EQUATIONS OF MOTION FOR A SYSTEM CONSISTING OF TWO TYPES OF INTERACTING SPINS

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SOLOMON[1] has found the equations of motion describing the magnetization of a system consisting of two types of interacting magnetic moments in parallel fields. Kurbatov and the author[2] have investigated the thermodynamic properties of a two-spin system, including the spin-spin and spin-lattice relaxations. The present note gives a simple thermodynamic derivation of the equations describing the behavior of such a system in a constant field $\mathbf{H}_0$ arbitrarily oriented with respect to an alternating field $\mathbf{h}$.

We shall start with the equations

$$\dot{M}_k^{(1)} = L_{ik}^{11}(H_i - H_i^{(1)}) + L_{ik}^{12}(H_i - H_i^{(2)}),$$
$$\dot{M}_k^{(2)} = L_{ik}^{21}(H_i - H_i^{(1)}) + L_{ik}^{22}(H_i - H_i^{(2)}), \quad (1)$$

where $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ are related to the magnetizations $\mathbf{M}^{(1)}$ and $\mathbf{M}^{(2)}$ of the spin subsystems by

$$\mathbf{M}^{(1)} = \chi_{01}\mathbf{H}^{(1)}, \quad \mathbf{M}^{(2)} = \chi_{02}\mathbf{H}^{(2)}. \quad (2)$$

The L_{ik} satisfy the Onsager relations. Assuming that in the absence of a field the medium is isotropic, we write

$$L_{ik}^{11} = \frac{\chi_{01}}{\tau_1}\delta_{ik} + \gamma_1\chi_{01}\varepsilon_{ikl}H_0; \quad L_{ik}^{12} = \frac{\chi_{02}}{\tau}\delta_{ik},$$
$$L_{ik}^{21} = \frac{\chi_{01}}{\tau}\delta_{ik}; \quad L_{ik}^{22} = \frac{\chi_{02}}{\tau_2}\delta_{ik} + \gamma_2\chi_{02}\varepsilon_{ikl}H_0, \quad (3)$$

where γ_1 and γ_2 are the gyromagnetic ratios for the spin subsystems, ϵ_{ikl} is the unit antisymmetric tensor, and $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}(t)$. Equations (1) now become*

$$\dot{\mathbf{M}}_1 + \mathbf{M}_1/\tau_1 + \mathbf{M}_2/\tau = (\chi_{01}/\tau_1 + \chi_{02}/\tau)\mathbf{H} + \gamma_1[\mathbf{M}_1\times\mathbf{H}],$$
$$\dot{\mathbf{M}}_2 + \mathbf{M}_2/\tau_2 + \mathbf{M}_1/\tau = (\chi_{01}/\tau + \chi_{02}/\tau_2)\mathbf{H} + \gamma_2[\mathbf{M}_2\times\mathbf{H}]. \quad (4)$$

In the absence of a transverse rf field in the steady state, as may have been expected, these equations lead to the relations given by (2).

For parallel fields, i.e., if $[\mathbf{H}_0 \times \mathbf{h}(t)] = 0$, Eqs. (4) are the same as those obtained by Solomon. If the second subsystem is missing, they become

$$\dot{\mathbf{M}} + \mathbf{M}/\tau = (\chi_0/\tau)\mathbf{H} + \gamma[\mathbf{M}\times\mathbf{H}].$$

Let us now require that $\mathbf{M}^{(1)}$ and $\mathbf{M}^{(2)}$ are of equal magnitudes; then multiplying Eqs. (4) by $\mathbf{M}^{(1)}$ and $\mathbf{M}^{(2)}$, respectively, we obtain

$$\frac{\mathbf{M}_1^2}{\tau_1} + \frac{(\mathbf{M}_1\cdot\mathbf{M}_2)}{\tau} = \left(\frac{\chi_{01}}{\tau_1} + \frac{\chi_{02}}{\tau}\right)(\mathbf{M}_1\cdot\mathbf{H}),$$
$$\frac{\mathbf{M}_2^2}{\tau_2} + \frac{(\mathbf{M}_1\cdot\mathbf{M}_2)}{\tau} = \left(\frac{\chi_{01}}{\tau} + \frac{\chi_{02}}{\tau_2}\right)(\mathbf{M}_2\cdot\mathbf{H}). \quad (5)$$

Eliminating χ_{01} and χ_{02} from (4) and (5), we obtain

$$\dot{\mathbf{M}}_1 = \gamma_1[\mathbf{M}_1\times\mathbf{H}] - \frac{\lambda_{11}}{M_1^2}[\mathbf{M}_1\times[\mathbf{M}_1\times\mathbf{H}]] - \frac{\lambda_{12}}{(\mathbf{M}_1\cdot\mathbf{M}_2)}[\mathbf{M}_1\times[\mathbf{M}_2\times\mathbf{H}]],$$
$$\dot{\mathbf{M}}_2 = \gamma_2[\mathbf{M}_2\times\mathbf{H}] - \frac{\lambda_{22}}{M_2^2}[\mathbf{M}_2\times[\mathbf{M}_2\times\mathbf{H}]] - \frac{\lambda_{21}}{(\mathbf{M}_1\cdot\mathbf{M}_2)}[\mathbf{M}_2\times[\mathbf{M}_1\times\mathbf{H}]], \quad (6)$$

where

$$\lambda_{11} = M_1^2/\tau(\mathbf{M}_1\cdot\mathbf{H}); \quad \lambda_{12} = (\mathbf{M}_1\cdot\mathbf{M}_2)/\tau(\mathbf{M}_1\cdot\mathbf{H});$$
$$\lambda_{21} = (\mathbf{M}_1\cdot\mathbf{M}_2)/\tau(\mathbf{M}_2\cdot\mathbf{H}); \quad \lambda_{22} = M_2^2/\tau(\mathbf{M}_2\cdot\mathbf{H}). \quad (7)$$

If $\lambda_{12} = \lambda_{21} = 0$ (that is in the limit as $\tau \to \infty$), Eqs. (6) go over into the Landau-Lifshitz equations for two noninteracting spin systems. They can be used to describe relaxation processes and resonance phenomena in antiferromagnets.

***Henceforth we shall write the indices denoting the subsystems as subscripts.**

[1] I. Solomon, Phys. Rev. **99**, 559 (1955).

[2] G. V. Skrotskii and L. V. Kurbatov, Izv. Akad. Nauk SSSR, Ser. Fiz. **21**, 833 (1957) [Columbia Techn. Transl. 21, 833 (1957)].

[3] G. V. Skrotskii and V. T. Shmatov, Изв. высших учебн. завед., физика (Bulletin of the Higher Inst. of Study, Physics) **2**, 138 (1958).

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CLEBSCH-GORDAN EXPANSION FOR INFINITE-DIMENSIONAL REPRESENTATIONS OF THE LORENTZ GROUP

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ONE of the authors has given[1] the explicit form of the Clebsch-Gordan coefficients for the expansion of the finite-dimensional representations of