

NUCLEAR BINDING ENERGIES IN THE REGION OF THE 82 PROTON AND 126 NEUTRON MAGIC NUMBERS

R. A. DEMIRKHANOV, T. I. GUTKIN and V. V. DOROKHOV

Submitted to JETP editor May 17, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 917-925 (October, 1958)

The results of measurements of masses of isotopes of bismuth, lead, thallium, and mercury, which were made with a mass spectrograph having a resolving power of 60,000 to 80,000, are presented. The masses of the isotopes were determined by direct comparison with the masses of appropriate organic compounds. The values of the masses of the Pb^{206} , Pb^{207} and Pb^{208} isotopes found from different doublets are internally consistent. The nuclear binding energies computed from the measured isotopic masses confirm the presence of a shell structure with completion of a shell at 82 protons and 126 neutrons. The difference in binding energy for nuclei with odd and even numbers of nucleons and its smoothing out as the shell is filled can be seen clearly. After the filling of the shell at $Z = 82$ and $N = 126$, the binding energy of the next neutron is greater than that of the next proton. The binding energy for a pair of neutrons, which gives Hg^{204} , is greater than the binding energy for a pair of protons, which gives Pb^{204} .

INTRODUCTION

THE determination of binding energies of nucleons in nuclei in the region of the 82 proton and 126 neutron magic numbers is of fundamental importance for explaining the structure of nuclei. On the basis of the shell model, the jump in binding energy of the next nucleon after the filling of the shell should be of the order of 2 or 3 Mev. In order to determine the size of the discontinuity in the mass region $A \sim 200$ to an accuracy of 10%, the relative accuracy of the measurement of $\Delta M/M$ must be better than 10^{-6} . The data available at present on masses of isotopes and nuclear binding energies in the mass region $A \sim 200$ have errors exceeding the required figure given above.¹⁻³ The small error given in reference 2 for the measurements of masses of the Pb^{207} , Pb^{206} , Pb^{204} , Tl^{205} , Tl^{203} , Bi^{209} , and Hg^{202} isotopes is explained by the fact that in determining the masses of these isotopes from the energy balance of nuclear reactions the mass of Pb^{208} was taken as a standard and was assigned zero error. The mass of Pb^{208} was taken from the mass spectrograph measurements of Duckworth.¹ A precision measurement of the masses of the lead isotopes is therefore of interest not only from the point of view of explaining the structure of nuclei near $A \sim 200$, but also in order to get a more precise value of the mass of the Pb^{208} isotope to serve as a standard in calculating masses of heavy isotopes from the energy

balance of nuclear reactions.

In addition, since Pb^{206} , Pb^{207} , Pb^{208} , and Bi^{209} are the final products in four of the radioactive series, their masses can be used as a base for calculating the masses of all the radioactive isotopes with $Z \geq 82$ from the energy balance of nuclear reactions. The masses of the Bi^{209} , Pb^{206} , Pb^{204} , Tl^{205} , Tl^{203} , Hg^{204} , Hg^{202} , Hg^{201} , Hg^{200} , Hg^{199} , and Hg^{198} isotopes have not been determined mass-spectrographically. The values calculated from energy balance of nuclear reactions for some of these isotopes contain a sizeable error ($\sim 3000 \mu \text{MU}$). The value of the binding energy for an additional neutron or proton determined from these values does not allow us to get a clear picture of nuclear structure, because of the large error in the measurements.

In carrying out the present work we used various means for increasing the accuracy of measurement. The measurement of the masses of the lead, mercury, bismuth, and thallium isotopes was done on a mass spectrograph which has been described earlier.^{4,5} The resolving power of the apparatus, as determined from doublet lines, was 60,000 to 80,000. To increase the accuracy of the measurements, the masses of the bismuth, lead, thallium, and mercury isotopes were determined by direct comparison with the masses of appropriate organic compounds containing H^1 , C^{12} , N^{14} , and O^{16} . The masses of these isotopes had been measured previously^{4,5} to sufficient accuracy. No

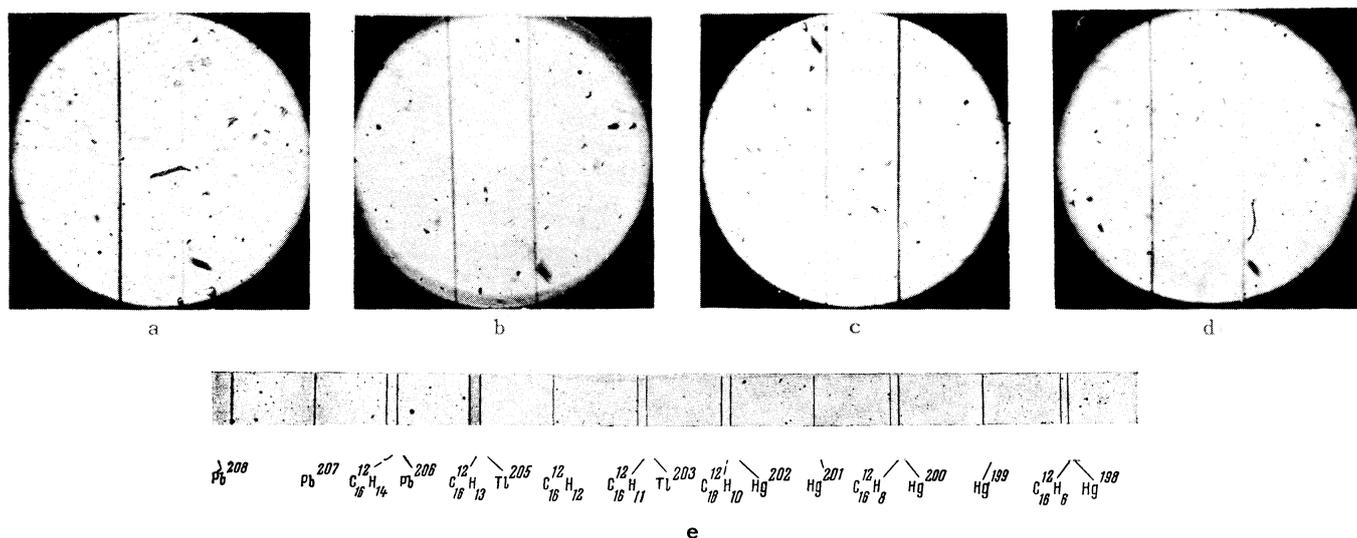


FIG. 1. Photographs of mass-spectrographic doublets and spectra of Pb, Tl, Hg and diphenylbutadiene: a) $C_{16}H_8 - Hg^{200}$ doublet ($\times 37$; M200); b) $C_{16}H_{10} - Hg^{202}$ doublet ($\times 37$; M202); c) $C_{16}H_{12} - Pb^{204}$ doublet ($\times 37$; M204); d) $C_{16}H_{15} - Pb^{207}$ doublet ($\times 37$; M207); e) spectrum of Pb, Tl, Hg, and di phenylbutadiene ($\times 6$; M206).

correction was made for content of rare isotopes in the hydrocarbons. The presence of a significant amount of C^{13} would have resulted in a marked broadening of the lines with a clearly visible asymmetry (a marked drop in intensity toward the low mass side). No such effect was noticed on the plates (cf. Fig. 1); this is even more correct for the rare isotopes D and N^{15} , which are less abundant. We can therefore state that errors due to the presence of rare isotopes are excluded.

MEASUREMENT OF MASSES OF ISOTOPES

Lead isotopes: Pb^{204} , Pb^{206} , Pb^{207} , and Pb^{208}
To check internal consistency, the masses of the lead isotopes were determined from various doublets. The lead ions were obtained by introducing tetramethyl lead vapor in one case and metallic lead vapor in another into the gas discharge region of a plasma ion source, using an evaporator of special construction.

In the first case we observed on the screen of the mass spectrograph spectra of both the lead isotopes and their hydrocarbon compounds, such as $PbCH_3$, $Pb(CH_3)_2$, $Pb(CH_3)_3$, and $Pb(CH_3)_4$. In this case the main discharge was supported by helium. Appropriate organic compounds were introduced into the gas discharge region by means of a heater of approximately the same construction as that for metallic lead. The evaporator construction enabled us to produce quickly a filling with a new sample of material without dismounting the source and breaking the vacuum in the system. A check of internal consistency was made not only with lead ions but also by measur-

ing the masses of the lead isotopes from doublets in various combinations with organic compounds, including the case where the molecular weight of the organic compound was equal to the atomic weight of lead. In this case we eliminate completely the phenomena that lead to the systematic measurement error due to the dissociation of the molecules both with respect to the hydrocarbons and with respect to the lead.⁶

In determining the mass of the Pb^{208} isotope, we used anthraquinone ($C_{14}H_8O_2$, $M = 208$) to form the doublet; this also gave us the doublet for Pb^{207} at mass 207. In another case the doublet line for Pb^{207} was obtained from the organic compound $C_{15}H_{12}O$. We used diphenylbutadiene ($C_{16}H_{14}$, $M = 206$) for the measurements of the masses of the Pb^{206} and Pb^{204} isotopes. The results of the measurements, obtained in each case from 18 to 20 mass-spectra, are given in Tables I and II.

The data of Tables I and II show that within the limits of error of the measurement there is internal consistency for the values of the doublets and the isotopic masses determined from different doublet combinations. The values of the masses of the isotopes were determined in the one case from metal vapor, in the other from the products of dissociation of tetramethyl lead. There was good agreement, within the statistical error of the measurement, when the doublet pair for a given lead isotope was formed by means of different organ compounds. This may serve as a confirmation of the absence of systematic errors in the measurement and of the reliability of the data.

TABLE I

Doublet	ΔM in 10^{-3} MU	Mass of lead isotope in MU	Average value of mass of lead isotope in MU	Product from which ions were obtained
$Pb^{204} - C_{16}H_{12}$	$120,472 \pm 44$	$204,038352 \pm 48$	$204,038352 \pm 48$	$C_{16}H_{12}$ from $C_{16}H_{14}$ Pb^{204} from metal vapor Pb^{206} from metal vapor
$Pb^{206} - C_{16}H_{14}$	$134,849 \pm 33$	$206,040259 \pm 39$	$206,040184 \pm 76$	Pb^{206} from $Pb(CH_3)_4$
$Pb^{206} - C_{16}H_{14}$	$135,000 \pm 66$	$206,040108 \pm 69$		
$Pb^{207} - C_{15}H_{11}O$	$105,273 \pm 35$	$207,041589 \pm 40$	$207,041574 \pm 35$	$C_{15}H_{11}O$ from $C_{15}H_{12}O$ Pb^{207} from metal vapor $C_{14}H_7O_2$ from $C_{14}H_8O_2$ Pb^{207} from metal vapor $C_{14}H_7O_2$ from $C_{14}H_8O_2$ Pb^{207} from $Pb(CH_3)_4$
$Pb^{207} - C_{14}H_7O_2$	$68,887 \pm 50$	$207,041587 \pm 54$		
$Pb^{207} - C_{14}H_7O_2$	$68,928 \pm 110$	$207,041546 \pm 112$		
$Pb^{208} - C_{14}H_8O_2$	$75,919 \pm 47$	$208,042697 \pm 51$	$208,042658 \pm 35$	Pb^{208} from metal vapor Pb^{208} from $Pb(CH_3)_4$
$Pb^{208} - C_{14}H_8O_2$	$75,998 \pm 59$	$208,042618 \pm 63$		

TABLE II

Author	Mass of isotope in MU			
	Pb^{204}	Pb^{206}	Pb^{207}	Pb^{208}
Duckworth ¹	—	—	$207,042900 \pm 1600$	$208,041600 \pm 1000$
Huizenga ²	$204,036859 \pm 130^*$	$206,038826 \pm 10^*$	$207,040580 \pm 10^*$	$208,041640 \pm 0^*$
Our data	$204,038352 \pm 48$	$206,040184 \pm 76$	$207,041574 \pm 35$	$208,042658 \pm 35$

*Errors marked with an asterisk should be increased by $\pm 1000\mu$ MU because of the error in the determination of the mass of Pb^{208} , as given by mass-spectrographic measurements.¹

TABLE III

Doublet	Value of ΔM in 10^{-3} MU	Masses of mercury isotopes in MU	
		from our data	from nuclear reactions*
$Hg^{198} - C_{16}H_8$	$80,259 \pm 68$	$198,029713 \pm 71$	$198,029000 \pm 3000$
$Hg^{199} - C_{13}H_{11}O$	$107,674 \pm 38$	$199,031548 \pm 45$	$199,030550 \pm 3050$
$Hg^{200} - C_{16}H_8$	$94,354 \pm 43$	$200,031902 \pm 47$	$200,031910 \pm 3010$
$Hg^{201} - Hg^{202}$	$100,398 \pm 32$	$201,034564 \pm 62$	$201,034000 \pm 3000$
$Hg^{202} - C_{16}H_{10}$	$107,588 \pm 48$	$202,034952 \pm 52$	$202,035341 \pm 620^{**}$
$Hg^{204} - C_{16}H_{12}$	$119,784 \pm 43$	$204,039040 \pm 47$	$204,037323 \pm 230^{**}$

*Huizenga² and Wapstra.³

**Data obtained using the mass of Pb^{208} as a standard.¹

TABLE IV

Doublet	Value of ΔM in 10^{-3} MU	Masses of thallium isotopes in MU	
		from our data	from nuclear reactions*
$Tl^{203} - C_{16}H_{11}$	$113,059 \pm 36$	$203,037623 \pm 41$	$203,035951 \pm 400^*$
$Tl^{205} - C_{16}H_{13}$	$127,061 \pm 38$	$205,039905 \pm 43$	$205,038480 \pm 130^*$

*Huizenga.²

TABLE V

Doublet	Value of ΔM in 10^{-3} MU	Mass of bismuth isotope in MU	
		from our data	from nuclear reactions*
$\text{Bi}^{209}-\text{C}_{14}\text{H}_{13}\text{N}_2$	127.516 ± 63	209.046864 ± 71	$209.045794 \pm 50^*$

*Huizenga².

TABLE VI

Isotope	M - Mass of isotope in MU from our data	M' - mass of isotope in MU from nuclear reac- tion data. The mass of Pb^{208} from the present work is taken as standard	$\Delta = M - M'$ in 10^{-3} MU
Hg^{198}	198.029713 ± 71	$198.029000 \pm 3000^*$	+0.713
Hg^{199}	199.031548 ± 45	$199.030550 \pm 3050^*$	+0.998
Hg^{200}	200.031902 ± 47	$200.031910 \pm 3010^*$	-0.008
Hg^{201}	201.034564 ± 62	$201.034000 \pm 3000^*$	+0.564
Hg^{202}	202.034952 ± 52	202.036359 ± 620	-1.407
Hg^{204}	204.039040 ± 47	204.038341 ± 230	+0.699
Tl^{203}	203.037623 ± 41	203.036969 ± 400	+0.654
Tl^{205}	205.039905 ± 43	205.039498 ± 130	+0.407
Pb^{204}	204.038352 ± 48	204.037877 ± 130	+0.475
Pb^{206}	206.040184 ± 76	206.039844 ± 10	+0.340
Pb^{207}	207.041574 ± 35	207.041598 ± 10	-0.024
Pb^{208}	208.042658 ± 35	208.042658 ± 0	0.000
Bi^{209}	209.046864 ± 71	209.046812 ± 50	+0.052

*The mass of the O^{16} isotope³ was used as a standard in the calculations. These values are given without correction.

Mercury isotopes: Hg^{198} , Hg^{199} , Hg^{200} , Hg^{201} , Hg^{202} , and Hg^{204} . The mercury ions were obtained by introducing mercury vapor into the gas-discharge region of the ion source. Diphenylbutadiene was used to form the doublet pairs. The values of the mass differences of these doublets and the masses of the mercury isotopes calculated from them are given in Table III. For comparison we give in the table the data obtained from nuclear reactions.

Thallium isotopes: Tl^{203} and Tl^{205} . The thallium ions were obtained from metallic thallium vapor. Diphenylbutadiene was used to form the doublet pair. The values of the mass differences of the doublets and the masses of the isotopes are given in Table IV. For comparison we also include data from nuclear reactions.

Bi^{209} isotope. Bismuth ions were obtained from vapors of metallic bismuth. Phenylhydrazone acetophenone ($\text{C}_{14}\text{H}_{13}\text{N}_2$, $M = 209$) was used to form the doublet pair. The mass difference of the doublet and the mass of the isotope calculated from these data are given in Table V.

Comparing the values of isotopic masses found in the present work with the corresponding data from nuclear reactions (cf. Tables II to V), we see that in general they agree with one another within the limits of error of the measurements. However, this agreement is achieved only as a

result of the large errors in determination of the masses of isotopes from the Q values of nuclear reactions. For the Hg^{198} , Hg^{199} , Hg^{200} , and Hg^{201} isotopes, for which in the last analysis the mass of O^{16} is taken as a base, the large error in the measurement results from using a large number of intermediate steps with their corresponding Q values. For all those isotopes for which the mass of the Pb^{208} isotope is used as a standard, the magnitude of the error of the measurement is determined both by the errors of the corresponding Q values and by the accuracy of measurement of the mass of Pb^{208} . In his tables, Huizenga² used for the Pb^{208} isotope the value $M = 208.041640 \pm 1000 \mu \text{MU}^*$ found by Duckworth.¹ For the whole set of isotopes presented here, the differences of the average values are sufficiently large ($\sim 1500 \mu \text{MU}$) as to be possibly due to the inaccuracy of the value of the mass of Pb^{208} which was taken as a standard.

In Table VI we give the masses of the isotopes calculated using Q -values with the value of the mass of the Pb^{208} isotope which was found in the present work as the standard. We then find agreement, within the limits of error of the measurements, between the present data and the calculated values for most of the isotopes, even though the

*The errors of measurements in the text and tables are given in μMU .

TABLE VII

Isotope	A	Z	N	Binding energy of nucleons in the nucleus in Mev	Binding energy per nucleon in Mev
Hg ¹⁹⁸	198	80	118	1566.100±0.066	7.90960±0.00033
Hg ¹⁹⁹	199	80	119	1572.758±0.043	7.90331±0.00021
Hg ²⁰⁰	200	80	120	1580.795±0.044	7.90397±0.00022
Hg ²⁰¹	201	80	121	1586.682±0.057	7.89394±0.00034
Hg ²⁰²	202	80	122	1594.685±0.048	7.89450±0.00024
Hg ²⁰⁴	204	80	124	1607.614±0.044	7.88046±0.00022
Tl ²⁰³	203	81	122	1599.782±0.038	7.88070±0.00019
Tl ²⁰⁵	205	81	124	1614.390±0.040	7.87507±0.00020
Pb ²⁰⁴	204	82	122	1606.685±0.045	7.87590±0.00022
Pb ²⁰⁶	206	82	124	1621.712±0.071	7.87238±0.00034
Pb ²⁰⁷	207	82	125	1628.784±0.033	7.86852±0.00016
Pb ²⁰⁸	208	82	126	1636.141±0.033	7.86606±0.00016
Bi ²⁰⁹	209	83	126	1639.806±0.066	7.84596±0.00034

*1 MU = 931,162 ± 24 kev.

TABLE VIII

Isotope	A	Z	N	Mass of isotope in MU	Binding energy of nucleons in the nucleus, in Mev	Binding energy per nucleon, in Mev
Bi ²⁰⁸	208	83	125	208.045699±90	1632.524±0.084	7.84867±0.00041
Pb ²⁰⁹	209	82	127	209.047492±70	1640.006±0.065	7.84692±0.00031
Bi ²¹⁰	210	83	127	210.051378±40	1643.969±0.037	7.82842±0.00018
Pb ²¹⁰	210	82	128	210.051447±40	1644.690±0.037	7.83185±0.00018
Po ²¹⁰	210	84	126	210.050115±40	1644.360±0.037	7.83028±0.00018

error in measurement of the mass of the standard Pb²⁰⁸ isotope is ~ 30 times smaller than the error in the determination of the Pb²⁰⁸ mass value used in the tabulated data.^{1,2} However, as one sees from Table VI, the values of the isotopic masses obtained by calculation from nuclear reaction data (cf. column 3) are essentially lower by 0.3 to 0.5 m MU than the present data, except for the mass of Hg²⁰².

BINDING ENERGY OF NUCLEONS IN THE NUCLEUS

The values of the masses of the bismuth, lead, thallium, and mercury isotopes found in the present work enable us to determine more precisely the binding energy of nucleons in the nucleus in the region of the 82 proton and 126 neutron magic numbers. The values of the nuclear binding energies and the binding energies per nucleon are given in Table VII. In addition to this, by using Q values from nuclear reactions and β decay, the masses of the Bi²⁰⁸, Pb²⁰⁹, Pb²¹⁰, Bi²¹⁰, and Po²¹⁰ isotopes were calculated (cf. Table VIII). In these calculations, the masses of Pb²⁰⁸ and Bi²⁰⁹ found in the present work were used as standards. The mass of the Pb²⁰⁹ isotope was calculated from the Pb²⁰⁸ (d, p) Pb²⁰⁹ reaction with the value Q = 1.64 ± 0.05 Mev.⁷ The value found for the mass of the Pb²⁰⁹ isotope is M_{Pb209} = 209.047492 ± 70 MU. The value for Pb²⁰⁹ was also calculated using the value Q = 0.63 ± 0.01 Mev⁸ obtained from the

⁸²Pb₁₂₇ → ⁸³Bi₁₂₆ β decay. The value of the mass of the Pb²⁰⁹ isotope found from this reaction was M_{Pb209} = 209.047544 ± 67, which within the limits of error of the measurement is in satisfactory agreement with the value found from the Pb²⁰⁸ (d, p) Pb²⁰⁹ reaction. The calculated values of nuclear binding energies are given in Table VIII. Figure 2 shows the curve for the binding energy per nucleon in the nucleus. The significant difference of the binding energies of the Bi²⁰⁸, Pb²⁰⁹, Bi²⁰⁹, Pb²¹⁰, and Po²¹⁰ compared with the Pb²⁰⁸ isotope is a confirmation of the existence of a nuclear shell structure with closing of shell at Z = 82 and N = 126. The difference in binding energy of nucleons in nuclei with even and odd

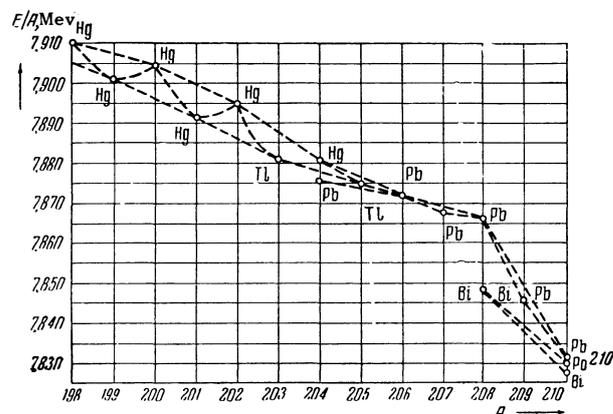


FIG. 2. Binding energy per nucleon in the region 198 ≤ A ≤ 210.

TABLE IX

Isotope	Z	N	Binding energy of the last neutron, in Mev	Isotope	Z	N	Binding energy of the last neutron, in Mev
Hg ¹⁹⁹ Hg ¹⁹⁸	80 80	119 118	6.658	Au ^{197*} Pt ^{196*}	79 78	118 118	6.105
Hg ²⁰⁰ Hg ¹⁹⁹	80 80	120 119	8.037	Au ^{198*} Pt ^{197*}	79 78	119 119	6.467
Hg ²⁰¹ Hg ²⁰⁰	80 80	121 120	5.887	Au ^{199*} Pt ^{198*}	79 78	120 120	5.691
Hg ²⁰² Hg ²⁰¹	80 80	122 121	8.003	Hg ²⁰¹ Au ^{200*}	80 79	121 121	6.395
Hg ^{203*} Hg ²⁰²	80 80	123 122	5.994	Hg ^{202*} Au ^{201*}	80 79	122 122	7.826
Tl ^{204*} Tl ²⁰³	81 81	123 122	7.362	Tl ^{202*} Hg ²⁰¹	81 80	121 121	6.322
Tl ²⁰⁵ Tl ^{204*}	81 81	124 123	7.246	Tl ²⁰³ Hg ²⁰²	81 80	122 122	5.095
Pb ²⁰⁷ Pb ²⁰⁶	82 82	125 124	7.072	Tl ^{204*} Hg ^{203*}	81 80	123 123	6.463
Pb ²⁰⁸ Pb ²⁰⁷	82 82	126 125	7.357	Tl ²⁰⁵ Hg ²⁰⁴	81 80	124 124	6.776
Pb ²⁰⁹ Pb ²⁰⁸	82 82	127 126	3.845	Pb ²⁰⁶ Tl ²⁰⁵	82 81	124 124	7.322
Pb ²¹⁰ Pb ²⁰⁹	82 82	128 127	4.684	Bi ^{207*} Pb ²⁰⁶	83 82	124 124	3.866
Pb ^{211*} Pb ²¹⁰	82 82	129 128	4.355	Bi ^{208*} Pb ²⁰⁷	83 82	125 125	3.644
Bi ²⁰⁹ Bi ²⁰⁸	83 83	126 125	7.282	Bi ²⁰⁹ Pb ²⁰⁸	83 82	126 126	3.665
Bi ²¹⁰ Bi ²⁰⁹	83 83	127 126	4.554	Bi ²¹⁰ Pb ²⁰⁹	83 82	127 127	4.354
Bi ^{211*} Bi ²¹⁰	83 83	128 127	5.280	Bi ^{211*} Pb ²¹⁰	83 82	128 128	4.390
Po ^{213*} Po ^{214*}	84 84	131 130	4.087	At ^{215*} Po ^{214*}	85 84	130 130	4.067
Po ^{216*} Po ^{215*}	84 84	132 131	5.806	At ^{216*} Po ^{215*}	85 84	131 131	4.568
Po ^{217*} Po ^{216*}	84 84	133 132	4.088	At ^{217*} Po ^{216*}	85 74	132 132	4.745
Po ^{218*} Po ^{217*}	84 84	134 133	5.608	At ^{218*} Po ^{217*}	85 84	133 133	5.175
Po ^{219*} Po ^{218*}	84 84	135 134	3.628	At ^{219*} Po ^{218*}	85 84	134 134	5.136

*Binding energies calculated from the data of references 2 and 3.

mass numbers¹⁰ is seen very clearly on the curve. As we see from Fig. 2, as the shell is filled this difference decreases. The effect of the nuclear shell structure can also manifest itself in the energy for binding the (Z + 1)-th proton and the (N + 1)-th neutron. The values of these quantities are given in Table IX. The tables show that there is a sharp discontinuity in the binding energy at 83 protons and 126 neutrons. A characteristic feature in the filling of the shells is the fact that

the binding energy for a neutron is greater than that for a proton. This can be seen directly from Fig. 2, by comparing the binding energies of Pb²⁰⁹ and Bi²⁰⁹ and Bi²⁰⁹ with those of Pb²¹⁰ and Po²¹⁰, and also from Table IX. It can be seen especially clearly from a comparison of the binding energies of the Pb²⁰⁴ and Hg²⁰⁴ isotopes. The proton shell is filled at Pb²⁰⁴, so that one would expect a maximum binding energy compared to other isotopes having A = 204. However, at the

stable isotope Hg^{204} , the binding energy proved to be greater, because of the larger value of the binding energy of a pair of neutrons compared to the corresponding pair of protons in Pb^{204} . This effect is washed out in the packing fraction curve. Thus for very accurate measurements the use of the packing fraction curve is not suitable for analysis of dependences of nuclear binding energies. We see from Table IX that the binding energy of the last neutron or proton satisfies the odd-even rule, i.e., the energy for binding each odd proton or neutron is less than that for binding each even one.

In addition to the jump in binding energy at the closing of the shell (i.e., at $N = 127$ and $Z = 83$), we also noted a non-monotonic behavior of the curve of binding energy of the last proton or neutron for the nuclei with $N = 121$ and $Z = 81$ (cf. Table IX), i.e., 6 neutrons and 2 protons from the start of the filling of the new shell. In order to check whether there is any regularity in the jumps of binding energy in this region, we used, in addition to the measured values of the masses of 13 isotopes and the corresponding nuclear binding energies, the values of binding energies from references 2 and 3, even though the uncertainties of the tabulated data of reference 2 and especially of reference 3 are so great that it is impossible to speak of regularities with any certainty on the basis of these data. From examination of these data, one also sees a non-monotonic behavior of the curve of binding energy of the last neutron or proton for some other nuclei which are shifted by 6 neutrons and 2 protons, i.e., for which $Z' = Z + 2$ and $N' = N + 6$.

With respect to the neutrons, this non-monotonicity of the curve of binding energies for a shift of 2 protons and 6 neutrons appears in the fact that the binding energy of the 115-th neutron for $Z = 78$, of the 121-st neutron for $Z = 80$, of the 127-th neutron for $Z = 82$ and of the 133-rd neutron for $Z = 84$ is markedly low. For protons this non-monotonic behavior for a shift of 2 protons and 6 neutrons manifests itself in a more complicated way. After Bi^{209} with $Z = 83$ and $N = 126$, the jump in the binding energy for $Z = 81$ and $N = 120$ cannot be used, since no nucleus exists for $Z = 81$ and $N = 120$. The nearest nucleus with $Z = 81$ and an even number of neutrons, i.e., with $N = 122$, has a minimal binding energy compared to its neighbors, as we see from Table IX. A markedly reduced

value of the binding energy of the last proton occurs at $Z = 79$ and $N = 120$, and at $Z = 77$ and $N = 114$. The uncertain drop in binding energy of the last proton at $Z = 85$ and $N = 132$ becomes completely clear on the curve of binding energy of the last proton as a function of neutron number in the region $130 \leq N \leq 134$.

The "anomaly" noted by Johnson and Nier⁹ in the binding energy of nuclei in the region of $N = 90$ may possibly be of this same type. Small nuclear deformations are known to be unstable. A change in the shape of the nucleus and, consequently, of the size of its surface results in a discontinuity. A change of the surface of the nucleus may be one of the reasons for jumps in the binding energy of even-odd and odd-even nuclei.

For the majority of the isotopic masses found in the present work, the nuclear binding energies are raised by approximately 1500 keV compared with those calculated from nuclear reactions,² in which the value of the mass of Pb^{208} found in reference 1 is used as a standard value.

In conclusion the authors must express their thanks to E. E. Baron', T. N. Lebsadza, K. A. Kovyrzina, and V. M. Shoniia for the preparation of the metal-organic compounds and heavy hydrocarbons, and also to P. S. Brostiuk, M. I. Dzkuia, and G. A. Dorokhova for practical assistance in the work.

¹H. E. Duckworth and R. S. Preston, *Phys. Rev.* **82**, 468 (1951). Stanford, Duckworth, Hogg and Geiger, *Phys. Rev.* **85**, 1039 (1952). B. G. Hogg and H. E. Duckworth, *Can. J. Phys.* **32**, 65 (1954).

²J. R. Huizenga, *Physica* **21**, 410 (1955).

³A. H. Wapstra, *Physica* **21**, 385 (1955).

⁴Demirkhanov, Gutkin, Dorokhov, and Rudenko, *Атомная энергия (Atomic Energy)* **1**, 21 (1956).

⁵Demirkhanov, Gutkin, and Dorokhov, *Атомная энергия (Atomic Energy)* **2**, 544 (1957).

⁶T. I. Gutkin, *Приборы и техника эксперимента (Instr. and Meas. Engg.)* No. 5, 46 (1957).

⁷D. M. Van Patter and W. Whaling, *Revs. Modern Phys.* **26**, 402 (1954).

⁸R. W. King, *Revs. Modern Phys.* **26**, 327 (1954).

⁹W. H. Johnson and A. O. Nier, *Phys. Rev.* **105**, 1014 (1957).

¹⁰V. A. Kravtsov, *Usp. Fiz. Nauk* **47**, 341 (1952).