ANGULAR DISTRIBUTION FOR THE DIFFRACTION SCATTERING OF DEUTERONS

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THE angular distribution of elastically scattered deuterons was considered in reference 1. However, as the authors themselves pointed out, their formula (10) is valid only for $\kappa' \ll p$. Here, κ is the transverse momentum acquired by the deuteron as a result of the scattering, $\kappa' = R$, R is the nuclear radius, R_d is the radius of the deuteron, and $p = R/R_d$. The theory of diffractional scattering is valid up to $\kappa \sim \mu c$, i.e., up to $\kappa' \sim 2p$. It is therefore of interest to obtain a formula covering the whole angular range.

We turn to the derivation of the angular distribution. We have

$$d\sigma = |A_\varkappa|^2 d\varkappa/(2\pi)^2,$$

where

$$A_\varkappa = \int e^{-i\varkappa\rho} d\rho \int dr\, \varphi^2_0(r) \left[\omega\left(\rho + \frac{r}{2}\right) + \omega\left(\rho - \frac{r}{2}\right)\right. \tag{1}$$

$$\left. - \omega\left(\rho + \frac{r}{2}\right)\omega\left(\rho - \frac{r}{2}\right)\right], \qquad \omega(\rho) = \begin{cases} 1 & \text{for } \rho < R \\ 0 & \text{for } \rho > R \end{cases}$$

Here, ρ is the radius vector of the center of mass of the deuteron in the plane perpendicular to the axis of the incident beam, $\mathbf{r}$ is the radius vector of the relative distance in the deuteron, $\varphi_0(r) = \sqrt{\alpha/2\pi}\, e^{-\alpha r}/r$ is the internal wave function of the deuteron in the approximation of zero-range nuclear forces ($\alpha = 1/2R_d$). We consider the case $R_d \ll R$, and replace the nucleus by a flat disk with a straight edge, as in references 2 and 3. We note that the expression inside the square bracket in (1) is equal to zero for $\rho < R$, while, for $\rho > R$, it equals unity in a band of width $2(\rho - R)$ and zero elsewhere. After these remarks, the integral (1) is easily evaluated with the result

$$A_\varkappa = 2\pi R^2 \left\{\frac{J_1(\varkappa')}{\varkappa'} + \frac{1}{2p}\int_0^\infty dx\, J_0\left[\varkappa'\left(1 + \frac{x}{2p}\right)\right]\mathscr{E}_1(x)\right\}$$

$$= 2\pi R^2 a_p(\varkappa'). \tag{2}$$

Here $J_0(x)$ and $J_1(x)$ are the Bessel functions, $\mathscr{E}_1(x)$ is the Gold integral:

$$\mathscr{E}_1(x) = \int_1^\infty e^{-xt}\, dt/t^2.$$

With the help of the formula

$$\int_0^\infty \varkappa' d\varkappa' J_0\left[\varkappa'\left(1 + \frac{x}{2p}\right)\right] J_0\left[\varkappa'\left(1 + \frac{x'}{2p}\right)\right] = \frac{2p\delta(x - x')}{1 + x/2p},$$

we easily verify that

$$\int d\sigma = \int |A_\varkappa|^2\, 2\pi\varkappa d\varkappa = \pi R^2 + \frac{2\pi}{3}(1 - \ln 2) R R_d.$$

This total cross section agrees with the total cross section of reference 1.

The function $a_p(\kappa')$ was tabulated for $p = R/R_d = 3$, which corresponds to $A = 216$:

$\varkappa' =$	0	1	2	3	4	5
$a_3(\varkappa') =$	0.5833	0.5691	0.3154	0.0656	—0.0796	—0.0797

$\varkappa' =$	6	7	8	9
$a_3(\varkappa') =$	0.0177	0.0356	0.0402	0.0088.

V. S. Popov pointed out to the author that the function (2) can be represented approximately (with accuracy $\sim 10\%$) by the following expression:

$$a_p(\varkappa') = \frac{J_1(\varkappa')}{\varkappa'} + pJ_0(\varkappa')\frac{\ln(1 + \varkappa'^2/4p^2)}{\varkappa'^2}$$

$$+ \frac{J_1(\varkappa')}{\varkappa'^2}\left(1 - \frac{2p}{\varkappa'}\tan^{-1}\frac{\varkappa'}{2p}\right).$$

In terms of the angular variables, the required angular distribution has the following form:

$$d\sigma(\theta) = 2\pi R^2 |a_p(p_0 R\theta/h)|^2 (p_0 R/h)^2\, \theta d\theta, \tag{3}$$

where p_0 is the momentum of the incident deuteron. This distribution decreases much more slowly with increasing angles than the distribution in reference 1. We note that the width of the secondary maxima in the distribution (3) is one half of that for the diffraction of point particles.

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[2] R. Glauber, Phys. Rev. **99**, 1515 (1955).

[3] R. Serber, Phys. Rev. **72**, 1008 (1947).

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