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Letters to the Editor

ANALOGY BETWEEN THE MECHANICAL AND THERMODYNAMICAL EQUATIONS OF MOTION AND THE ONSAGER RECIPROCITY RELATION

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1. In many works by Popov[1*] and Karanikolov,[2] the Onsager reciprocity relations[3] are derived from a proposed analogy between classical mechanics and irreversible thermodynamics. The fundamental equations of irreversible thermodynamics can be written[4] in the following convenient notation:

$$X = -g\alpha, \tag{1}$$

$$\dot{\alpha} = LX, \quad X = R\dot{\alpha}\,(R = L^{-1}). \tag{2}$$

Here X is the force vector, α the state vector, L(R) the conductivity ("resistance") matrix, and g is a symmetrical matrix,

$$g = \tilde{g}, \tag{3}$$

(the tilde denotes transposition), whose elements, are the negatives of the second-order derivatives

$$g_{ik} = -\partial^2 S/\partial\alpha_i\partial\alpha_k \tag{4}$$

of the entropy

$$S = S(\alpha_1, \alpha_2, \ldots, \alpha_n) = S(\alpha). \tag{5}$$

We note that in matrix notation X and α are columns, while their transposed values $(\tilde{X}, \tilde{\alpha})$ are rows

Since Eqs. (1) and (2) do not permit a determination of the symmetry properties of the matrix L, Popov proposes that the following equation holds:†

$$X = \ddot{\alpha}. \tag{6}$$

Using Eq. (6), Popov and Karanikolov[1,2] succeed, through rather lengthy computations, in deriving the reciprocity relation of Onsager

$$L = \tilde{L}. \tag{7}$$

In the present article we prove the following.

(1) Popov's results can be obtained in a considerably simpler form.

(2) Relation (6) of Popov is a very restrictive and, generally speaking, unfounded condition, which reduces to

$$L^{-2} = g, \quad L = g^{-1/2}. \tag{8}$$

(3) Instead of (6), it is necessary to use a general relation that can be considered with full justification as the thermodynamic analogy of Newton's equations of motion. Equation (6) and its equivalent, the expressions in (8), can be valid only in a few rather particular cases. It will be shown that Eq. (7) unfortunately no longer follows from the above-mentioned general relation.

2. The Onsager relation can be obtained very simply from Eqs. (6), (1), (2), and (3). We differentiate Eqs. (1) and (2) with respect to time

$$\dot{X} = -g\dot{\alpha}, \quad \dot{X} = L^{-1}\ddot{\alpha};$$

we use again Eq. (2)

$$L^{-1}\ddot{\alpha} = -g\dot{\alpha} = -gLX$$

and furthermore

$$-X = (LgL)^{-1}\ddot{\alpha} = Rg^{-1}R\ddot{\alpha}. \tag{9}$$

It is obvious that by deriving this relation we have solved the above problem. In fact, since Eq. (9) follows from (1) and (2), it is equivalent to these two equations. Furthermore, (9) has the same form as Newton's equation of motion

$$F = m\ddot{r}.$$

It is clear that Popov's relation holds only when

$$LgL = 1,$$

hence,

$$L^2 = g^{-1}, \quad L = g^{-1/2}, \quad |g| \neq 0.$$

It is obvious that if $g = \tilde{g}$, $g^{-1/2}$ is also symmetrical, i.e.,

$$L = g^{-1/2} = (\widetilde{g^{-1/2}}) = \tilde{L}.$$

Since the equation of motion (9) results from Eqs. (1) and (2), and since the symmetry relation (7) does not follow from (1) and (2), it cannot follow from (9). Relation (7) is obtained only if we assume that L is a symmetrical function of the matrix g. At the present time we know of not one thermodynamic system for which L depends on g, so that this assumption is quite unfounded.

In analogy with Eq. (9), or with Eqs. (1) and (2), with Newtonian mechanics it can be followed by making the following comparisons:

F	$-X$
m	$(LgL)^{-1}$
r	α
$F = m\ddot{r}$	$-X = (LgL)^{-1}\ddot{\alpha}.$

Further comparisons pertain to the motion of a point under the influence of friction:

$$F = -\varkappa \ddot{r} \qquad -X = -R\dot{\alpha}$$
$$\varkappa \qquad R.$$

The resistance matrix corresponds to the coefficient of friction.

There exists also an analogy in that respect, that the positive definite matrices $(LgL)^{-1}$ and R correspond, in the case of $|g| \neq 0$ and $|L| \neq 0$ to positive values of m and κ. The analogy remains in full force also in that case when we consider, instead of the motion of a point under the influence of friction, the rotation of a solid body in the presence of an anisotropic friction force.

Thus, the mechanical model of irreversible thermodynamics comprises rotation of a solid about a certain point in the presence of anisotropic friction. This model, to the extent that it is practically realizable, can be used to integrate the equation of motion of irreversible processes.

*For other work by Popov on this subject see the literature of reference 1.

†See Eq. (10) of reference 1.

[1] K. Popov, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 257 (1955), Soviet Phys. JETP **1**, 336 (1955).

[2] Kh. Karanikolov, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 283 (1955), Soviet Phys. JETP **1**, 265 (1955).

[3] L. Onsager, Phys. Rev. **37**, 405 (1931), **38**, 2265 (1931).

[4] S. R. de Groot, Thermodynamics of Irreversible Processes, Amsterdam, North Holland Publ. Co., 1952.

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*ON THE POSSIBILITY OF APPLYING THE BELEN'KII-TAMM EQUILIBRIUM SPECTRUM TO THE DETERMINATION OF (γ, n) REACTIONS**

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THE measurement of photonuclear cross sections in the energy range of 25 to 80-100 Mev with presently-known methods involves many difficulties. These difficulties are in a large measure due to the smallness of cross section of the (γ, n) reaction and to the relatively high neutron background in experiments of this kind.

A further difficulty is connected with the necessity of knowing the sensitivity of the monitor as a function of the photon energy. In contrast to earlier work, where the (γ, n) excitation functions were determined with thin specimens, we propose a new method of determination of photonuclear cross sections. The basic idea of this method is the utilization of the equilibrium photon spectrum obtained when the original monoenergetic electron beam strikes a target so thick that practically the full electron-photon shower can develop.

The number of photoneutrons thus generated in the thick block of the investigated material is connected with the equilibrium photon spectrum and with the cross section $\sigma_{\gamma n}(E)$ by the integral equation (see, e.g., reference 1)

$$Q(E_0) = \int_{E_t}^{E_0} \sigma_{\gamma n}(E)\,\Gamma_p(E_0E)\,dE,$$

which in our case has a solution

$$\sigma_{\gamma n}(E_0) = \sigma_T(E_0) f(0)\left[\frac{E_0\beta}{f(0)}\frac{d^2Q(E_0)}{dE_0^2} + E_0\frac{dQ(E_0)}{dE_0} - Q(E_0)\right], \qquad (1)$$

where $\sigma_{\gamma n}(E_0)$ is the photoneutron production cross section, $\sigma_T(E_0)$ the total photon absorption cross section at energy E_0, β the critical energy, $Q(E_0)$ the neutron yield per primary electron of energy E_0, $f(0) = 2.29$, E_t the energy of the (γ, n) threshold in the material under investigation, $\Gamma_p(E_0E)$ the so-called equilibrium photon spectrum[2] which results when electrons of energy E_0 bombard a target of such thickness that practically the full electron-photon shower can develop.

Equation (1) was obtained by using the explicit expressions for $\Gamma_p(E_0E)$ given in reference 2.

It was reported in reference 3 that the uncertainties connected with the monitor problem alone can lead to errors in the cross section of (γ, n) reactions of the order of 100% in the 30 to 80 Mev energy region. In the same energy range, the cross sections can have additional errors of 50% due to errors in the determination of the photoneutron yield. In the presently proposed method the monitoring can be performed on the electron beam with greater accuracy (e.g., by means of