

for the sake of convenience, put $L = 1$ or put L^3 equal to the volume of the elementary crystal cell. The meaning of \mathbf{P}_l is then that it is the matrix element of the dipole moment per unit volume, or, respectively, of the elementary cell. Below we shall consider only excited states for which $\mathbf{P} \neq 0$.

The coefficients of the linear combination of the degenerate functions c_l are determined from the condition that $\mathcal{C}(\mathbf{k})$ or the quantity $|(\mathbf{P} \cdot \mathbf{s})|$ is extremum under the supplementary condition $\sum |c_l|^2 = 1$.

If the excited state of the cube (cell) is non-degenerate, the single coefficient c is equal to 1. This occurs for all excited states of a rhombic crystal and also, for instance, for those states of a tetragonal crystal for which \mathbf{P}_l is directed along the tetragonal axis. Then α is the angle between the uniquely determined direction in the crystal $\mathbf{P} = p_l$ and the direction \mathbf{s} . Depending on the direction of \mathbf{s} , $\cos^2 \alpha$ takes on all values from 0 to 1.

Apart from the just mentioned states there are in a tetragonal crystal still two-fold degenerate excitations of the cube for which \mathbf{P} lies in the plane perpendicular to the tetragonal axis, z . There are thus two sets of extremal values c_1 and c_2 and correspondingly two exciton states for a given direction of \mathbf{s} :

$$(1) \mathbf{P} \perp \mathbf{s}, u = 0; (2) \mathbf{P} \text{ coplanar with } \mathbf{s} \text{ and } z, \\ u = 4\pi L^{-3} |\mathbf{P}|^2 \sin^2(\mathbf{s}, z). \quad (7)$$

In both cases $|\mathbf{P}|$ is unique and is independent of the direction of \mathbf{s} .

In cubic crystals there are two kinds of exciton for any given direction of \mathbf{s} :

$$(1) \mathbf{P} \perp \mathbf{s} \text{ and otherwise arbitrary in direction;} \\ u = 0, \\ (2) \mathbf{P} \parallel \mathbf{s} \text{ and } u = 4\pi L^{-3} |\mathbf{P}|^2. \quad (8)$$

In both cases $|\mathbf{P}|$ is unique and is independent of the direction of \mathbf{s} .

The dependence of $u(\mathbf{k})$ on the direction of \mathbf{k} as $|\mathbf{k}| \rightarrow 0$, just proven, means that the function $\mathcal{C}(\mathbf{k})$ has a discontinuity at $\mathbf{k} = 0$. One must thus introduce the corresponding corrections in many earlier papers in which $\mathcal{C}(\mathbf{k})$ was expanded in a power series in k_x, k_y, k_z , in particular in reference 2. However, for each given direction $\mathcal{C}(\mathbf{k})$ can be considered to be an analytical function of the absolute magnitude k and can be expanded in a power series in it. In this way one obtains all basic results of reference 2, including Eqs. (39) to (44). One must only consider the parameter $\mathcal{C}(0)$ to be a constant for a given direction of \mathbf{s} , and generally speaking, a quantity depending on the angle between the direction of \mathbf{s} and $\mathbf{P}_{0\mathbf{k}}(0)$.

The discontinuous dependence of the exciton energy on k of the above discussed type was earlier obtained for particular models for the exciton: for a Frenkel exciton in molecular crystals (naphthalene and anthracene)³ and for a plasmon in an anisotropic medium.⁴

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ON THE STABILITY OF A FERMI LIQUID

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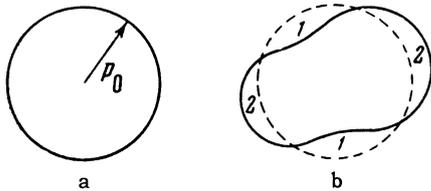
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IN the theory of the Fermi liquid proposed by Landau^{1,2} the energy of the system is a functional of the distribution of the excitations among the quantum states (in particular, among values of the momentum). When the distribution function of the excitations has only small deviations δn from the equilibrium distribution corresponding to the filling up of all states with $p < p_0$ (all the notations are the same as in reference 1), the energy functional E has the form

$$E = \sum_{\sigma} \int \varepsilon(p) \delta n(p) \frac{d^3 p}{(2\pi\hbar)^3} \\ + \frac{1}{2} \sum_{\sigma} \sum_{\sigma'} \int \int f(\mathbf{p}\mathbf{p}') \delta n(p) \delta n(\mathbf{p}') \frac{d^3 p d^3 p'}{(2\pi\hbar)^6} \quad (1)$$

(σ is the excitation spin).

Stability will exist for small δn if $E > 0$ for arbitrary δn . The values of p actually involved in the integrals of Eq. (1) are those close to p_0 (the temperature is zero). Therefore $\varepsilon(p) = (d\varepsilon/dp)_{p_0}(p - p_0) = v_0(p - p_0)$. The variations δn appearing in Eq. (1) are due to deformations of the Fermi surface, and we shall find the criterion for stability with respect to such deformations (see diagram).



(a) unperturbed distribution (all states inside occupied).
 (b) perturbed distribution; in regions 1, $\delta n = -1$, in regions 2, $\delta n = 1$; inside the solid line all states are occupied.

Let us expand the momentum p corresponding to the solid line of diagram b in a series of spherical surface harmonics:

$$p = p_0 + \sum_{lm} \Phi_{lm} Y_{lm}(\theta, \varphi), \quad Y_{l0}(0) = 1. \quad (2)$$

The first term in Eq. (1) takes the following form:

$$\frac{2v_0 p_0^2}{(2\pi\hbar)^3} \int_0^{p-p_0} (p' - p_0) dp' = \frac{p_0^2 v_0}{(2\pi\hbar)^3} \sum_{lm} \Phi_{lm}^2 \frac{4\pi}{2l+1} \frac{(l+m)!}{(l-m)!} \quad (3)$$

The potential energy reduces to the expression

$$2p_0^4 / (2\pi\hbar)^6 \iint [p(\theta_1, \varphi_1) - p_0][p(\theta_2, \varphi_2) - p_0] d\omega_1 d\omega_2 f(\theta_{12}), \quad (4)$$

$$\cos \theta_{12} = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2).$$

We now expand $f(\theta_{12})$ in a series of surface harmonics:

$$f(\theta_{12}) = \sum_l f_l P_l(\cos \theta_{12})$$

$$= \sum_l \sum_{m'} Y_{lm'}(\theta_1, \varphi_1) Y_{lm'}(\theta_2, \varphi_2) \frac{(l-m')!}{(l+m')!} f_l, \quad (5)$$

$$f_l = \frac{2l+1}{4\pi} \int f(\theta) P_l(\cos \theta) d\omega.$$

Substituting Eq. (5) into (4) and combining with Eq. (3), we get the total energy functional

$$E = \frac{p_0^2 v_0}{(2\pi\hbar)^3} \sum_{lm} \Phi_{lm}^2 \frac{4\pi}{2l+1} \frac{(l+m)!}{(l-m)!} + \frac{2p_0^4}{(2\pi\hbar)^6} \sum_{lm} \Phi_{lm}^2 \left(\frac{4\pi}{2l+1} \right)^2 f_l \frac{(l+m)!}{(l-m)!}. \quad (6)$$

The conditions for stability are written separately for each l, m :

$$1 + \frac{8\pi}{2l+1} \frac{p_0^2 f_l}{v_0 (2\pi\hbar)^3} > 0, \quad 1 + \frac{2p_0^2}{v_0 (2\pi\hbar)^3} \int f_l P_l(\cos \theta) d\omega > 0. \quad (7)$$

For $l=0$ and $l=1$ these conditions agree with the conditions (11) and (18) of reference 1, where they characterized the positiveness of the square of the speed of sound and of the effective mass.

So far we have supposed that $f(p, p')$ does not depend on the mutual orientation of the spins of the excitations. If we take into account this dependence,

then instead of f we have the function

$$\hat{f}(pp') + (\sigma_1 \sigma_2) g(p_1 p_2). \quad (8)$$

The perturbation of the Fermi surface is now expressed by the formula

$$p = p_0 + \sum_{lm} \sigma \Phi_{lm} Y_{lm}(\theta, \varphi). \quad (9)$$

instead of Eq. (2). Since E involves the trace with respect to σ and σ' , we get instead of Eq. (6):

$$E = \frac{p_0^2 v_0}{2(2\pi\hbar)^3} \sum_{lm} \Phi_{lm}^2 \frac{4\pi}{2l+1} \frac{(l+m)!}{(l-m)!} + \frac{p_0^4}{2(2\pi\hbar)^6} \sum_{lm} \Phi_{lm}^2 \left(\frac{4\pi}{2l+1} \right)^2 \frac{(l+m)!}{(l-m)!} g_l, \quad (10)$$

where g_l are the expansion coefficients of the function $g(p_1 p_2) = g(\cos \theta_{12})$ (all taken from the Fermi surface) in a series of Legendre polynomials:

$$g(\cos \theta_{12}) = \sum_l g_l P_l(\cos \theta_{12}). \quad (11)$$

The stability conditions following from Eq. (10) have the form

$$1 + \frac{p_0^2}{v_0 (2\pi\hbar)^3} \int g(\cos \theta) P_l(\cos \theta) d\omega > 0. \quad (12)$$

For $l=0$ this condition means the absence of ferromagnetism and is contained in the condition $\chi > 0$ (Eq. (26) of reference 1).

In conclusion I express my gratitude to L. D. Landau for a discussion of this note.

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ON THE MECHANISM OF THE DAMPING OF FREE OSCILLATIONS IN A CYCLIC ACCELERATOR

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REFERENCES 1 and 2 contained a statement about the absence of damping of betatron oscillations, which is connected with the emission of