THE ACCELERATION OF COSMIC RAYS IN A FLUCTUATING MAGNETIC FIELD

V. M. BIAKOV and R. G. AVALOV

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A mechanism for the acceleration of cosmic rays is examined which involves their passage through regions of fluctuating magnetic field in the galaxy or the atmospheres of certain types of stars. During the passage of a stream of particles through a region of fluctuating magnetic field the spread of the particles in energy is increased, and a systematic acceleration occurs. Estimates show that the mechanism considered here may be more effective than the Fermi acceleration mechanism.

1. In the present paper we consider the acceleration of cosmic-ray particles by magnetic fields that vary with time. Such fields accompany turbulent motions of the interstellar medium, and also the atmospheric motions in certain types of stars. In an ideally-conducting magnetized medium (such as cosmic space) the magnetic lines of force are attached to the matter, so that a change of the density means at the same time a change of the magnetic field.

In the interstellar medium the hydrodynamic velocities are as a rule never small in comparison with the speed of sound. Under these conditions velocity fluctuations bring about large fluctuations of the density, and we can expect also large fluctuations of the magnetic field.

We may consider two possibilities for the state of the magnetic field: it is either increasing on the average on account of the turbulent motion of the magnetized medium, or it is stationary (or relatively slowly changing), so that basically the only changes are fluctuations.

The acceleration of cosmic rays in a magnetic field that is increasing because of the turbulent motions of the interstellar gas has been studied by Logunov and Terletskii,[1] and acceleration by magnetohydrodynamic waves in a spiral arm of the galaxy has been studied by Davis.[2]

Here we shall start with the second possibility and examine the acceleration of charged particles occurring on account of fluctuations of the magnetic field.

Charged particles moving in a fluctuating magnetic field will sometimes get into regions where the field is increasing, and at other times into regions where it is falling off, with corresponding increases and decreases of their energy. It is obvious that as a result of fluctuations in the numbers of positive and negative changes of energy experienced by individual particles, some of the particles will be accelerated and will acquire considerable energies under suitable conditions. Generally speaking, in addition to this spread in particle energies there will also occur a systematic increase of the energy of the particles. On certain assumptions, which are indicated below, this acceleration mechanism can be more effective than the mechanism proposed by Fermi.[3]

2. Let us estimate the order of magnitude of the effect to be expected.

The change of the energy of a particle of charge e moving in a fluctuating magnetic field depends on the rate of change of the field with time and on its degree of homogeneity in space. If the momentum p of the particle satisfies the condition

$$p \ll (e/c) H^2 / |\mathrm{grad}\, H|, \tag{1}$$

its motion takes place along a spiral trajectory, which is characterized by the magnitude of the momentum and the angle ϑ between the direction of the momentum and that of the magnetic field intensity vector H (c is the speed of light).

In interstellar space, with field intensities $\leq 10^{-5}$ oersteds, for particles with energies corresponding to the main mass of the cosmic radiation, the condition (1) is as a rule well satisfied within the limits of a single fluctuation region. In this case, since the change of the field in the time of a single turn around the spiral is negligible, we can base the calculation of the change in the particle's energy on the condition of conservation of the adiabatic invariant

$$p^2 \sin^2 \vartheta / H = p_1^2 \sin^2 \vartheta_1 / H_1. \tag{2}$$

The index 1 is affixed to quantities taken at the time of entrance of the particle into the field region in question.

The changing magnetic field changes only the component of the particle's momentum perpendicular to the field; the momentum component parallel to the field remains unchanged during the motion:

$$p\cos\vartheta = p_1\cos\vartheta_1. \tag{3}$$

For a nonrelativistic particle this equation means that the speed of the motion along the field, $v\cos\vartheta$, is constant, and the time t of passage of the particle through a region of linear dimension l is given by

$$t = l/v\cos\vartheta. \tag{4}$$

At relativistic energies, for which the speed of the particle is of the order of that of light, $v\cos\vartheta$ does not remain constant, and we can use the relation (4) as an approximate one for small changes of the energy of the particle.

Equations (2) and (3) lead to

$$\Delta p^2 = p^2 - p_1^2 = p_1^2\sin^2\vartheta_1(H/H_1 - 1),$$
$$\Delta p = p - p_1 = p_1[\sqrt{1+\sin^2\vartheta_1(H/H_1-1)} - 1],$$
$$(\Delta p)^2 = p_1^2[\sin^2\vartheta_1(H/H_1-1)$$
$$-2(\sqrt{1+\sin^2\vartheta_1(H/H_1-1)}-1)] = \Delta p^2 - 2p_1\Delta p.$$

We shall consider only small changes of the magnetic field, for which Δp and $(\Delta p)^2$ can be represented in the forms:

$$\Delta p = \frac{p_1}{2}\sin^2\vartheta_1\left(\frac{H}{H_1}-1\right) - \frac{p_1}{8}\sin^4\vartheta_1\left(\frac{H}{H_1}-1\right)^2,$$
$$(\Delta p)^2 = \frac{1}{4}p_1^2\sin^4\vartheta_1\left(\frac{H}{H_1}-1\right)^2. \tag{5}$$

At the boundaries of the fluctuating regions or "clouds," which are evidently determined by shock waves,[4,5] the strong field gradients can cause scattering of the particles, which destroys the correlation between values of the angle ϑ in different regions of the medium. We shall assume that this scattering preserves the isotropic character of the cosmic-ray flux.

The rate of acceleration can be characterized by the average values of the quantities

$$\frac{dp}{dt} = \frac{v}{l}\Delta p, \quad \frac{dp^2}{dt} = \frac{v}{l}\Delta p^2,$$

which, in the absence of correlation between the values of v/l and Δp (or Δp^2), can be written in the form

$$\overline{\frac{dp}{dt}} = \overline{\frac{v}{l}}\,\overline{\Delta p}, \quad \overline{\frac{dp^2}{dt}} = \overline{\frac{v}{l}}\,\overline{\Delta p^2}. \tag{6}$$

Here the average distance between fluctuations is taken to be of the order of magnitude of the average dimensions of the fluctuations themselves, and $\overline{\Delta p}$ and $\overline{\Delta p^2}$ denote the average changes of the momentum and of its square on passage of the particle through a single fluctuation.

The averaging has to be carried out over all possible values of the angles of entrance, or, what is the same thing in view of relation (4), over all values of the time of flight through a fluctuation region, and also over all times of entrance t_1. If 2τ is the period of variation of the field (see below), the averaging process gives

$$\overline{\Delta p^2} = \frac{l}{2v\tau}p^2J_1, \quad \overline{\Delta p} = \frac{l}{4v\tau}pJ_1 - \frac{1}{16}\frac{l}{v\tau}pJ_2,$$
$$\overline{(\Delta p)^2} = \frac{l}{8v\tau}p^2J_2 = \overline{\Delta p^2} - 2p\overline{\Delta p},$$

where

$$J_1 = \int_0^{2\tau}dt_1\int_{l/v}^{\infty}\left[1-\left(\frac{l}{vt}\right)^2\right]\frac{dt}{t^2}\left[\frac{H(t+t_1)}{H(t_1)}-1\right],$$
$$J_2 = \int_0^{2\tau}dt_1\int_{l/v}^{\infty}\left[1-\left(\frac{l}{vt}\right)^2\right]^2\frac{dt}{t^2}\left[\frac{H(t+t_1)}{H(t_1)}-1\right]^2.$$

Choosing the origin for counting the time so that at time zero the value of the field strength was a minimum, we can write:

$$0\leqslant t\leqslant\tau \qquad H = H_{\min}[1+nt/\tau],$$
$$\tau\leqslant t\leqslant 2\tau \qquad H = H_{\min}[1+n(2\tau-t)/\tau],$$

where

$$\frac{n}{\tau} = \frac{1}{H_{\min}}\left\langle\frac{dH}{dt}\right\rangle_{\text{av}}, \quad n = \frac{H_{\max}-H_{\min}}{H_{\min}}.$$

If we assume that the average time τ for variation of the magnetic field is given in order of magnitude by l/V, where V is the speed of the turbulent motions, then for sufficiently fast particles $l/v\tau = V/v \ll 1$. Calculating the integrals with neglect of terms of the order $l/v\tau$ and of higher-order terms, and assuming that $n^2/(1+n) \ll 3$, we have

$$J_1 = n^2/2(1+n), \quad J_2 = 2J_1,$$
$$\overline{\Delta p^2} = \frac{1}{4}\frac{n^2}{1+n}\frac{l}{v\tau}p^2 = \frac{1}{4}\frac{n^2}{1+n}\frac{V}{c}p^2,$$
$$\overline{\Delta p} = \frac{1}{16}\frac{n^2}{1+n}\frac{V}{c}p, \quad \overline{(\Delta p)^2} = \frac{1}{8}\frac{n^2}{1+n}\frac{V}{c}p^2. \tag{7}$$

The comparatively large values of $\overline{\Delta p}$ and $\overline{(\Delta p)^2}$ are due to the possibility of taking independent averages in the formulas (6). The existence of correlations between the angles leads to a lowering of the rate of the acceleration.[2]

3. Let us compare the effectiveness of this mechanism of acceleration with that of the Fermi acceleration mechanism.[3] According to Fermi, in the relativistic case the average rate of increase of the energy of a particle is

$$\left(\frac{dE}{dt}\right)_{\text{F}} = \frac{v}{l}\left(\frac{V}{c}\right)^2E,$$

(E is the total energy of the particle), whereas according to Eqs. (6), (7)

$$\left(\frac{dE}{dt}\right) = \frac{1}{16}\frac{n^2}{1+n}\frac{v}{l}\frac{V}{c}E. \tag{8}$$

From this we have

$$\left(\frac{\overline{dE}}{dt}\right) \Big/ \left(\frac{\overline{dE}}{dt}\right)_F = \frac{1}{16} \frac{n^2}{1+n} \frac{c}{V}.$$

Since for the interstellar medium $V \leq 100$ km/sec, already for $n > 0.1$ the fluctuational changes of the field can play an important part in the acceleration of particles.

We have not given detailed consideration to effects associated with the passage of particles through the boundaries of the fluctuation regions. Reflection of particles from these boundaries, and also possible anisotropy of the flux can, generally speaking, reduce the acceleration below the above estimates. Nevertheless the data at present available clearly indicate that these effects are not so large that our estimate needs to be changed greatly. We hope later on to examine this question in more detail.*

Let us estimate the average value of the magnetic field fluctuation that can be expected in a magnetized turbulent medium. This can be done if we assume, following Batchelor,[5] that the average energy density of the magnetic field is of the same order of magnitude as the energy of the smallest turbulent eddies. Then the magnetic effects manifest themselves only in the domains of the smallest eddies, and can be neglected for the larger eddies.

Neglecting also the influence of viscous forces, we get for the average value of the fractional density fluctuation the expression

$$\Delta\rho / \rho = (V/u)^2,$$

where u is the speed of sound and ρ is the mean density. If we assume that the compressions and rarefactions of the gas occur isotropically, then $H \sim \alpha\rho^{2/3}$, and

$$\Delta H / H = {}^2/_3 (V/u)^2.$$

As we mentioned above, speeds of the interstellar gas, small in comparison with the speed of sound, are never of common occurrence. Therefore we can expect considerable fluctuations of the magnetic field strength.

In conclusion we remark that the mechanism considered here may turn out to be important in treating the acceleration of cosmic rays in the expanding shells of supernovas and novas, where in a number of cases the Fermi mechanism gives a retardation.[6]

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250