BREMSSTRAHLUNG FROM π MESONS INTERACTING WITH NUCLEI

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Bremsstrahlung from π^+ mesons in a nuclear force field was observed in a freon bubble chamber in the course of study of the interactions of π^+ mesons of energies from 80 to 300 Mev with carbon, fluorine, and chlorine nuclei. The bremsstrahlung cross section both in the cases of inelastic and elastic scattering of π^+ mesons amounts, on the basis of 20 observed events, to $(4.5^{+1.2}_{-2.0}) \times 10^{-27}$ cm^2 per fluorine nucleus in the energy range indicated above. Three events have been observed of bremsstrahlung accompanying the absorption of a π^+ meson by a nucleus, and two events have been observed which may be attributed to bremsstrahlung accompanying charge exchange scattering of π^+ mesons by nuclei. The bremsstrahlung cross section for various nuclear processes has been calculated in the quasi-classical approximation. The values of the cross sections computed from these formulas are in good agreement with the experimental results.

1. INTRODUCTION

BECAUSE of the strong nuclear interaction between π mesons and nucleons, bremsstrahlung from π mesons, i.e., the emission of γ quanta arising as a result of a change of velocity or of a loss of charge, must be determined primarily by nuclear rather than Coulomb forces. Bremsstrahlung in a nuclear force field from mesons of various energies has been treated in several theoretical papers.[1-4] In particular, Solov'ev[3] has discussed bremsstrahlung from π mesons at energies close to the π-meson rest energy. It was found that the bremsstrahlung cross section in this energy range should in order of magnitude amount to approximately 10^{-28} cm^2 per nucleon.

The relatively large theoretically-expected value of the bremsstrahlung cross section for π mesons in a nuclear force field makes it probable that this phenomenon should be observed experimentally. However, up until now no bremsstrahlung from π mesons has been found experimentally. In the present work bremsstrahlung from π mesons was found in the course of study of the interactions of π^+ mesons with light nuclei in the range of energies close to the rest energy.

The experiments were carried out in the external π^+-meson beam of the synchrocylclotron of the Laboratory for Nuclear Problems of the Joint Institute for Nuclear Research. π^+ mesons of definite energy were introduced into a freon bubble chamber[5] whose useful observation volume amounted to approximately 17 l. The working liquid in the chamber was a mixture of two freons — CCl_2F_2 and $CClF_3$. Bremsstrahlung γ quanta were observed by means of conversion into electron-positron pairs. The energies of the charged products of the interaction of π^+ mesons in the chamber were determined mainly from the range of the particles. The range curves have been computed earlier[5] according to the theory of ionization losses. In some cases the energy of π mesons after interaction was found by measuring the ionization losses along the length of the tracks or by multiple scattering. The remaining details of the method are given in the paper by Blinov et al.[6]

2. CLASSIFICATION OF THE VARIOUS FORMS OF BREMSSTRAHLUNG AND COMPETING PROCESSES

We shall distinguish the following cases of bremsstrahlung that can result from the interaction of π mesons with nuclei.

1. Radiation arising during inelastic scattering of π mesons by nuclei. In this case the principal track of the π meson in bubble-chamber photographs shows a kink at the point of interaction, and at the same time the photograph shows an electron-positron pair pointing towards the kink. The scheme of the process $\pi^+ + A \rightarrow \pi^+ + \gamma + A'$, where A and A′ are the initial and final states of the nucleus. The transformation of nucleus A into nucleus A′ is accompanied by the emission of protons and neutrons from nucleus A, which in the majority of cases can be seen in the photograph as proton tracks leaving the point of interaction. Such a photograph can alternately be in-

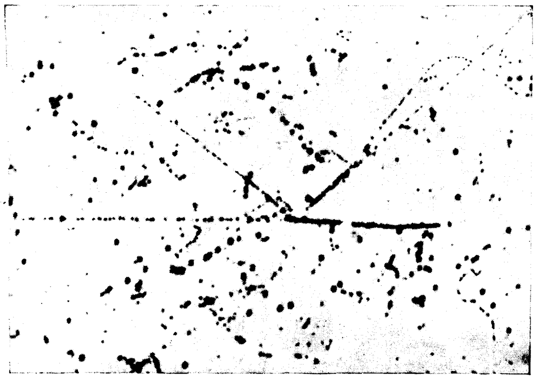

FIG. 1. Bremsstrahlung accompanying inelastic scattering.

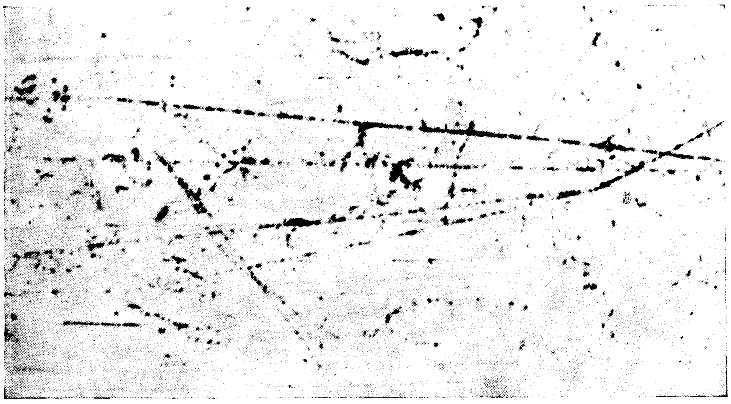

FIG. 2. Bremsstrahlung accompanying scattering (elastic or inelastic).

FIG. 3. Bremsstrahlung accompanying absorption.

terpreted as the creation of a neutral π meson in accordance with $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$, with subsequent decay of the π^0 meson into two γ quanta, only one of which is converted within the region of observation. One may discriminate between this competing process and bremsstrahlung on the basis of the following energy considerations. We denote the kinetic energy of the π meson before interaction by E_0, after interaction by E_1, and the energy of the protons leaving the nucleus by E_p. It is then evident that if the following condition is fulfilled

$$E_0 - (E_1 + E_p) < \mu c^2,$$

where μ is the π^0-meson mass, this event should be attributed to bremsstrahlung. The inverse condition

$$E_0 - (E_1 + E_p) > \mu c^2$$

can be evidence either of the process $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$, or of bremsstrahlung, and therefore such cases require additional investigation.

2. Radiation arising during elastic or diffraction scattering of a π meson by a nucleus in accordance with the scheme $\pi^+ + A \rightarrow \pi^+ + \gamma + A$. In the photographs this case differs from the preceding one by the absence of proton tracks at the kink, and by a relatively small angle of scattering of the π meson. Just as in the preceding case, a competing process is the process of creation of a π^0 meson by a π^+ meson according to the reaction $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$, under the condition that the transformation of nucleus A into nucleus A′ is not accompanied by visible tracks of reaction products, so that the measured value is $E_p = 0$. It is evident that if the inequality $E_0 - E_1 < \mu c^2$ is satisfied, the event should be attributed to bremsstrahlung.

3. Radiation arising when a π meson is absorbed in accordance with $\pi + A \rightarrow \gamma + A'$. In the photograph the π-meson track ends in a star or disappears ("prongless" star). At the same time the photograph shows a pair directed towards the center of the star or towards the end of the track. A competing and a considerably more probable process in this case is the process of charge-exchange scattering of the π^+ meson in accordance with the reaction $\pi^+ + A \rightarrow \pi^0 + A'$ under the condition that, as in the preceding two cases, only one of the two γ quanta into which the π^0 meson decays is converted within the region of observation. As before, discrimination between these two processes is possible only in individual cases, on the basis of energy considerations.

4. Radiation arising in charge-exchange scattering of the π^+ meson according to the reaction $\pi^+ + A \rightarrow \pi^0 + \gamma + A'$. The track of the π meson ends in a star or stops, while electron-positron pairs directed toward the point of interaction can be seen in the photograph. If three pairs can be seen in the photograph, then the process $\pi^+ + A \rightarrow \pi^0 + \pi^0 + A'$ is possible, i.e., the creation of a π^0 meson accompanied by a simultaneous charge exchange of the π^+ meson is possible, with only three of the four γ quanta from the π^0 meson decay being converted.

In all the above four cases the bremsstrahlung can be distinguished from the competing process not only on the basis of energy balance, but also by considering the angular distribution of the γ quanta. As will be shown in Sec. 4, the theoretical angular distribution of the bremsstrahlung γ quanta is such that most γ-quanta should be emitted forward, while the angular distribution of the γ quanta coming from the decay of π^0 mesons must be considerably more isotropic.

3. EXPERIMENTAL RESULTS

More than 7000 photographs obtained in three series of experiments with different initial π^+-meson energies beam have been analyzed. In these photographs there were observed 20 events of bremsstrahlung accompanying inelastic and elastic scattering, three events accompanying absorption, and two events which may be attributed either to bremsstrahlung accompanying charge exchange, or to the process $\pi^+ + A \rightarrow \pi^0 + \pi^0 + A'$.

Typical photographs of bremsstrahlung processes accompanying scattering and absorption are given in Figs. 1 to 3. The photograph shown in Fig. 2 can apparently be interpreted as bremsstrahlung accompanying elastic scattering, on the basis of the absence of a star at the point of interaction and of the small angle of scattering. Naturally these features are not sufficient to distinguish this event from bremsstrahlung accompanying inelastic scattering, since inelastic interaction might for one reason or another, not be accompanied by visible proton tracks in the photograph.

Since there exists no reliable criterion for distinguishing between eleastic and inelastic scattering, we have classified all the 20 observed events as bremsstrahlung accompanying scattering. These events are given in the table.

The values of the energies E_0 and E_p given in the table are obtained from measurements of the ranges of the particles. The precision with which the energies E_0 are determined is approximately 5%. The energy E_1 of events 9 and 12 has been determined from the energy of the prongs of the star in which the π^+-meson track ends after scattering. In two events (10 and 19) the energy E_1 is also found from the multiple scattering of the π^+ meson. The γ-quanta energy E_γ was determined from the ranges of the electron and positron of the pair.

As can be seen from the table, for the first 13 events the inequality $E_0 - (E_1 + E_p) < 140$ Mev holds, so that these events cannot be interpreted as the result of the creation of a π^0 meson in accordance with the reaction $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$, and must definitely be attributed to bremsstrahlung. In the remaining seven events (14 to

Observed events of bremsstrahlung emitted by π^+-mesons during scattering (energies given in Mev)

No.	π^+-meson energy: Before scattering, E_0	After scattering, based on: Range	Ionization	Other Considerations	Energy of star prongs E_p	Energy of γ quanta, E_γ	$E_0 - (E_1 + E_p)$
1	237	>60	>100		>120	~ 10	<17
2	276	>55	>70		173	>30	<33
3	237	>50	>100		100	>23	<37
4	170	>110			—	>37	<60
5	183	63			54	>23	66
6	239	>40			120	>29	<79
7	213	>60	>100		20	>22	<93
8	147	>54			—	>13	<93
9	262			>80	77	>26	<105
10	266	>54	>160	160 ± 80	—	>17	<106
11	279	>165			—	>28	<114
12	235			>69	36	>55	<130
13	250	>65	>100		18	>60	<132
14	265	>12	>25		35	~ 15	<205
15	152	<5			~ 7	>24	<145
16	256	65			40	>30	151
17	247	55			20	>70	<172
18	261	>40			46	>24	<175
19	249	>54		~ 60	—	>35	<195
20	259	>35			—	>20	<224

20) $E_0 - (E_1 + E_p)$ may exceed 140 Mev, which indicates the possibility of creation of a π^0 meson. The identification of these events as bremsstrahlung requires a separate investigation.

Event 14 should be attributed to bremsstrahlung, since the γ-quantum energy determined from the ranges of the electron and the positron of the pair, and also from the angle between their paths, is approximately 15 Mev. A γ-quantum of such energy cannot be the result of π^0-meson decay and must be interpreted as being due to bremsstrahlung.

In event 15 the initial energy of the π^+ meson is $E_0 = 152$ Mev, i.e., very close to the energy threshold of creation of a π meson by a π meson. In view of the very low probability of observing this process at such a value of E_0, this case must also be attributed to bremsstrahlung.

The last five events (16 to 20) should be ascribed, with a high degree of probability, to bremsstrahlung on the basis of the following two considerations. Firstly, not more than one out of the five events can be ascribed to the competing process $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$, the cross section of which can be estimated with an accuracy sufficient for present purposes from the results of Blinov et al.[6] Secondly, measurements of the angles of emission of the γ quanta, have shown the radiation in all five events to be directed forward, principally within the 10 to 40° range, while in the case of the process $\pi^+ + A \rightarrow \pi^+ + \pi^0 + A'$ a more isotropic angular distribution of the γ quanta is expected.

On the basis of the 20 events given in the table, we computed the total cross section for bremsstrahlung accompanying scattering (elastic and inelastic) in the energy range from 80 to 300 Mev, and found it to be $\sigma_S^\gamma = (4.5^{+1.2}_{-2.1}) \times 10^{-27}$ cm^2 per fluorine nucleus.

As can be seen from the table we did not observe a single case of bremsstrahlung in the energy range from 80 to 150 Mev. This can be explained, first, by the experimental condition, since in the energy range indicated the number of π mesons passing through the chamber will be smaller by a factor of several fold than at higher energies. Second, theoretical estimates (Sec. 4) show that the cross section for bremsstrahlung accompanying scattering at a π-meson energy of 100 Mev must be approximately one-half that obtained at an energy of 250 Mev.

4. COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

A theoretical estimate of the bremsstrahlung from π mesons may be made with sufficient accuracy in the quasi-classical approximation. The quasi-classical formula is obtained if the exact expression for the differential bremsstrahlung cross section $d\sigma_\gamma(\omega, \beta)$ is expanded in terms of the energies $\hbar\omega$ of the γ quantum. In this expansion the first term has the quasi-classical form, i.e., it is the product of the cross section $d\sigma(\beta)$ of the corresponding nonradiative process by the probability dW_γ of radiation of a γ quantum for a given state of motion of the particles. We have denoted by β the complete set of initial and final velocities of the interacting particles. As shown by Low,[7] a general form of the second term in the expansion of the exact cross section $d\sigma_\gamma(\omega, \beta)$ can also be derived. However, in making estimates of the total radiation there is no point in using the second term of the expansion, since when $\hbar\omega/E \ll 1$, where E is the energy of the colliding particles, it gives rise to a correction of the order of $\hbar\omega/E$, which is of no significance to us, while in the case $\hbar\omega/E \sim 1$ the contribution of this term is of the same order of magnitude as that of subsequent terms in the expansion, and therefore we cannot be sure that the accuracy is improved by using the second term of the expansion in the upper part of the spectrum of the γ quanta. Moreover, this term also contains a derivative (with respect to energies and angles of the amplitude of the nonradiative process) which is difficult to determine with sufficient accuracy from the available experimental data. With these remarks in mind, we shall restrict the comparison of our results with theory only to the first term in the expansion of $d\sigma_\gamma(\omega, \beta)$.

The advantage of the quasi-classical estimate of bremsstrahlung is that in this estimate the formulas are expressed in terms of the exact nonradiative process $d\sigma(\beta)$ independently of the model used for the interaction of the π meson with the nucleus. This is a very important fact, since there exist no reliable methods of making calculations in the case of strong interaction. With respect to the high-energy part of the spectrum of the γ quanta, where the quasi-classical approximation is inapplicable, we can see that at our π-meson energies this part of the spectrum makes no significant contribution to the total bremsstrahlung cross section.

If we take the nucleus to be infinitely heavy and if we denote by β_0 and β_1 the initial and the final velocities of the π meson without taking into account the energy-momentum loss due to radiation, we obtain in the quasi-classical approximation:

$$d\sigma_\gamma(\omega, \beta_0, \beta_1) \approx dW_\gamma d\sigma(\beta_0, \beta_1) = \frac{1}{137}\left[\left(\frac{\boldsymbol{\beta}_0}{(1-\boldsymbol{\beta}_0\cdot\mathbf{n})} - \frac{\boldsymbol{\beta}_1}{1-\boldsymbol{\beta}_1\cdot\mathbf{n}}\right)\mathbf{n}\right]^2 \frac{d\Omega_\gamma}{4\pi^2}\frac{d\omega}{\omega} d\sigma(\beta_0, \beta_1), \tag{1}$$

where $\mathbf{n}$ is the unit vector in the direction of radiation of γ quanta, $d\Omega_\gamma$ is an element of solid angle of the γ quanta, and $d\sigma(\beta_0, \beta_1)$ is the cross section of some nonradiative process.

To compute the total cross section for any one of the classes of bremsstrahlung defined in Sec. 2, we should substitute into formula (1) the expression for the differential cross section of the corresponding nonradiative process and integrate over the energies and directions of the γ quanta and over the final velocities of the π meson. Assuming an isotropic angular dependence, we express the cross section for the inelastic scattering of a π meson by a nucleus in the form

$$d\sigma_i(\beta_0, \beta_1) = \frac{\sigma_i(\beta_0, \beta_1)}{4\pi}\, d\Omega, \tag{2}$$

where $\sigma_i(\beta_0, \beta_1)$ is the total cross section for inelastic scattering, for which the final velocity of the π meson is given by β_1.

To obtain the cross section for the elastic scattering of a π meson, we make use of the diffraction type of angular dependence and write

$$d\sigma_d(\beta_0, \beta_1) = \frac{\sigma_d(\beta_0)}{\pi}\left|\frac{I_1(R\theta/\lambda)}{\theta}\right|^2 d\Omega, \tag{3}$$

where R is the nuclear radius, $\cos\theta = \boldsymbol{\beta}_0\boldsymbol{\beta}_1/\beta_0\beta_1$, $\beta_0 = \beta_1$, λ is the wavelength of the π meson, and $\sigma_d(\beta_0)$ is the total cross section for elastic scattering.

We shall combine the cross sections for π-meson absorption and charge-exchange scattering into a total cross section for processes in which the π-meson charge comes to rest or, more accurately, the charge is transferred to nonrelativistic nucleons:

$$d\sigma_r(\beta_0) = d\sigma_a(\beta_0) + d\sigma_e(\beta_0). \tag{4}$$

Replacing $d\sigma(\beta_0, \beta_1)$ in formula (1) by one of the expressions (2), (3), or (4) and integrating, we obtain the required expressions for the total cross sections of radiative processes. For bremsstrahlung accompanying inelastic scattering we obtain:

$$\sigma_i^\gamma = \frac{1}{137}\ln\frac{xE_0}{\hbar\omega_{\min}} \times \sum_{\beta_1}\left(\frac{1}{\beta_0}\ln\frac{1+\beta_0}{1-\beta_0} + \frac{1}{\beta_1}\ln\frac{1+\beta_1}{1-\beta_1} - 4\right)\frac{\sigma_i(\beta_0, \beta_1)}{\pi} \tag{5}$$

$$= \frac{1}{137}\ln\frac{xE_0}{\hbar\omega_{\min}}\left(\frac{1}{\beta_0}\ln\frac{1+\beta_0}{1-\beta_0} + \frac{1}{\beta_1}\ln\frac{1+\beta'}{1-\beta'} - 4\right)\frac{\sigma_i(\beta_0)}{\pi}.$$

In this formula $\hbar\omega_{\min}$ is the minimum measured energy of the γ quanta; in our experiments $\hbar\omega_{\min} \approx 10$ Mev, as can be seen from the table; β' is some average π-meson velocity after scattering (in the calculations we assume that β' corresponds to the kinetic energy $E_0/2$); x is a certain parameter describing the cutting off of the spectrum of the γ-quanta. If we take $x = 1$, i.e., the spectrum is not cut off, the quasi-classical formulas gives too high a value for the total cross section, compared with the exact formula. A correct choice of x can make the quasi-classical formulas as accurate as $\int d\sigma_\gamma(\omega, \beta)\, d\omega$. At our π-meson energies it is best to take $x \approx 1/2$.

Similarly, we obtain the total cross section for bremsstrahlung accompanying elastic scattering:

$$\sigma_d^\gamma = \frac{1}{137}\ln\frac{xE_0}{\hbar\omega_{\min}} I(\beta_0, \mu cR/\hbar)\,\frac{\sigma_d(\beta_0)}{\pi}. \tag{6}$$

where the integral $I(\beta_0, \mu cR/\hbar)$ must in general be evaluated numerically, but in our case we can use the approximate value of this integral for $(1-\beta^2)^{-1} \gg 1$ and $2\mu cR/\hbar \gg 1$, which turns out to be equal to $2.3/(\mu cR/\hbar)$.

Finally, the total bremsstrahlung cross section in the case when the charge is "stopped" is given by:

$$\sigma_r^\gamma = \frac{1}{137}\ln\frac{xE_0}{\hbar\omega_{\min}}\left(\frac{1}{\beta_0}\ln\frac{1+\beta_0}{1-\beta_0} - 2\right)\frac{\sigma_r(\beta_0)}{\pi}, \tag{7}$$

where $\sigma_r = \sigma_a + \sigma_e$, the sum of the cross sections for the absorption and for charge-exchange scattering of π mesons.

To compare our results with theory we take into account the fact that the average π^+-meson energy in the observed 20 events of bremsstrahlung accompanying scattering is 234 Mev. The cross sections σ_i, σ_d and σ_r required for our calculations have been measured by Dzhelepov et al.[8] for 230-Mev π^- mesons.

Calculations with formulas (5) and (6) yield, at 230 Mev, a cross section for bremsstrahlung accompanying scattering $\sigma_s^\gamma = \sigma_i^\gamma + \sigma_d^\gamma = 3.5 \times 10^{-27}$ cm^2 per fluorine nucleus, which agrees with our value $\sigma_s^\gamma = (4.7^{+1.2}_{-2.0}) \times 10^{-27}$ cm^2. The cross section for bremsstrahlung accompanying the stopping of the charge, calculated according to formula (7), is $\sigma_r^\gamma = 2.5 \times 10^{-27}$ cm^2 which is also in agreement with our results in the five observed events of this type of bremsstrahlung.

In conclusion, let us compare the theoretical and the experimentally-observed angular distributions of the bremsstrahlung γ quanta. We have measured the angles of emission of the γ quanta in all the 20 events listed in the table. Only in 2 events is the bremsstrahlung directed backwards, while in 18 events it is directed forwards. Such a distribution is in agreement with the one expected theoretically, since formulas (1) to (3) give a value

≈ 10 for the forward-backward ratio.

In conclusion, we express our deep gratitude to I. Ia. Pomeranchuk for his interest in our work.

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188