

sibly a new system of relations, which is valid also in the general case. In this connection it is interesting to note a paper by Moses,<sup>14</sup> in which a formal procedure is developed for the construction of a potential from the known backward scattering amplitude for all energies and all directions of the incident particles over a hemisphere. Moses points out the analogy between these data and the reflection coefficient in the case of a uni-dimensional barrier. It is of interest to note that these data consist of two real functions of the parameter  $E$  and a unit vector describing a hemisphere, while the potential depends on the parameter  $r$  and on a unit vector describing a full sphere.

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## ROTATIONAL STATES OF NONAXIAL NUCLEI

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A theory is developed for the energy states of nuclei which have no axial symmetry, and for the electromagnetic transitions between these states. It is shown that the breakdown of axial symmetry, though it does not appreciably change the rotational states from those for axial nuclei, leads to the appearance of new energy states. Comparison of theory with experiment shows that the so-called  $\gamma$ -vibrational levels of even-even nuclei should be regarded as rotational levels. The levels of some nuclei with the spin sequence 0, 2, 2, 3 should be assigned to this same type.

## INTRODUCTION

ON the basis of the uniform nuclear model of A. Bohr and Mottelson,<sup>5,6</sup> the present authors have investigated<sup>1-4</sup> the energy levels of nonspherical nuclei corresponding to collective excitations in which there is no breakdown of the axial symmetry of the nucleus. It was shown that the rotation-vibration energy of the collective excited states of the nucleus is a function of only two parameters — the frequency of the surface oscillations of the

nucleus and the ratio of the equilibrium deformation to the amplitude of the zero-point vibrations. The question naturally arises as to the extent to which these results remain valid when one takes account of possible breakdown of axial symmetry of the nucleus.

The question of the breakdown of axial symmetry of the nucleus has already been discussed qualitatively in some papers.<sup>5,7,8</sup> Recently it has even become customary (cf. for example references 9 and 10) to assign certain excited nuclear

states to the so-called  $\gamma$ -vibrations. Such an assignment is usually based on the values of the spins of the levels and the occurrence of a high probability for electromagnetic transitions, which confirms the collective nature of the levels. No quantitative theory of  $\gamma$  vibrations exists.

In the present paper we investigate the energy levels corresponding to rotation of the nucleus without change of its internal state. It will be shown that, with the breakdown of axial symmetry of even-even nuclei, the rotational spectrum corresponding to the axial nucleus is changed comparatively little but that new rotational states with total angular momentum  $J = 2, 3, 4 \dots$  appear. For small deviations from axial symmetry these levels lie very high and are not excited, but with increasing deviation from axial symmetry some of the additional levels are lowered appreciably. So, for example, the ratio of the second excited level with spin 2 to the first level, which is also present in an axial nucleus, changes from infinity to two. The probabilities of electromagnetic transitions between rotational states of nonaxial nuclei are calculated in Sec. 2. From a comparison of the theory with experimental data (Sec. 3) one can conclude that the properties of the experimentally observed energy states of even-even nuclei are satisfactorily explained if one assumes that these nuclei do not have axial symmetry.

### 1. ROTATIONAL LEVELS OF NONSPHERICAL EVEN-EVEN NUCLEI

On the basis of the uniform model, let us consider the nuclear levels corresponding to rotation of the nucleus as a whole without change of its internal state. The operator for the rotational energy of the nucleus has the form

$$H = \frac{A}{2} \sum_{\lambda=1}^3 \frac{\hat{J}_{\lambda}^2}{\sin^2(\gamma - 2\pi\lambda/3)}, \quad (1.1)$$

where  $A = \hbar^2/4B\beta^2$  is a quantity having the dimensions of an energy;  $\gamma$  varies between 0 and  $\pi/3$  and determines the deviation of the shape of the nucleus from axial symmetry; the  $\hat{J}_{\lambda}$  are the operators for the projections of the angular momentum of the nucleus on the axes of a coordinate system fixed in the nucleus. The commutation rules for these operators differ from those for the corresponding operators relative to a fixed coordinate system by a change in sign of their right-hand side.

According to (1.1), for  $\gamma \neq 0$  or  $\pi/3$  the nucleus should be treated as an asymmetric top. In the stationary states of the asymmetric top, none of the projections of the total angular momentum

on the axes 1, 2, 3 of the coordinate system fixed in the nucleus can have definite values, so that the energy levels cannot be classified by means of the value of  $K = J_3$ . In the asymmetric top there are  $2J + 1$  different energy levels for each value of the total angular momentum. These levels can be classified in terms of the irreducible representations of the group  $D_2$  (with symmetry elements  $C_2^1, C_2^2, C_2^3$ , corresponding to rotation through  $\pi$  around each of the coordinate axes 1, 2, 3), since the operator (1.1) and the commutation relations of the  $\hat{J}_{\lambda}$  are invariant under this group of transformations. Thus the energy levels of the asymmetric top split into four types, corresponding to the four irreducible representations of the group  $D_2$  (cf. reference 11, § 101, and reference 12).

In the case of even-even nuclei, of the  $2J + 1$  different energy levels with given  $J$ , only those can occur which correspond to the completely symmetric representation of  $D_2$ . For  $J = 1$ , there are no rotational levels of the required symmetry. For  $J = 2$  there are two such states, there is one for  $J = 3$ , three for  $J = 4$ , two for  $J = 5$ , four for  $J = 6$ , etc.

If we express the energy in units of  $A$ , the energies of the two states that have the required symmetry for  $J = 2$  are given by

$$\begin{aligned} \varepsilon_1(2) &= \frac{9(1 - \sqrt{1 - 8/9 \sin^2 3\gamma})}{\sin^2 3\gamma}, \\ \varepsilon_2(2) &= \frac{9(1 + \sqrt{1 - 8/9 \sin^2 3\gamma})}{\sin^2 3\gamma} \end{aligned} \quad (1.2)$$

The energy levels with angular momentum  $J = 3$  are given by the formula

$$\varepsilon(3) = \sum_{\lambda=1}^3 2/\sin^2\left(\gamma - \frac{2\pi}{3}\lambda\right) = 18/\sin^2 3\gamma. \quad (1.3)$$

The three energy levels with spin 4 are determined by the roots of the cubic equation

$$\begin{aligned} \varepsilon^3 - \frac{90}{\sin^2 3\gamma} \varepsilon^2 + \frac{48}{\sin^4 3\gamma} [27 + 26 \sin^2 3\gamma] \varepsilon \\ - \frac{640}{\sin^4 3\gamma} [27 + 7 \sin^2 3\gamma] = 0. \end{aligned}$$

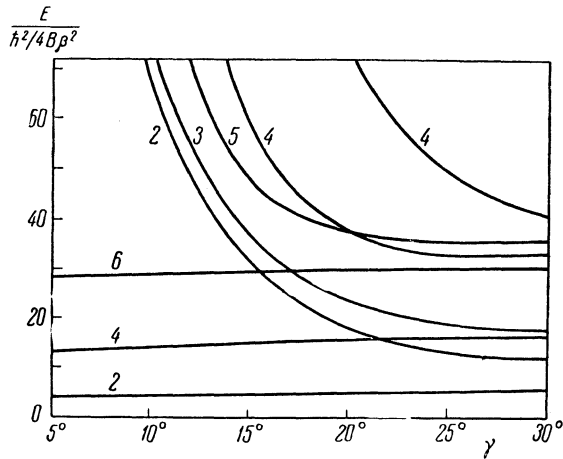
The two energy levels with spin 5 are given by

$$\varepsilon_{\tau}(5) = [45 \pm 9 \sqrt{9 - 8 \sin^2 3\gamma}] / \sin^2 3\gamma. \quad (1.4)$$

In (1.4),  $\tau = 1$  for the minus sign on the square root and  $\tau = 2$  for the plus sign. The energy of the rotational states with angular momentum equal to six is determined from the solution of a quartic equation which we shall not give here.

From (1.2) and (1.3) we get the simple relation

$$\varepsilon_1(2) + \varepsilon_2(2) = \varepsilon(3), \quad (1.5)$$



which relates the energies of the levels with spins 2 and 3.

The energy levels of even-even nuclei, calculated on the basis of these formulas, are shown in the figure as functions of the parameter  $\gamma$ . For  $\gamma = 0$ , the energy spectrum coincides with that for an axially symmetric nucleus. For fixed  $\beta$ , the breakdown of the axial symmetry leads to an increase in energy of the levels which occur in the axial nucleus. This increase in energy corresponds to a reduction of the effective nuclear momentum of inertia or the effective deformation parameter  $\beta_{\text{eff}}$ . For the first excited state with spin 2, the effective deformation parameter can be defined by the formula

$$\beta_{\text{eff}} = \beta \left( \frac{4 \sin^2 3\gamma}{9 - \sqrt{81 - 72 \sin^2 3\gamma}} \right)^{1/2}$$

In addition to the comparatively small change in energy of those levels which occur in axially symmetric nuclei, the breakdown of axial symmetry results in the appearance of new energy levels  $\epsilon_2(2)$ ,  $\epsilon(3)$ ,  $\epsilon_2(4)$ , ... . By using the dependence of  $\epsilon_2(2)/\epsilon_1(2)$  on  $\gamma$ , one can from the experimental value of this ratio determine the appropriate value of  $\gamma$ .

Then we can use the figure to determine the sequence of spins and energies of the other collective levels of the nucleus. Comparison of the theory with experimental data will be made in Sec. 3.

## 2. ELECTROMAGNETIC TRANSITIONS BETWEEN ROTATIONAL STATES OF NUCLEI

It is well known that measurements of probabilities of transitions between nuclear states enables one to obtain valuable information concerning the nature of the excited states. In particular, to clarify the nature of the second excited  $2^+$  state in even-even nuclei, one can study the relative

probability of transitions from this level directly to the ground state ( $0^+$ ) and to the first excited  $2^+$  state. It has been assumed in various papers that the first two experimentally observed levels with spin 2 correspond to one-phonon and two-phonon oscillation of the nuclear surface. Then the transition from the second spin 2 level to the ground state can occur only because of breakdown of the oscillator approximation. However no one has as yet succeeded in giving a quantitative estimate of this effect.

Starting from the assumption that both of the spin-2 levels are to be assigned as rotational levels, we shall calculate the ratio of the reduced probabilities for the transitions  $\epsilon_2(2) \rightarrow \epsilon_1(2)$  and  $\epsilon_2(2) \rightarrow \epsilon(0)$  as a function of the parameter  $\gamma$  and, consequently, as a function of the ratio  $\epsilon_2(2)/\epsilon_1(2)$ , since the latter depends on  $\gamma$ .

To calculate the probability of E2 transitions between rotational states we must express the nuclear quadrupole moment operator

$$\hat{Q}_{2\mu} = 2e \sqrt{4\pi/5} \sum_{i=1}^Z r_i^2 Y_{2\mu}(\vartheta_i \varphi_i)$$

in terms of the Euler angles which determine the orientation of the nucleus and the collective coordinates measured relative to axes fixed in the nucleus. Making use of the transformation

$$Y_{2\mu}(\vartheta \varphi) = \sum_{\nu} D_{\mu\nu}^2(\theta) Y_{2\nu}(\vartheta' \varphi')$$

of the spherical functions when we go to a coordinate system fixed in the nucleus, and expressing the proton coordinates  $r$ ,  $\vartheta_1$ ,  $\varphi_1$  in this system (assuming that they are uniformly distributed inside the nucleus) in terms of collective coordinates  $a_\nu$ , where

$$a_0 = \beta \cos \gamma, \quad a_1 = a_{-1} = 0, \quad a_2 = a_{-2} = \frac{\beta}{\sqrt{2}} \sin \gamma,$$

we can use the formula

$$a_\nu = \frac{4\pi}{Z} \sum_{i=1}^Z \left( \frac{r_i}{R} \right)^2 Y_{2\nu}(\vartheta'_i \varphi'_i),$$

to obtain the following expression for the  $\mu$ -th component of the electric quadrupole-moment operator:

$$\hat{Q}_{2\mu} = eQ_0 \left( D_{\mu 0}^2 \cos \gamma + \frac{1}{\sqrt{2}} (D_{\mu 2}^2 + D_{\mu, -2}^2) \sin \gamma \right), \quad (2.1)$$

where

$$Q_0 = 3ZR^2\beta / \sqrt{5\pi} \quad (2.2)$$

is the intrinsic quadrupole moment of an axial nucleus with deformation parameter  $\beta$ ;  $D_{\mu\nu}^2$  are

generalized spherical functions, dependent on the Euler angles, determine the unitary transformation from the coordinate system fixed in space to the coordinate system fixed in the nucleus.

The wave functions of the rotational states of the nonaxial nucleus which we want can be written in the form

$$\begin{aligned}\psi_{001} &= (1/8 \pi^2)^{1/2} \varphi(\beta\gamma), \\ \psi_{2m_1} &= (5/8 \pi^2)^{1/2} \varphi(\beta\gamma) \left[ a_1 D_{m_0}^2 + \frac{b_1}{\sqrt{2}} (D_{m_2}^2 + D_{m, -2}^2) \right], \\ \psi_{2M_2} &= (5/8 \pi^2)^{1/2} \varphi(\beta\gamma) \left[ a_2 D_{M_0}^2 + \frac{b_2}{\sqrt{2}} (D_{M_2}^2 + D_{M, -2}^2) \right],\end{aligned}\quad (2.3)$$

where  $\varphi(\beta\gamma)$  is a function describing the internal state of the nucleus, which is assumed to be the same in all three rotational states;

$$\begin{aligned}a_1 N_1 &= -[\sqrt{9 - 8 \sin^2 3\gamma} + \sin \gamma \sin 3\gamma + 3 \cos \gamma \cos 3\gamma], \\ b_1 N_1 &= 3 \sin \gamma \cos 3\gamma - \cos \gamma \sin 3\gamma, \\ N_1^2 &= 2 \sqrt{9 - 8 \sin^2 3\gamma} [\sqrt{9 - 8 \sin^2 3\gamma} \\ &\quad + \sin \gamma \sin 3\gamma + 3 \cos \gamma \cos 3\gamma], \\ a_2 N_2 &= \sqrt{9 - 8 \sin^2 3\gamma} - \sin \gamma \sin 3\gamma - 3 \cos \gamma \cos 3\gamma, \\ b_2 N_2 &= 3 \sin \gamma \cos 3\gamma - \cos \gamma \sin 3\gamma, \\ N_2^2 &= 2 \sqrt{9 - 8 \sin^2 3\gamma} [\sqrt{9 - 8 \sin^2 3\gamma} \\ &\quad - \sin \gamma \sin 3\gamma - 3 \cos \gamma \cos 3\gamma].\end{aligned}\quad (2.4)$$

As already mentioned,  $\gamma$  varies between the limits 0 and  $\pi/3$  and determines the deviation of the nucleus from axial symmetry. The axes of the ellipsoid by means of which the shape of the nucleus is approximated are expressed in terms of  $\gamma$  by means of the formulas

$$R_\lambda = R [1 + \beta \sqrt{5/4} \pi \cos(\gamma - 2\pi\lambda/3)], \quad \lambda = 1, 2, 3.$$

For  $\gamma = 0$ , the nucleus is a prolate ellipsoid of rotation with symmetry axis along 3. For  $\gamma = \pi/3$ , the nucleus is an oblate ellipsoid with symmetry axis along 2. The rotational states as determined by the operator (1.1), and the probabilities of electromagnetic transitions between them are the same for values of  $\gamma$  equal to  $\gamma_1$  and  $\pi/3 - \gamma_1$ . We therefore give all quantities only in the interval  $0 \leq \gamma \leq \pi/6$ .

In connection with the above, it should be remarked that the measurement of energies of rotational states and of electromagnetic transitions between them cannot give any indication as to whether the nucleus is a prolate or an oblate ellipsoid. The answer to this question could be obtained by measurement of average values of electric quadrupole moments in stationary states ( $J; M = J$ ). In even-even nuclei, the average values of electric quadrupole moments in the ground state ( $J = 0$ ) are zero. In the first ex-

cited state with spin 2 the average value of the quadrupole moment is

$$Q_1 = -Q_0 6 \cos 3\gamma / 7 \sqrt{9 - 8 \sin^2 3\gamma},$$

where  $Q_0$  is defined in (2.2). The average electric quadrupole moment of the second excited state with spin 2 has the opposite sign:  $Q_2 = -Q_1$ .

The reduced probability for an electric quadrupole transition ( $J\tau \rightarrow J'\tau'$ ), averaged over initial states of polarization of the nucleus, is

$$B(E 2; J\tau \rightarrow J'\tau') = \frac{5}{16\pi(2J+1)} \sum_{M, m, u} |(J'm\tau' | \hat{Q}_{2u} | JM\tau)|^2. \quad (2.5)$$

Since we are assuming that there is no change in the internal state of the nucleus in the transition, the reduced transition probability can be expressed in terms of average values of  $\beta$  and  $\gamma$  in the state  $\varphi(\beta, \gamma)$ . Substituting (2.3) in (2.5) and using (2.4), we find the following values for the reduced probabilities of electric quadrupole transitions, expressed in units of  $e^2 Q_0^2 / 16\pi$  which is the reduced probability for electric quadrupole transition in an axially symmetric nucleus between rotational states with spins 2 and 0:

$$b(E 2; 21 \rightarrow 0) = \frac{B(E 2; 21 \rightarrow 0)}{(e^2 Q_0^2 / 16\pi)} = \frac{1}{2} \left[ 1 + \frac{3 - 2 \sin^2 3\gamma}{\sqrt{9 - 8 \sin^2 3\gamma}} \right]; \quad (2.6)$$

$$b(2E; 22 \rightarrow 0) = \frac{1}{2} \left[ 1 - \frac{3 - 2 \sin^2 3\gamma}{\sqrt{9 - 8 \sin^2 3\gamma}} \right]; \quad (2.7)$$

$$b(E 2; 22 \rightarrow 21) = \frac{10}{7} \frac{\sin^2 3\gamma}{9 - 8 \sin^2 3\gamma}. \quad (2.8)$$

In Table I we give, for several values of  $\gamma$ , the ratio  $\epsilon_2(2)/\epsilon_1(2)$ , the values of the reduced probabilities of the transitions (2.6) to (2.8) and the ratio of the reduced probabilities of the electric quadrupole transitions  $b(E 2; 22 \rightarrow 21)/b(E 2; 22 \rightarrow 0)$ . From the data of Table I it follows that with the breakdown of axial symmetry of the nucleus, the reduced probability of the transition from the first excited level to the ground state changes little. The reduced probability (2.7) of the transition from the second excited level with spin 2 directly to the ground state is equal to zero for  $\gamma = 0$  and  $30^\circ$ , and for  $\gamma = 15 - 24^\circ$  amounts to approximately 5 to 7% of the probability of the corresponding transition between the first excited state and the ground state. The reduced probability of the E2 transition from the second excited state with spin 2 to the first excited level is very small for  $\gamma \approx 0$ , but it then increases rapidly and in the region of  $\gamma \sim 20^\circ$  it amounts to approximately 40%, in the region  $\gamma \sim 30^\circ$  to approximately 140% of the reduced probability of the ground state

TABLE I

$\gamma^\circ$	$\frac{\epsilon_2(2)}{\epsilon_1(2)}$	$b(E2; 21 \rightarrow 0)$	$b(E2; 22 \rightarrow 0)$	$b(E2; 22 \rightarrow 21)$	$\frac{b(E2; 22 \rightarrow 21)}{b(E2; 22 \rightarrow 0)}$
0	$\infty$	1.000	0	0	1.43
5	64.2	0.993	0.0074	0.011	1.49
10	15.9	0.972	0.028	0.051	1.70
15	6.85	0.947	0.053	0.143	2.70
20	3.73	0.933	0.067	0.357	5.35
22.5	2.93	0.937	0.0625	0.563	9.02
24	2.59	0.948	0.052	0.782	15.1
25	2.41	0.955	0.0425	0.865	20.6
26	2.26	0.968	0.0324	1.01	31.2
28	2.07	0.99	0.010	1.28	126
29	2.01	0.996	0.004	1.41	363
30	2.00	1.000	0.000	1.43	$\infty$

transition in the axial nucleus.

Especially interesting is the ratio of the reduced probabilities  $b(E2; 22 \rightarrow 21)/b(E2; 22 \rightarrow 0)$ , since this quantity is independent of the occupation of the level  $\epsilon_2(2)$  and can be measured directly.

Using the explicit form of the wave function for the level with energy  $\epsilon(3)$ ,

$$\psi_{3m_1} = \sqrt{7/16} \pi^2 \varphi(\beta\gamma) [D_{m_2}^3 - D_{m, -2}^3],$$

we can calculate the reduced probabilities (in our units) of the electric quadrupole transitions

$$b(2E; 3 \rightarrow 22) = \frac{25}{28} \left( 1 + \frac{3 - 2 \sin^2 3\gamma}{\sqrt{9 - 8 \sin^2 3\gamma}} \right),$$

$$b(2E; 3 \rightarrow 21) = \frac{25}{28} \left( 1 - \frac{3 - 2 \sin^2 3\gamma}{\sqrt{9 - 8 \sin^2 3\gamma}} \right).$$

The values of these probabilities and their ratio are given in Table II.

TABLE II

$\gamma^\circ$	$b(E2; 3 \rightarrow 21)$	$b(E2; 3 \rightarrow 22)$	$\frac{b(E2; 3 \rightarrow 21)}{b(E2; 3 \rightarrow 22)}$
10	0.0132	1.77	0.0075
15	0.095	1.69	0.056
20	0.12	1.67	0.072
25	0.079	1.70	0.044
29	0.0696	1.72	0.039
30	0	1.78	0

### 3. COMPARISON WITH EXPERIMENT

The results obtained in the preceding sections are based on the assumption that the internal state of the nucleus is not changed during its rotation. This assumption can be satisfied only approximately, the better the farther the rotational levels are from those levels (with the same J, parity, etc) corresponding to excitation of the internal state of the nucleus.

In Table III we give experimental data on the ratio of the energy of the second level with spin 2 to the first. This ratio enables us to calculate the value of the parameter  $\gamma$  using formulas (1.2). Using this value of  $\gamma$ , we can calculate the ratio  $N = b(E2; 22 \rightarrow 21)/b(E2; 22 \rightarrow 0)$  from (2.7) and (2.8). Comparison of these ratios with the experimentally observed values (column 7) shows that the theory gives a good description of the experimentally-observed marked change in the ratio of reduced probabilities when we go from one nucleus to another. Comparison of the experimental values of the sum of the energies of the two  $2^+$  levels with the energy of the  $3^+$  level, as we see from comparison of columns 4 and 5 of Table III, confirms the identity (1.5) which is demanded by the theory to within 1% for all elements except  $\text{Cd}^{114}$ , where a deviation of 5% is observed. In

TABLE III

Nucleus	$\frac{\epsilon_2(2)}{\epsilon_1(2)}$	$\gamma^\circ$	Exper. (kev)		N		Reference
			$\epsilon_2(2) + \epsilon_1(2)$	$\epsilon(3)$	Theoret.	Exptl.	
$\text{Pt}^{192}$	1.94	30	929	920.9	$\infty$	163	[13]
$\text{Xe}^{128}$	2.0	30	1445	—	$\infty$	197	[14]
$\text{Cd}^{114}$	2.17	26.75	1771	1860	78	77.9	[15]
$\text{Se}^{76}$	2.2	26.6	1760	—	68	23.5	[16]
$\text{Te}^{182}$	2.25	26.5	1760	—	66	78.2	[17]
$\text{Gd}^{154}$	8.21	13.7	1133	1140	2.0	—	[18]
$\text{W}^{186}$	8.08	13.9	1000	1000	2.0	—	[19]
$\text{Sm}^{152}$	8.9	13.48	1206	1226	1.9	1.7	[18, 22]
$\text{Dy}^{160}$	11	12.2	1051	1047	1.9	2.38	[10]
$\text{W}^{182}$	12.1	11.6	1322	1331	1.6	1.59	[20]
$\text{Pu}^{238}$	23.4	8.13	1074	1076	1.2	1.3—1.5	[19, 23]
$\text{Pu}^{240}$	23.7	8.0	1063	1060	1.2	—	[19]

the case of  $\text{Cd}^{114}$ , in the region of the second excited  $2^+$  level there are still three nearby levels  $4^+$ ,  $0^+$ ,  $2^+$ , which may have an effect. From the figure we see that for  $\gamma < 21.5^\circ$ , the second excited spin-2 level should lie above the  $4^+$  level, while it lies below for  $\gamma > 21.5^\circ$ . The spin-3 level should always lie higher than the  $4^+$  level. These rules are satisfied for all those nuclei of Table III for which the positions of the levels with spins  $2^+$ ,  $4^+$ , and  $3^+$  are known.

It is interesting to note that for  $\gamma = 30^\circ$  the theory leads to equal separations between the levels  $\epsilon_1(2)$ ,  $\epsilon_2(2)$ ,  $\epsilon(3)$ . Such an arrangement of levels also follows from the oscillator approximation for the energy of the surface oscillations. It is true that in the latter case the levels  $\epsilon_2(2)$  and  $\epsilon(3)$  should be degenerate with spin values 0, 2, 4, and 0, 2, 3, 4, 6, respectively.

Unfortunately there is little experimental information from which one could determine the ratio  $b(E2; 3 \rightarrow 21)/b(E2; 3 \rightarrow 22)$ . This ratio is known only for  $\text{Kr}^{82}$  and is equal to  $0.016^{21}$ , which corresponds to a value of  $\gamma$  a little greater than  $29^\circ$ . The value  $\gamma \sim 30^\circ$  is in agreement with the observed value of  $\epsilon_2(2)/\epsilon_1(2) = 1.9$  if we assume that the adiabatic conditions are broken down in this nucleus.

Thus a comparison of the results of the theory with the known experimental data confirm the assumption that certain even-even nuclei do not possess axial symmetry.

It should be noted that Gursky<sup>24</sup> in 1955, using a three-dimensional harmonic oscillator model, showed that the minimum energy of the nucleons corresponds to a nonaxial shape of the nucleus. In Fig. 6 of reference 7 the data of Gursky for the energy of the nucleons in the nucleus with  $Z = 55$  and  $N = 91$  are shown as a function of  $\beta$  and  $\gamma$ . From the figure we see that the minimum energy corresponds to  $\gamma = 7.5^\circ$ .

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