$$\psi_{I_0M} = \sum_{Il\Omega} \sqrt{\frac{2I+1}{2I_0+1}} C^{I_0, I_0+\Omega}_{l\Omega; II_0} \frac{f_{Il}(r)}{r} Y_{l\Omega}(\theta, \varphi) \widetilde{D}^{(I_0)}_{M, I_0+\Omega}(\Theta_i). \tag{A.8}$$

On the nuclear surface, where $r = R(\theta)$, the wave function should not depend on φ. This will be the case only if, when we substitute $r = R(\theta)$ in (A.8), all terms in the sum over Ω except the term for $\Omega = 0$ vanish.

It then follows from (A.8) that at the nuclear surface

$$\psi_{I_0M}[R(\theta), \theta, \varphi; \Theta_i] = \chi(\theta) \widetilde{D}^{(I_0)}_{MI_0}(\Theta_i), \tag{A.9}$$

where

$$\chi(\theta) = \sum_{Il} \sqrt{\frac{2I+1}{2I_0+1}} C^{II_0}_{l0; II_0} \frac{f_{Il}[R(\theta)]}{R(\theta)} Y_{l0}(\theta)$$

is some function of θ whose specific form will depend on the structure of the α-particle function in the interior of the nucleus. Substituting $r = R(\theta)$ in (A.8), equating the right sides of (A.8) and (A.9), multiplying both sides of the resulting equations by $\{\widetilde{D}^{(I_0)}_{M,I_0+\Omega}(\Theta_i)\}^*$ and integrating over Θ_i, we get the boundary condition (2.5).

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Translated by M. Hamermesh
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SOVIET PHYSICS JETP VOLUME 35(8), NUMBER 1 JANUARY, 1959

PROPAGATION OF DETONATION WAVES IN THE PRESENCE OF A MAGNETIC FIELD

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Submitted to JETP editor February 13, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 203-207 (July, 1958)

It is shown that relativistic detonation waves in a magnetic field possess properties similar to those of the ordinary waves. Solutions of the equations at the discontinuity are presented for the relativistic and nonrelativistic cases.

SHOCK waves in a plasma situated in a magnetic field have been discussed frequently in recent times. In some of this work, for example that of Hoffman and Teller,[1] use was made of the relativistic hydrodynamic equations.

In the present article we consider "perpendicular" detonation waves, i.e., waves propagated at right angles to the direction of the magnetic field. One may expect that the influence of the magnetic field will become noticeable when its energy per unit mass of the medium becomes comparable with the energy liberated in the medium. The calculations are made in a relativistic manner, although for those thermonuclear fuels which are now known and for fields which can be achieved at the present time there is no necessity to take relativistic effects into account. Nevertheless, a relativistic treatment is interesting because it indicates the behavior of the different quantities in the case of more powerful fuels and fields, and gives their limiting values.

1. EQUATIONS AT THE DISCONTINUITY

We now undertake the derivation of the relation between the physical parameters of the cold and of the burnt gas in a detonation wave, leaving aside for the time being the question of the structure of the wave.

We assume that the shock wave, which forms the front of the detonation wave, converts the medium into a plasma with infinite conductivity and with magnetic permeability $\mu = 1$. Moreover, we assume that the width of the wave is sufficiently large that we do not need to take losses into account.

The equations at the discontinuity express the continuity of the components of the energy-momentum tensor. In writing them down we should take into account the fact that the detonation energy is obtained at the expense of converting a certain fraction (α) of the rest mass of the initial gas. In the reference system in which the wave is at rest, they have the following form:

$$\rho_1 v_1 / \gamma_1 = \rho_2 v_2 / \gamma_2 (1-\alpha) = j, \tag{1}$$

$$\gamma_1^{-2}\left[\left(e_1 + \frac{H_1^2}{2}\right)\frac{v_1^2}{c^2} + p_1 + \frac{H_1^2}{2}\right] = \gamma_2^{-2}\left[\left(e_2 + \frac{H_2^2}{2}\right)\frac{v_2^2}{c^2} + p_2 + \frac{H_2^2}{2}\right], \tag{2a}$$

$$\frac{v_1}{c\gamma_1^2}\left[e_1 + \frac{H_1^2}{2} + p_1 + \frac{H_1^2}{2}\right] = \frac{v_2}{c\gamma_2^2}\left[e_2 + \frac{H_2^2}{2} + p_2 + \frac{H_2^2}{2}\right], \tag{3a}$$

$$H_1/\rho_1 = H_2/\rho_2 (1-\alpha), \quad \gamma = \sqrt{1 - v^2/c^2}. \tag{4}$$

Here p is the pressure, $\sqrt{4\pi}$ H is the intensity of the magnetic field, e the internal energy per unit volume in the proper reference system, v the velocity of the medium, c the velocity of light, ρ the rest-mass density in the proper reference system (the one associated with the gas), and j the flux density of the rest mass. Equation (1) expresses conservation of rest mass, (2a) the continuity of the momentum flux, (3a) conservation of energy, and (4) expresses continuity of the field. Equation (4) is valid in virtue of the assumed infinite conductivity of the plasma, and expresses the "freezing" of the lines of force into the medium.

Let us reduce these equations to a more usual form by introducing the quantities

$$p^* = p + H^2/2, \qquad w^* = e + p + H^2. \tag{5}$$

On calculating v and γ from (1), and on substituting these expressions into (2a), we obtain

$$p_2^* - p_1^* = -(j/c)^2 [w_2^* (1-\alpha)/\rho_2^2 - w_1^*/\rho_1^2]. \tag{2}$$

Repeating the same calculations with respect to (3a), we obtain the equation for the detonation adiabatic

$$p_2^* - p_1^* = \frac{(1-\alpha)^2 w_2^{*3}/\rho_2^2 - w_1^{*3}/\rho_1^2}{(1-\alpha)^2 w_2^{*2}/\rho_2^2 + w_1^{*2}/\rho_1^2}. \tag{3}$$

To Eqs. (1) to (4) one should also add the equation of state, which can be written in the form

$$w^* = w^*(p^*, \rho, H_1). \tag{6}$$

2. THE JOUGUET POINT

Equations (1) to (6) form a complete system of equations at the discontinuity. In order to solve it we must also give one further additional relation, which must be found from the boundary conditions. As is well known, the latter lead often (e.g. in the case of spontaneous detonation) to the fact that the detonation wave is propagated with the least possible velocity. The system then corresponds to the so called Jouguet point on the detonation adiabatic.

We can derive certain general relationships which hold at the Jouguet point. To do this, we shall utilize the method given by Zel'dovich. It is well known that the usual detonation wave consists of two regions: of a shock wave of not very great width in which the material is heated to the required temperature, and of a considerably more extensive zone of combustion. In such a case Eq. (2) is valid for all the intermediate states if α is assumed variable.

In the case of stronger detonation waves, a strict separation into a shock wave and into a combustion zone loses its meaning. Combustion begins inside the shock wave, which is considerably broadened by ordinary and radiant thermal conductivity and by slow establishment of equilibrium between electrons and ions. By utilizing the schematic representations of the structure of the shock wave developed by Zel'dovich[2] and Shafranov,[3] we can represent the detonation wave in the form shown in Fig. 1.

Within the region $1 - 1'$ a gradual heating of the gas takes place owing to the thermal conductivity and to the slowing down in the gas of un-

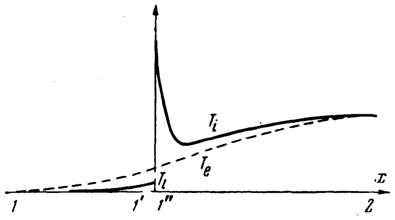

FIG. 1

charged particles formed during combustion. Between 1′ and 1″ the ionic (and, possibly, also the electronic) temperature undergoes a sharp discontinuity. Combustion, the establishment of electron-ion equilibrium, and the slowing down of uncharged particles occur simultaneously in the region 1″ − 2.

In order that the following arguments be valid, it is necessary that combustion should be the slowest of all these processes, for otherwise it would be continued in the region in which energy is absorbed. The final state for the shock adiabatic is the state 1″, in which the combustion starts. This adiabatic is situated below the detonation adiabatic. In order to find it, we must generally solve the whole problem of the structure of the wave.

We shall represent the state of the gas on the plane $(p^*, (1-\alpha)^2 \omega^*/\rho^2)$. In this plane the detonation adiabatic is determined by Eq. (3). As may be seen from Eq. (2), the detonation process is represented in this plane by a straight line passing through the point 1 (cf. Fig. 2) situated

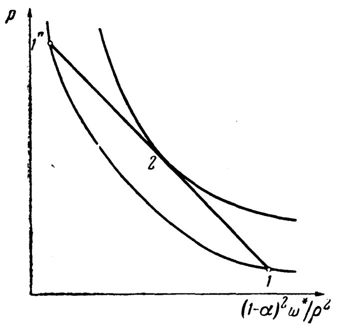

FIG. 2

on the shock adiabatic and intersecting the detonation adiabatic. A displacement along the segment 1 − 1″ corresponds to the shock wave. Combustion is represented by the segment 1″ − 2. The speed of the wave is determined by the slope of this straight line. The wave with the minimum speed corresponds to the tangent to the detonation adiabatic constructed at the point 1; the point of tangency is the Jouguet point. As is well known, the part of the adiabatic below this point has no physical meaning.

On differentiating (2) and (3), and on taking into account the identity

$$d(w^*/\rho) = dp^*/\rho + T ds \tag{7}$$

(s is the entropy per unit rest mass), we obtain the following relation:

$$\left[\frac{(1-\alpha)^2 w_2^*}{\rho_2^2} - \frac{w_1^*}{\rho_1^2}\right]^2 \frac{dj^2}{c^2} = 2(1-\alpha)^2 \frac{w_2^*}{\rho_2} T_2 ds_2. \tag{8}$$

From this we see that at the Jouguet point, where $dj = 0$, we must have

$$ds_2 = 0. \tag{9}$$

From Eq. (3) we now obtain the flux

$$\frac{j^2}{c^2} = -\frac{1}{(1-\alpha)^2}\left[\frac{\partial p_2^*}{\partial (w_2^*/\rho_2^2)}\right]_s. \tag{10}$$

This relation gives us immediately

$$v_2^2/c^2 = (\partial p_2^*/\partial e_2^*)_s. \tag{11}$$

The derivative should be taken after replacing H_2 by its value from Eq. (4).

As is well known, $c(\partial p/\partial e)_s^{1/2}$ is the relativistic expression for the speed of sound. The right-hand side of (11) is the speed of sound in the magnetic field. We obtain the usual result: the wave moves over the detonation products with the speed of sound.

3. SOLUTION OF THE EQUATIONS

For a definite detonation process we know the parameter α, the thermodynamic state of the gas 1, and the magnetic field H_1. To be able to find the remaining quantities, we must express explicitly thermodynamic functions for the detonation products. We give below the solution of Eqs. (1) to (4) and (6) for the detonation wave which corresponds to the Jouguet point in the case when the detonation products may be regarded as an ideal gas.

For a relativistic ideal gas the following equation of state remains valid

$$p = \rho RT = \rho c^2 \sigma, \tag{12}$$

where $\sigma = RT/c^2$. The heat function has the form

$$w = \rho c^2 g(\sigma)$$

and the speed of sound is given by

$$a = c\left[\frac{H^2/\rho c^2 + f(\sigma)}{H^2/\rho c^2 + g(\sigma)}\right]^{1/2}$$

We introduce the dimensionless variables

$$R = \rho_2/(1-\alpha)\alpha_1, \quad h = H_1^2/\rho_1 c^2 \tag{13}$$

The functions $f(\sigma)$ and $g(\sigma)$ which depend on the kind of gas have, generally speaking, a fairly complicated form for a gas heated to tem-

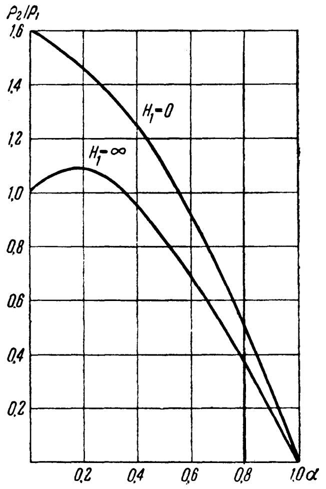

FIG. 3

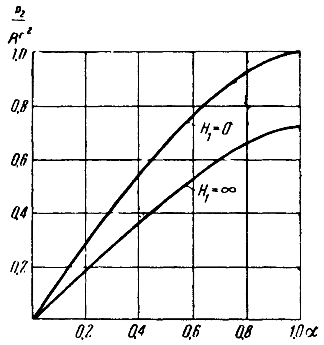

FIG. 4

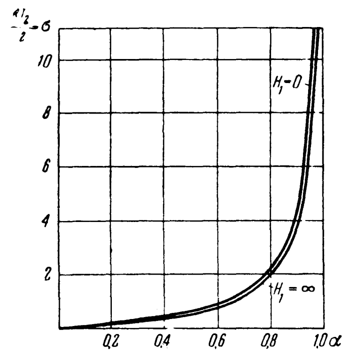

FIG. 5

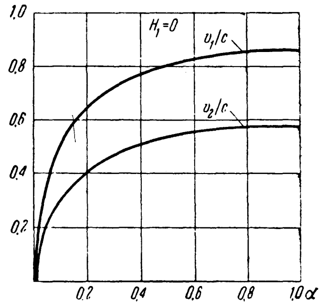

FIG. 6

peratures at which relativistic effects come into play (for the electronic component of the gas this occurs at $T \sim 10^{9}$°K). It is therefore convenient to express the solution of the system of equations in parametric form (the parameters are R and σ). It may be written down in the form of a quadratic equation for h:

$$h^2[-3R^2+2(3g-f)R+2(g-f)(\sigma-g)-g^2]$$
$$+h[-2R^2+(3g-4f)R$$
$$+4gf+4\sigma(g-f)-g^3/R] \qquad (14)$$
$$+2[-fR+gf+\sigma(g-f)]=0$$

and of the equation

$$\frac{1}{1-\alpha^2}=(g+hR)^2-\frac{hR+f}{g-f}\,\frac{R-g}{(1+h)^2}[g+(2h+1)R]. \qquad (15)$$

The results of sample numerical calculations are shown in Figs. 3 to 6.

4. THE NONRELATIVISTIC CASE

We shall examine in greater detail the case when the energy liberated and the energy of the magnetic field are both low in comparison with the energy of the gas. We may set in Eqs. (1) to (4) and (6)

$$v_1/c\ll 1;\quad v_2/c\ll 1;\quad \alpha\ll 1; \qquad (16)$$
$$H_1^2/\rho_1c^2\ll 1;\quad \alpha\rho_1c^2=q;\quad w\to\rho c^2+w.$$

Here q denotes the quantity of heat liberated per unit mass, while the nonrelativistic heat function

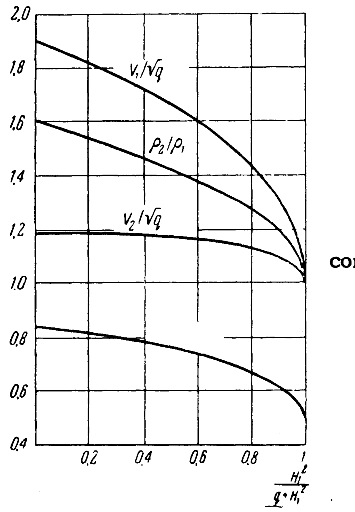

FIG. 7. The lower curve corresponds to RT_2/q.

is denoted by the same letter w. The system (1) to (4) and (6) now assumes the usual form:

$$\rho_1v_1=\rho_2v_2,\quad p_1+\rho_1v_1^2+H_1^2/2=p_2+\rho_2v_2^2+H_2^2/2,$$
$$q+w_1+v_1^2/2+H_1^2/\rho_1=w_2+v_2^2/2 \qquad (17)$$
$$+H_2^2/\rho_2,\quad H_1/\rho_1=H_2/\rho_2.$$

The speed of sound is given by

$$v_2=(\varkappa p_2/\rho_2+H_2^2/\rho_2)^{1/2}. \qquad (18)$$

For those cases which are of interest in connection with the foregoing, the detonation can always be regarded as strong, i.e., we can set $p_1 = w_1 = 0$. Moreover, we shall assume that the product of detonation is an ideal gas.

We introduce the dimensionless variables

$$\rho_2=\rho_1R_2,\quad v_1=\sqrt{q}V_1,\quad v_2=\sqrt{q}V_2,$$
$$H_1^2=\rho_1qh_1,\quad H_2^2=\rho_2qh_2,\quad p_2=\rho_2qz_2.$$

The solution may be given in parametric form (parameter R_2):

$$\frac{1}{h_1} = \frac{[1-(2-\varkappa) R_2^2 (1-R_2)] [(\varkappa/2)(1-R_2^2) + \varkappa/(\varkappa-1)]}{2R_2 [\varkappa(1-R_2)+1]} + \frac{2-\varkappa}{4} R_2 (1-R_2^2) - \frac{2-\varkappa}{2(\varkappa-1)} R_2 - 1; \quad (19)$$

$$z_2 = h_1 [1-(2-\varkappa) R_2^2 (1-R_2)] / 2R_2 [\varkappa(1-R_2)+1];$$

$$V_2^2 = \varkappa z_2 + \frac{2-\varkappa}{2} h_1 R_2, \quad V_1 = R_2 V_2,$$

where κ is the adiabatic coefficient. This solution is given in Fig. 7 for $\kappa = 5/3$.

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Translated by G. Volkoff
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SOVIET PHYSICS JETP VOLUME 35 (8), NUMBER 1 JANUARY, 1959

A NONLINEAR THEORY OF VECTOR FIELDS

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Submitted to JETP editor February 17, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 208-215 (July, 1958)

In analogy with Einstein's theory of gravitation, the existence of a physical vector field is treated as the curvature in a "non-Pythagorean" space whose metric is $ds = \gamma_i dx^i$. This leads to nonlinear field equations which in the linear approximation (for weak fields) become the usual equations. In spite of its potential being singular, the total energy in the field of a point charge is finite.

1. STATEMENT OF THE PROBLEM

As a rule, the nonlinear generalizations of field theories are constructed in an attempt to eliminate divergences in ordinary field theory. It is known, however, that this can be done also in other ways (nonlocal interactions, the use of higher drivatives, space quantization, etc.). Furthermore, one likes to hope that it is possible to go on without essentially leaving the framework of the existing theory. This would seem to be why the nonlinear theories do not receive wide recognition at present.

We wish to emphasize that a nonlinear generalization of field theory is necessary, independently of the divergences in the linear theory and independently of the possible necessity for other generalizations. Our argument is based on the following considerations.

(a) In principle, nonlinearity in field theory follows unavoidably from the fact of pair creation and annihilation, which leads to nonlinear effects such as scattering of light by light. It is not, therefore, merely accidental that when dealing with interactions in modern field theory we arrive at nonlinear equations. Nonlinearity whould therefore not be considered simply one of many possible methods for eliminating the difficulties of the theory, but as a reflection of the objective properties of the field.

(b) It is well known that in all nonlinear theories there appears a characteristic length which, although it has different meanings in different versions, is always of the order of magnitude of the "classical radius" $r_0 = e^2/m_0c^2$ of the field's source. This parameter serves as a criterion that can be used to define weak fields (with $r \gg r_0$) for which nonlinear effects can be neglected. It turns out that in electrodynamics all real processes take place in a region for which $r \gg r_0$ (weak fields and low energies), and the linear theory is insufficient only when one needs to consider virtual processes with arbitrary energies. The situation is different, however, where in mesodynamcs, weak fields (in the above sense