SOVIET PHYSICS JETP VOLUME 35(8), NUMBER 4 APRIL, 1959

ON SOME PECULARITIES OF CYCLOTRON RESONANCE IN METALS WITH NON-CONVEX FERMI SURFACES

E. A. KANER

Institute of Radio Physics and Electronics, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor April 27, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 962-964 (October, 1958)

It is shown that "logarithmic" cyclotron resonance should take place at Larmor frequencies near the hyperbolic limiting points of a non-convex Fermi surface.

IN previous papers[15] it was shown that cyclotron resonance takes place in metals with an arbitrary non-quadratric dispersion law $\epsilon(\mathbf{p})$* at Larmor frequencies that are extremal in p_H:

$$\Omega_{\text{ext}} = eH/cm_{\text{ext}} = 2\pi eH/c\,(\partial S/\partial\varepsilon)_{\text{ext}}$$

and at values of Ω at elliptic limiting points.†

However, in the investigation of Larmor resonant frequencies in the case of non-convex Fermi surface, resonance was not observed at Larmor frequencies that correspond to hyperbolic limiting points.

It is clear that for certain directions of the constant magnetic field $\mathbf{H}$, besides the elliptic limiting points (B, B′, C, C′ — see drawing), limiting points of the hyperbolic type A and A′ (saddle points) are present on a non-convex surface. Near these points, the effective mass M as a function of p_x ($Ox \parallel \mathbf{H}$) experiences discontinuities, since $m = (1/2\pi)\,\partial S/\partial\epsilon$ is determined in region 1 ($|p_x| < p_x^A$) by the areas $S(\zeta_0, p_x)$ of the cross section of the entire Fermi surface, while in region 5, for example, the effective mass is determined by the area $S(\zeta_0, p_x)$ only in this same region. Therefore, the Larmor frequency $\Omega(p_x) \sim 1/m(p_x)$ also points to a discontinuity of the function at $p_x = p_x^A, p_x^{A'}$, which reduces to cyclotron resonance for $\omega = q\,\Omega(p_x^A)$ (q = integer) and $\ln(\omega/\nu_0) \gg 1$. The present communication is devoted to a clarification of the properties of resonance at the hyperbolic limiting points.

Resonance at hyperbolic limiting points is logarithmic and for $\omega \gg \nu_0$ [$\nu_0 = \bar{\nu}(p_x^A)$], "sharpness" of the resonance is determined by $\ln(\omega/\nu_0)$. If the frequency ω and the magnetic field H satisfy the condition $\omega = q\,\Omega_4(p_x^{A'} - 0)$ (Ω_4 is determined by the cross-sectional area only in region 4) then, with logarithmic accuracy, the principal values of the tensor of the surface impedance $Z_{\mu\nu}$ are equal to

$$Z_\alpha = \sqrt{3}h\left(\frac{\pi\omega^2 q}{e^2c^4}\right)^{1/3}\left[\frac{\ln(\omega/\nu_0) + id}{a_\alpha}\right]^{1/3}, \tag{1}$$

where

$$a_\alpha = \left.\frac{n_\alpha^2}{K\partial\ln m/\partial\varphi}\right|_{\varphi_2-0} - \left.\frac{n_\alpha^2}{K\partial\ln m/\partial\varphi}\right|_{\varphi_1+0} \tag{2}$$

($0 < d \sim 1$). For $a_\alpha < 0$:

$$\operatorname{Re} Z_\alpha \sim \left(\ln\frac{\omega}{\nu_0}\right)^{-4/3}, \quad \operatorname{Im} Z_\alpha \sim \left(\ln\frac{\omega}{\nu_0}\right)^{-1/3}. \tag{2a}$$

If $a_\alpha > 0$,

$$Z_\alpha \sim \left(\ln\frac{\omega}{\nu_0}\right)^{-1/3}. \tag{2b}$$

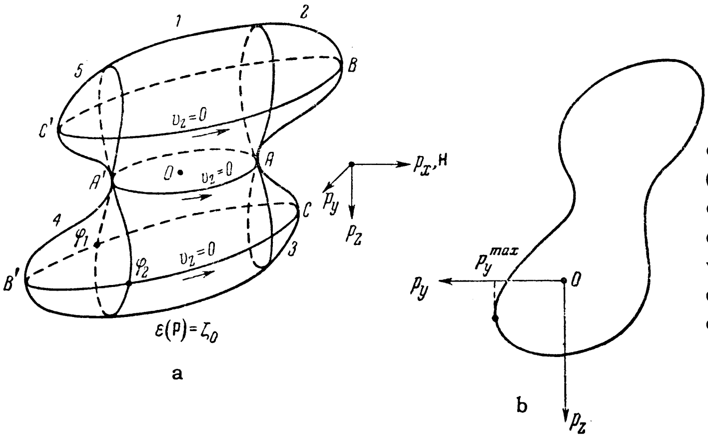
There are similar formulas for resonance (per effective mass) from the other side of the intersection $p_x = p_x^{A'} + 0$. In this case there remains in the expression (2) for a_α only the one term (one of the principal values of a_α is equal to 0; consequently, the resonance takes place at the chosen polarization of the external electromagnetic field along the direction of the velocity at the point $p_x = p_x^{A'}$; $\varphi = \varphi_2$, $\epsilon = \zeta_0$ (for details see references 3 and 5).

Cyclotron resonance at the remaining (extremal) Larmor frequencies in the case of a non-convex Fermi surface obviously possesses the same pecularities as for the convex surfaces (see references 1 to 5).

Calculations confirm the assertions made without proof in references 2 and 5 that, for a non-convex surface, not all curves $\nu_z \equiv -m^{-1} \times dp_y/d\tau = 0$ on the Fermi surface give a resonance contribution, but only those which correspond to

*We follow the notation of reference 2.

†At the limiting point, the velocity of the electrons (normal to the Fermi surface) is parallel to the magnetic field H.



a) non-convex Fermi surface $\varepsilon(p) = \zeta_0$; B, B′, C, C′ – elliptic limiting points; A, A′ – hyperbolic limiting points (saddle points). The regions 1 to 5 correspond to the regions of discontinuity of the effective mass $m(p_x) = 1/2\pi\, \partial S(\varepsilon, p_x)/\partial\varepsilon$; b) the intersection of the non-convex surface $\varepsilon(p) = \zeta_0$ with the plane p_x = const for $|p_x| < p_x^{(A)}$ (the trajectory of the electron in momentum space or the projection of the trajectory of the electron in the metal on the zy plane: $z \to -p_y$, $y \to p_z$).

a real minimum in the quantity $z = \Omega^{-1}\int v_z d\tau = -p_y/m\Omega$, i.e., a maximum $p_y(\tau)$ (τ/Ω is the period of rotation of the electron in its orbit). This condition guarantees a multiple hit of the electron in the skin depth and physically corresponds to a situation in which the highest point of the trajectory of the electron in the metal is located in this layer. The remaining extrema of $p_y(\tau)$ do not lead to resonance, inasmuch as the corresponding points of the trajectory of the electron in the metal lie outside of the skin layer, where the electron undergoes virtually no interactions with the high frequency field.

Equation (1) for Z_α is valid with logarithmic accuracy, inasmuch as there is no small parameter close to resonance. If $\ln(\omega/\nu_0) \sim 1$, Eq. (1) no longer holds, but it allows us to judge the qualitative picture of resonance at hyperbolic limiting points. Admitting all the difficulties associated with achieving large values of $\ln(\omega/\nu_0)$, we point out that for very pure specimens even at the present time, the values of $\omega/\nu_0 \sim 400$-600 [$\ln(\omega/\nu_0) \sim 5$-6] have been obtained. For example, for the samples of tin investigated in the research of Kip and others[6] at $\omega = 2\pi \cdot 24{,}000$ Mcs the value of ω/ν_0 was greater than 200.

In connection with the effect pointed out here, there are definite possibilities for determining the structure of the Fermi surface.

[1] M. Ia. Azbel' and E. A. Kaner, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 811 (1956), Soviet Phys. JETP **3**, 772 (1956).

[2] M. Ia. Azbel' and E. A. Kaner, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 896 (1957), Soviet Phys. JETP **5**, 730 (1957).

[3] E. A. Kaner and M. Ia. Azbel', J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1461 (1957), Soviet Phys. JETP **6**, 1126 (1958).

[4] E. A. Kaner, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1472 (1957), Soviet Phys. JETP **6**, 1135 (1958).

[5] M. Ia. Azbel' and E. A. Kaner, J. Phys. Chem. Solids, **6** (1958).

[6] Kip, Landenberg, Rosenblum, and Wagoner, Phys. Rev. **108**, 494 (1957).

Translated by R. T. Beyer.
199