where $\epsilon = E/E_0$, E is the initial total energy of the particle, $E_0 = mc^2$ is the rest energy, $r_0 = e^2/mc^2$ and $d\Omega = \sin\theta d\theta d\varphi$.

In the nonrelativistic case ($k \ll k_0, \kappa$) (7) gives the classical Rutherford formula, while in the extreme-relativistic case ($\kappa \ll k, k_0$) only transitions in which the final states have spin projections $s' = \pm 2$ are important and

$$d\sigma = \frac{z^2}{15} r_0^2 \varepsilon^2 \cos^4 \frac{\theta}{2} d\Omega. \tag{8}$$

The effect of the field due to the electron cloud around the nucleus can be evaluated with the Thomas-Fermi method;[1] for no screening ($qa \gg 1$) the result is

$$d\sigma = \frac{z^2 r_0^2}{960\varepsilon^4 (1 - 5\varepsilon^2 + 4\varepsilon^4)^2 \sin^4(\theta/2)} (7 - 14\varepsilon^2 - 166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) d\Omega, \tag{9}$$

while for maximum screening we have

$$d\sigma = \frac{0.32 z^{2/3} \hbar^4}{r_0^2 \varepsilon^4 (1 - 4\varepsilon^2)^2 m_e^4 c^4} (7 - 14\varepsilon^2 - 166\varepsilon^4 + 722\varepsilon^6 - 41\varepsilon^8 + 832\varepsilon^{10} + 756\varepsilon^{12} + 64\varepsilon^{14}) d\Omega. \tag{10}$$

At low velocities ($p \ll \hbar z^{1/3}/2a_0$, $a_0 = \hbar^2/m_e e^2$, m_e = rest mass of the electron) there is total shielding at all scattering angles and we must use formula (10), but can neglect the small quantities ϵ:

$$d\sigma = (2.24 z^{2/3} \hbar^4 / r_0^2 \varepsilon^4 m_e^4 c^4) d\Omega. \tag{11}$$

At high velocities ($p \gg \hbar z^{1/3}/2a_0$) the screening effect is noticeable only at small angles θ, defined by[1]

$$\sin\frac{\theta}{2} < \hbar z^{1/3}/2a_0 p. \tag{12}$$

Hence for large velocities and at angles satisfying (12), formula (10) should be used, while at angles not satisfying (12) the proper formula to use is (9).

In the extreme relativistic case (when the condition $p \gg \hbar z^{1/3}/2a_0$ will be satisfied if $m \geq m_e$) we need consider only higher order terms in ϵ, and for small angles satisfying (12), Eq. (10) gives

$$d\sigma = (1.28 z^{2/3} \varepsilon^6 \hbar^4 / r_0^2 m_e^4 c^4) d\Omega, \tag{13}$$

while for angles which do not satisfy (12), Eq. (9) reduces to

$$d\sigma = \frac{z^2 r_0^2 \varepsilon^2}{240 \sin^4(\theta/2)} d\Omega. \tag{14}$$

For ϵ close to unity, Eq. (9) gives the classical Rutherford formula (for $p \gg \hbar z^{1/3}/2a$, and $\sin\theta/2 > \hbar z^{1/3}/2a_0 p$).

I am deeply grateful to Prof. F. I. Fedorov, under whose guidance this work was carried out.

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HYDRODYNAMIC ANALYSIS OF THE COMPRESSION OF A RAREFIED PLASMA IN AN AXIALLY-SYMMETRIC MAGNETIC FIELD

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We consider a totally ionized plasma (neglecting collisions) confined in a narrow magnetic tube by an axially-symmetric magnetic field (the axis of the tube is the axis of symmetry of the magnetic field). Assuming quasi-neutral motion of the plasma along the tube, we can write the hydrodynamic equations:[1,2]

$$m\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = -\frac{W_\perp}{H}\frac{\partial H}{\partial x} - \frac{1}{n}\frac{\partial}{\partial x}(2W_\parallel n),$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \tag{1}$$

$$m = m_e + m_i,\ W_\perp = W_{\perp e} + W_{\perp i},\ W_\parallel = W_{\parallel e} + W_{\parallel i}.$$

Here the subscript e refers to the electrons and the subscript i refers to the ions, $W_\perp$ and $W_\parallel$ are the mean energies due to thermal motion of the electrons (ions) (perpendicular and parallel to the tube); $n \equiv N/H$ is a quantity proportional to the number of electrons (ions) per unit length of the tube, and N is the density per unit volume. We consider two modes of longitudinal compression of the plasma along the magnetic tube.

(a) Isothermal compression. Let $W_\perp$ and $W_\parallel$ be constant in time and over the length of the tube, with $W_\perp > 2W_\parallel$ (the latter condition can be realized by using a magnetic field which increases sharply). The magnetic field is assumed to be of the form:

$$H(x) = H^* \exp(x^2/2a^2). \tag{2}$$

At a given time the density distribution is given by the expression:

$$N/N_0 = (1/\psi)\exp[-(\psi^{-2} - 1)x^2 2a^2], \tag{3}$$

where $\psi(t)$ satisfies the equation:

$$\ddot{\psi} + (W_\perp/ma^2)(\psi - 2W_\parallel/W_\perp\psi) = 0,$$
$$\psi(0) = 1, \quad \dot{\psi}(0) = 0. \tag{4}$$

It follows from Eq. (4) that ψ is a periodic function of time (period T) which varies between the limits ψ_1 and 1. As an approximation we have:

$$T \approx \begin{cases} (W_\perp/ma^2)^{1/2} & \text{for} \quad W_\perp/2W_\parallel \sim 1, \\ \pi(W_\perp/ma^2)^{1/2} & \text{for} \quad W_\perp/2W_\parallel \gg 1; \end{cases} \tag{5}$$

$$\psi_1 \approx \begin{cases} (2W_\parallel/W_\perp)^{1/2} & \text{for} \quad W_\perp/2W_\parallel \sim 1, \\ \exp(-W_\perp/4W_\parallel) & \text{for} \quad W_\perp/2W_\parallel \gg 1. \end{cases} \tag{6}$$

According to Eqs. (3) to (6), the plasma undergoes periodic longitudinal compression at the point of minimum magnetic field and expands to a uniform distribution of density N_0.

(b) Adiabatic stratification when a magnetic gradient, which is periodic along the tube, is turned on quasi-statically. Let the magnetic field be of the form:

$$H(x, t) = H_1(t) - H_2(t)\cos(\pi x/a),$$
$$H_1(t) > H_2(t), \quad H_2(0) = 0. \tag{7}$$

We neglect the inertia term in Eq. (1) and introduce the adiabaticity equations:[1]

$$W_\perp/H = W_{\perp 0}/H_0, \quad W_\parallel/n^2 = W_{\parallel 0}/n_0^2. \tag{8}$$

The density is:

$$(n/n_0)^2 = A + B\cos(\pi x/a), \quad B = (W_{\perp 0}/3W_{\parallel 0})H_2(t)/H_0,$$
$$(2/\pi)(A+B)^{1/2}E(k) = 1, \quad k^2 = 2B/(A+B), \tag{9}$$

where $E(k)$ is a complete elliptic integral of the second kind. At a given time

$$\frac{n}{n_0} = \frac{\pi}{2}\left|\cos\frac{\pi x}{2a}\right|. \tag{10}$$

According to Eq. (9) the plasma is stratified along the magnetic field, forming bunches which are concentrated about the points of minimum magnetic field.

In conclusion we wish to thank Professor Ia. P. Terletskii for his interest in this work.

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THE RADIATION FROM A CHARGED PARTICLE PASSING THROUGH A PLATE

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LET a charged particle moving in vacuum along the positive z direction pass through a plate of thickness a made of a material with dielectric constant ϵ. We proceed as in reference 1, the only difference being that in the region $z < 0$ the radiation field will consist of reflected waves only, and the field in the region $z > a$ of waves moving in the positive z direction, while inside the plate there are both kinds of waves; we then get for the Fourier components of the radiation field in the spaces before and after the plate the formulas

$$\mathbf{E}''_{0t}(\mathbf{k}) = \frac{ei}{2\pi^2}\frac{\varkappa}{F}\left\{\left(\frac{\varepsilon}{\lambda} + \frac{1}{\lambda_0}\right)^2 \alpha e^{-i\lambda a} + \left(\frac{\varepsilon}{\lambda} - \frac{1}{\lambda_0}\right)\beta e^{i\lambda a} + \frac{2\varepsilon}{\lambda}\gamma e^{ik_z a}\right\}, \tag{1}$$

$$\mathbf{E}'_{1t}(\mathbf{k}) = -\frac{ei}{2\pi^2}\frac{\varkappa}{F}\left\{\left(\frac{\varepsilon}{\lambda} - \frac{1}{\lambda_0}\right)\alpha e^{i\lambda a} + \left(\frac{\varepsilon}{\lambda} + \frac{1}{\lambda_0}\right)\beta e^{-i\lambda a} + \frac{2\varepsilon}{\lambda}\delta e^{-ik_z a}\right\}, \tag{2}$$

where

$$\lambda_0^2 = \frac{\omega^2}{c^2} - \varkappa^2; \quad \lambda^2 = \frac{\omega^2}{c^2}\varepsilon - \varkappa^2;$$
$$F = \left(\frac{\varepsilon}{\lambda} + \frac{1}{\lambda_0}\right)^2 e^{-i\lambda a} - \left(\frac{\varepsilon}{\lambda} - \frac{1}{\lambda_0}\right)^2 e^{i\lambda a}, \tag{3}$$

$$\left.\begin{matrix}\alpha \\ \beta\end{matrix}\right\} = \frac{\pm\varepsilon/\lambda - v/\omega}{k^2 - \omega^2/c^2} + \frac{\mp 1/\lambda + v/\omega}{k^2 - \omega^2\varepsilon/c^2};$$
$$\left.\begin{matrix}\gamma \\ \delta\end{matrix}\right\} = \frac{\mp 1/\lambda_0 + v/\omega}{k^2 - \omega^2/c^2} + \frac{\pm 1/\lambda_0\varepsilon - v/\omega}{k^2 - \omega^2\varepsilon/c^2}. \tag{4}$$

Taking $R \gg a$, where R is the distance from the place the particle enters the plate (or where it leaves the plate) to the point of observation, we can obtain for the radiation emitted (as has been done in reference 1) a formula agreeing with that obtained by Pafomov.[2*] We only remark that the formula given in reference 2 does not hold for $a \lesssim R$. In this case we can obtain (cf. reference 1)