa) = i\kappa\sin\theta + i\lambda_0\cos\theta, \qquad \varphi(\kappa) = -i\kappa\sin\theta + i\lambda_0\cos\theta,$$

and $E''_{0,t}(\mathbf{k})$ is determined from Eq. (7).

The saddle point for the function $f(\kappa)$ is $(\omega/c)\cdot\sin\theta$. However, because of the presence in

$E''_{0,t}(\mathbf{k})$ of the exponentials in the expressions for q_1 and q_2, which are raised to the m-th power, it can be shown that the line of steepest descent of the integrand is shifted. This does not occur if

$$m(a+b) \ll R. \tag{9}$$

In the highly relativistic case, in which we shall be interested exclusively, the transition radiation is directed in the backward direction, or in the forward direction, at an angle $\theta_0 \sim \mu/E$, where μ and E are the mass and energy of the particle.

If we now impose the condition

$$a,\ b \ll \lambda\!\!\!^{-} E^2/\mu^2, \tag{10}$$

($\lambda\!\!\!^{-}$ is the wave length divided by 2π), after integration by the method of steepest descent it is possible to set $\theta = 0$ everywhere in λ and λ_0, with the exception of those expressions in which m appears in the exponents. Furthermore, we limit ourselves to those frequencies for which $|q| = 1$; this value encompasses essentially all frequency values if $(\sqrt{\epsilon}+1)^2/(\sqrt{\epsilon}-1)^2 \gg 1$ (when $\epsilon = 1.5$ this ratio is approximately 100). Also, taking $a\sqrt{\epsilon} = b$, we obtain the following expression for the transition radiation emitted in the backward direction:

$$W_0^- = \frac{e^2}{\pi c}\left(\ln\frac{2}{1-\beta} - 1\right)\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2 (\omega_2-\omega_1)\,2m, \tag{11}$$

if $m \ll (\sqrt{\epsilon}+1)^2\,(\sqrt{\epsilon}-1)^2$. Thus, if the conditions given in (10) are satisfied the intensity of the backward transition radiation will, under certain conditions, increase with the number of slabs, provided this number does not exceed the limiting value $(\sqrt{\epsilon}+1)^2/(\sqrt{\epsilon}-1)^2$. As the limiting number of slabs is approached, in order-of-magnitude terms the radiation intensity becomes equal to the radiation intensity for one boundary between a vacuum and an ideal conductor.[1] Any further increase in the number of slabs results in a reduction in the intensity of the transition radiation.

Using an approach similar to that above it can be shown that the intensity of the transition radiation emitted in the forward direction does not increase; the intensity of this component is, in order-of-magnitude terms, equal to the radiation for one slab of material.

3. Consider another case; in place of (10) we have

$$a\mu^2/E^2 \ll \lambda\!\!\!^{-} \ll b\mu^2/E^2. \tag{12}$$

Limiting ourselves to frequencies for which $|q| = 1$, we can show that the intensity of the transition radiation, in both the forward and backward directions, is equal to the transition radiation for one slab multiplied by $(2m-1)$. In computing the Poynting-vector flux in this case it is necessary to take account of the fact that the exponentials with $i(\omega/c)b\cos\theta$ give a zero contribution when averaged over the angle θ because of their oscillatory nature. The appearance of the factor $(2m-1)$ is easily understood because in addition to the m slabs we have $(m-1)$ voids, which also act as radiators of transition radiation.

Thus, if the conditions in (12) are satisfied the transition radiation can be increased by a desired factor, where, in accordance with Ref. 4, the radiation in the forward direction is a factor of $(\sqrt{\epsilon}+1)^2/(\sqrt{\epsilon}-1)^2$ larger than in the backward radiation.

Let (λ_1, λ_2) define a wavelength region which satisfies (12); we denote the corresponding frequencies by (ω_1, ω_2). The number of photons emitted in the forward direction in this frequency range is (cf. reference 4):

$$N(\omega_1, \omega_2) = \frac{2}{\pi}\frac{e^2}{\hbar c}\left(\ln\frac{2}{1-\beta} - 1\right)\ln\left(\frac{\omega_2}{\omega_1}\right)(2m-1). \tag{13}$$

It is apparent from this formula that the number of photons is independent of the point of the spectrum at which the frequency range (ω_1, ω_2) is taken. Here, in accordance with reference 4 the relation $|\sqrt{\epsilon}-1| \gg c/\omega a$ must be satisfied. In the optical region this relation is always satisfied. The dangerous frequencies are those which are above the optical region, where $\sqrt{\epsilon} = 1 - 2\pi n e^2/m\omega^2$ (n is the number of electrons per unit volume). Using this expression for $\sqrt{\epsilon}$ we obtain the following condition:

$$(c^2/a)\,m/2\pi n e^2 \ll \lambda\!\!\!^{-}. \tag{14}$$

When the values of a and b are fixed, in accordance with Eq. (12), as the particle energy increases photons of higher and higher energy must be used if we wish to measure the particle energy from the intensity of the transition radiation. However, when $E/\mu \gg (ae/c)\sqrt{2\pi n/m}$ [in accordance with Eq. (14)] the radiation in the frequency region given by Eq. (12) cannot be computed from Eq. (13).

Taking $a = 10^{-3}$ cm and $b = 10^{-1}$ cm, we can measure the particle energy up to $E/\mu = 10^3$. When $E/\mu = 10^2$, the number of photons in the wavelength region $(1 \text{ to } 3)\times 10^6$ cm will be 95 if the total number of slabs $m = 10^3$. When $E/\mu = 10^3$ the number of photons in the wavelength region $(1 \text{ to } 3) \times 10^{-8}$ cm is 145 for the same number of slabs. The total length of the stack is 101 cm.

As is well known, the transition radiation is emitted in a narrow cone with opening angle approximately μ/E in the forward direction. Ordinary bremsstrahlung is emitted in the same solid angle. Using the well known expression,[5] for particle energies of $E/\mu = 10^2$ and $E/\mu = 10^3$ we obtain 0.15 bremsstrahlung photons in the frequency region indicated ($Z = 14$, $N = 5 \times 10^{22}$).

It can also be shown that in the one-particle case the coherent radiation produced by virtue of macroscopic radiation, such as that considered by Askar'ian[6] for a particle bunch, is negligibly small.

In the optical region the Cerenkov radiation is emitted at large angles; in regions above the optical region in general there is no Cerenkov radiation (since $\epsilon < 1$).

To carry out the measurements indicated above it would be necessary to be able to make differential intensity measurements of electromagnetic radiation over a wide frequency range.

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