$$d\sigma = \frac{e^2}{(2\pi)^2}\frac{p^2}{v^2}|a_4(\mathbf{q})|^2 d\Omega\left\{1+\frac{v^2}{3}\left(1+\frac{8}{3}\frac{p^2}{M^2}\right)\sin^2\frac{\theta}{2} + \frac{8}{9}\frac{v^2p^4}{M^4}\sin^4\frac{\theta}{2}\cos^2\frac{\theta}{2}\right\},$$

where $\theta$ is the scattering angle and $q = 2p\sin(\theta/2)$.

For a pure Coulomb field $a_4(\mathbf{q}) = -i4\pi Ze/q^2$ and the factor preceding the curly brackets yields $(Ze^2/2pv)^2 d\Omega = d\sigma_{\text{Ruth.}}$, the Rutherford cross section.

Thus the expression in the curly brackets specifically gives the spin correction. We recall for the sake of comparison the cross section for particles of spin $1/2$ (reference 1)

$$d\sigma = d\sigma_{\text{Ruth}}[1 - v^2\sin^2\theta/2]$$

and for spin 1:[2]

$$d\sigma = d\sigma_{\text{Ruth}}[1 + (v^2p^2/6M^2)\sin^2\theta].$$

It can be seen that for spin 1 and $3/2$ the correction grows with energy, this is especially marked in the case of spin $3/2$ as it includes a factor $p^4$. At sufficiently high energies this may yield differences from the usual scattering picture even at relatively small angles. It should however be noted that if $q > 1/R$ where R is the nuclear radius, it is necessary to include the effect of the spread out nuclear charge.

[1] N. F. Mott, Proc. Roy. Soc. **124**, 425 (1929).
[2] H. S. W. Massey and H. C. Corben, Proc. Cambr. Phil. Soc., **35**, 463 (1939).

Translated by M. A. Melkanoff
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## *USE OF CYCLOTRON RESONANCE IN SEMICONDUCTORS FOR THE AMPLIFICATION AND GENERATION OF MICROWAVES*

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As is well known,[1-3] when cyclotron resonance occurs in certain semiconductors (Ge or Si), additional lines are observed along with the main absorption lines at frequencies that are integral multiples of the fundamental cyclotron frequency. This effect is associated with the anharmonic nature of the motion of holes, which leads to the breakdown of the usual selection rules $\Delta n = \pm 1$ for the Landau levels and to the appearance of nonvanishing dipole moments for transitions with $\Delta n = \pm 2$ or 3.

This phenomenon can be utilized for the construction of regenerative amplifiers or generators at microwave frequencies, for example, by using the following method. The semiconductor sample, placed into a constant magnetic field $H_0$, is acted upon by the high-frequency electric "pumping" field $E_p$ of frequency $\omega_p = n\omega_c = neH_0/m^*c$ ($n = 1, 2, 3, \ldots$, $m^*$ is the effective mass of the hole ($m_h^*$) or of the electron ($m_e^*$)) polarized in the plane perpendicular to $H_0$.

If the intensity of the field $E_p$ is sufficiently great, so that during a thermal relaxation time $\tau$ a sufficiently large number of carriers goes over into the upper energy levels, then in this system excitation (or amplification) can occur at frequencies given by $\omega_s = l\omega_p/n = l\omega_c$, $l = 1, 2, \ldots$, which can be both less than ($l < n$) and greater than ($l > n$) the "pumping" frequency. The excitation of such oscillations is facilitated by placing the semiconductor into a resonator for which $\omega_p$ and $\omega_s$ are eigenfrequencies.

According to Basov and Prokhorov,[4] in order to excite a maximum number of carriers at the frequency $\omega_p$ it is necessary to expend per unit volume of the semiconductor an amount of power given (in order of magnitude) by: $P_p \approx (10^{-7} \times 3\hbar^2\omega_p/4\pi\tau^2|d_n|^2 Q_p)$ w/cm$^3$, where $h = 2\pi\hbar$ is Planck's constant, $|d_n|$ and $Q_p$ are the dipole moment for the transition and the figure of merit of the resonator at the frequency $\omega_p = n\omega_c$.

In the special case $\omega_s = \omega_c = \omega_p/2$ (corresponding to the resonance of the second kind in a nonlinear oscillating system) at a temperature $T \approx 2$ to 4°K we have $\tau \approx 6 \times 10^{-11}$ sec, $|d_2| \approx 10^{-15}$ cgs Esu,[2,3] from which we obtain $P_p \approx (10^{-11}\omega_p/Q_p)$ w/cm$^3$.

For the excitation (or amplification) of oscillations of frequency $\omega_s = \omega_c = \omega_p/2$ it is necessary that the density of active carriers attain the value $N_{act} \approx 3\hbar/4\pi Q_1\tau|d_1|^2$. In the case under discussion $|d_1| \approx 10^{-14}$ cgs Esu (reference 3) and $N_{act} \approx (3 \times 10^{10}/Q_1)$ cm$^{-3}$. The maximum radiated power is

$$P_s = N_{act}\hbar\omega_s/2\tau \ll P_p,$$

which in the case $N_{act} \approx 10^{10}$ cm$^{-3}$ and $\omega_s = 2\pi \times 10^{11}$ cps will amount to approximately 5 mw/cm$^3$, i.e., to a significantly greater value than in the usual molecular or paramagnetic generators. This effect is due to the fact that the electric dipole moment for the transition in the case of cyclotron resonance exceeds approximately by a factor of $10^6$ the magnetic moment of the electron, and by a factor of $10^4$ the electric dipole moment of the molecule.

An important advantage of cyclotron resonance in comparison with paramagnetic resonance is the possibility of generating microwaves of shorter wavelengths, in the millimeter and the submillimeter ranges, since the low values for the effective mass of the carriers compared to the electron rest mass $m_0$ allow us to use lower magnetic fields. Thus, for example, in Ge $m_h^* \approx 0.04\, m_0$, and at a frequency of $1 \times 10^{12}$ cps, the resonance value of the magnetic field is approximately $1.4 \times 10^4$ Oe instead of the $3.5 \times 10^5$ Oe in paramagnetic resonance.

If the anharmonicity of cyclotron oscillations of the carriers is not strongly pronounced or is completely absent, it is possible to achieve amplification or frequency conversion of the oscillations by the following method. A linearly polarized electromagnetic wave of frequency $\omega_p = 2\omega_c/n$, in which the vector $\mathbf{H}_p$ is oriented parallel to the constant field while its amplitude is somewhat less than $H_0$, is incident on a semiconductor sample situated in the constant magnetic field $H_0$. Such a system is potentially unstable with respect to the high-frequency signal of cyclotron frequency polarized in the plane perpendicular to the constant magnetic field. Parametric amplification or generation of high frequency oscillations is possible at this frequency. The power of the "pumping" signal depends on the value of $m^*$. In the case $\omega_c = 2\pi \times 3 \times 10^9$ cycles/sec and $m^* \approx 0.01\, m_0$ (for example, in the case of InSb) the resonance value of the magnetic field is given by $H_0 \approx 10$ Oe and $P_p \approx 10^4/Q_p$ watts.


[1] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

[2] Dexter, Zeiger, and Lax, Phys. Rev. **104**, 637 (1956).

[3] Zeiger, Lax, and Dexter, Phys. Rev. **105**, 495 (1957).

[4] N. G. Basov and A. M. Prokhorov, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 431 (1954).

[5] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 556 (1955).


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## *SEVERAL POSSIBILITIES ASSOCIATED WITH THE SEPARATION OF CHARGED PARTICLES IN AN INHOMOGENEOUS HIGH-FREQUENCY ELECTROMAGNETIC FIELD*


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THE nonrelativistic motion of charged particles in an inhomogeneous electromagnetic field $\mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r})e^{i\omega t}$ has been investigated,[1-5] and it was shown that by means of high-frequency potential wells

$$\Phi(\mathbf{r}) = (\eta/2\omega)^2 |\mathbf{E}(\mathbf{r})|^2$$

($\eta$ is the ratio of charge to mass) it is possible to control within wide limits the average (over the period $2\pi/\omega$) motion of the particles: reflection from potential barriers, focusing of beams, acceleration of plasma blobs, confinement in a bounded region of space, and so forth. The field, however, had been considered as given, i.e., the reaction of the motion of the particles upon the magnitude and shape of the potential $\Phi(\mathbf{r})$ had been ignored, which is permissible only for low charge densities and if the particles do not leave the interaction region. Taking into account the finite charge density leads to the introduction of the effective dielectric constant of the region occupied by charges, which can markedly change the structure[6] of the potential $\Phi(\mathbf{r})$. Sometimes the second factor is also important; associated with it are a series of interesting new possibilities which are illustrated below on a simple example of sufficient generality.

Let a rectilinear beam of particles move with velocity $z = v_0$ into the side of a potential $\Phi(z)$, monotonically increasing from zero and produced by an inhomogeneous high-frequency field. If the period $2\pi/\omega$ of this field is much less than the particle transit time over a distance $L \sim E/|\nabla E|$, along which the amplitude of $E$ changes substantially, then the kinetic energy of a particle, being composed of the energy of the average (over the period $2\pi/\omega$) motion and of the average energy of the oscillatory motion (of frequency $\omega$), remain constant.[1,2] Consequently the particles advance only up to the plane $z = z_0$, such that $\Phi(z_0) = v_0^2/2$, whereupon in the turning point of the beam