

the contributions of the terms in the trial functions drop off rapidly with increasing n and m , and that $a_4 = 6.3 \times 10^{-13} \text{ cm}$ and $a_2 = 1.1 \times 10^{-13} \text{ cm}$ coincide within the limits of error with the experimental values (1) for the scattering lengths.

We are now in the course of calculation of the energy dependence of the s -phase, the depth of the potential and the type of exchange force, as well as the analysis of low energy proton-deuteron scattering, by the present method.

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19

NONEQUILIBRIUM PROCESSES IN IMPURITY SEMICONDUCTORS

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A treatment is given of the kinetic equations for impurity semiconductors, which describe transitions from the impurity levels to the conduction band. On the assumption that the distribution function of the free electrons (or holes) has the form of an equilibrium distribution function with a certain effective temperature that can be determined from the equations, explicit expressions are given for the energy and kinetic coefficients for cases in which the lifetime of the electrons in the conduction band is determined by photorecombination and triple-collision recombination processes. Nonradiative transitions other than those occurring in triple recombination are included by a phenomenological method. In this case the kinetic and energy coefficients can be expressed in terms of the lifetime of the electrons against such transitions in the equilibrium state. The equations obtained make it possible to determine the electron temperature and the number of electrons in the conduction band in various non-equilibrium processes.

1. THE KINETIC EQUATIONS FOR THE FREE ELECTRONS, INCLUDING EFFECTS OF RE-COMBINATION AND IONIZATION

THE kinetic equations for the distribution function \bar{n} of electrons or holes, including effects of their possible heating up, have the form [cf. e.g., Eqs.

(3.1) ff. in reference 1]

$$\frac{\partial n_0}{\partial t} + \frac{v}{3} \nabla \hat{n}_1 + \frac{1}{N_e} \frac{\partial}{\partial \epsilon} (N_e S_1) + \Sigma_{ee} (\bar{n}_0, \bar{n}_0) + \varphi(\epsilon) = 0, \quad (1.1)$$

$$\frac{\partial \bar{n}_1}{\partial t} + v \nabla \bar{n}_0 + e E v \frac{\partial \bar{n}_0}{\partial \epsilon} + \frac{e}{mc} [\mathbf{H} \times \bar{\mathbf{n}}_1] + \frac{v}{L} \bar{\mathbf{n}}_1 = 0. \quad (1.2)$$

Here

$$\begin{aligned} S_1 &= \frac{1}{3} e \mathbf{E} \cdot \bar{\mathbf{n}}_1 v - S \\ &= \frac{1}{3} e \mathbf{E} \cdot \bar{\mathbf{n}}_1 v - (2m v_s^2 \epsilon v / l) [\bar{n}_0 (1 - \bar{n}_0) / kT + \partial \bar{n}_0 / \partial \epsilon] \end{aligned} \quad (1.3)$$

is the energy flux density in the energy space. The notations are as follows: \bar{n}_0 is the part of the distribution function that is symmetric in the momenta (the average number of electrons in a given energy level); $\mathbf{v} \cdot \bar{\mathbf{n}}_1 / v$ is the part added to the symmetric part of the distribution function — together these two parts give the total function \bar{n} ; ϵ is the energy, $v = \partial \epsilon / \partial p$ is the speed, m the effective mass, e the charge of an electron (or hole), v_s the velocity of sound, and T the lattice temperature. The reciprocal of the mean free path L in which the electron loses its momentum is given by

$$1/L = 1/l_{el} + 1/l_{ei} + 1/l_{ee}, \quad (1.4)$$

where $l = l_{el}$ is the mean free path against loss of momentum by scattering by phonons, l_{ei} by scattering by impurities, and l_{ee} by electrons and holes. \mathbf{E} and \mathbf{H} are the external electric and magnetic fields, c the speed of light, and N_e the doubled (because of spin degeneracy) density of electron (or hole) levels. The expression Σ_{ee} describes the interaction between the electrons in the conduction band. The term $\varphi(\epsilon)$ in Eq. (1.1), which was not included in reference 1, describes the change of the average number of electrons (or holes) with the energy ϵ on account of transitions between the conduction band, the valence band, and the impurity levels. We neglect the analogous term in Eq. (1.2). Together with the normalization condition

$$n = \int \bar{n}_0 N_e d\epsilon \quad (1.5)$$

the system of equations completely determines the distribution function

$$n = n_0 + \mathbf{v} \cdot \bar{\mathbf{n}}_1 / v. \quad (1.6)$$

We shall choose the concrete form of $\varphi(\epsilon)$ for impurity semiconductors for conditions such that transitions from one band to another, and also between a band and local levels far removed from it can be neglected. Thus for semiconductors with electron conductivity we include only transitions between donor levels and the conduction band, and for semiconductors with hole conductivity only transitions between acceptor levels and the valence band. The construction of the scheme for semiconductors with hole conductivity does not differ at all in principle from the treatment of electronic semiconductors. Therefore for definiteness we shall speak hereafter about an electronic semi-

conductor in which in addition to the donors there are N_a acceptors. In the ground state the acceptors will be filled up with electrons which have gone to them from the donor levels. We shall denote by N_d the number of donor levels that are filled in the ground state. Thus we arrive at a Neiburov scheme of a semiconductor with N_d donors and N_a acceptors, located at the distance ϵ_0 below the bottom of the conduction band.

Ejection of electrons from the donor levels into the conduction band cannot be accomplished by the phonons, because their maximum energy is considerably smaller than ϵ_0 . Thermal fluctuations, under the action of which impurity electrons get ionized, can occur in the form of photons of black-body radiation. The dispersion formula for this excitation branch is given by the equation $\omega^2 = \omega_0^2 + v_\nu K^2$, where v_ν is the speed of light of frequency ν in the medium, K is the wave vector of the photon, and $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the Debye frequency determined by the total number n of free electrons. For semiconductors in which n is small compared with the value in metals, the quantity ω_0 can be neglected for frequencies that can produce transitions ($\omega \gtrsim \epsilon_0/h$). In addition to the processes of ionization by black-body radiation, ionization can be produced by fast electrons with energies larger than ϵ_0 . The inverse processes are those of photorecombination, accompanied by the emission of a photon, and of triple-collision recombination, i.e., recombination with an empty impurity by one of two electrons colliding with it simultaneously. For a number of semiconductors in which the numbers of free electrons are small, and for which it is known that in equilibrium one can neglect processes of ionization by collision and of radiationless triple-collision recombination, radiative recombination alone cannot explain the short lifetimes of electrons in the free state that are observed experimentally. The source and character of the perturbations that cause the nonradiative transitions in these semiconductors have not been ascertained as yet. We shall for the time being leave this class of semiconductors to one side, so that we confine ourselves to the consideration of semiconductors in which the lifetime of the free electrons in the equilibrium state is fixed either by radiative recombination or by nonradiative triple recombination.

Let $\tilde{Z}(\epsilon) - \tilde{Z}'(\epsilon)$ denote the increase of the number of free electrons with the energy ϵ per unit volume and unit time owing to processes of photoionization and photorecombination, and $Z(\epsilon) - Z'(\epsilon)$ that owing to processes of ionization by

collision and triple recombination. Then

$$\varphi(\varepsilon) = -\tilde{Z} + \tilde{Z}' - Z + Z'. \quad (1.7)$$

We confine ourselves to the consideration of semiconductors of relatively small conductivity, in which the number of electrons is so small ($n < 6 \times 10^{14} T^{3/2}$) that they obey classical statistics. Then for the photoprocesses we have

$$\tilde{Z}(\varepsilon) = (N_d - n) \kappa_v \bar{N}_v d\Gamma_v / d\Gamma_e, \quad (1.8)$$

$$\tilde{Z}'(\varepsilon) = (N_a + n) \kappa_e v (1 + \bar{N}_v) \bar{n}, \quad (1.9)$$

where \bar{n} is the average number of free electrons in the level in question, n is the total number of free electrons, $d\Gamma_e = N_e(\epsilon) d\epsilon$ is the number of electronic levels with the energy ϵ , $d\Gamma_v$ is the number of photon levels, κ_v is the cross-section for photoionization by a photon of frequency given by $h\nu = \epsilon_0 + \epsilon$, and κ_e is the cross-section for photorecombination. The cross-sections κ_v and κ_e are connected by the relation

$$\kappa_v = \kappa_e \frac{d\Gamma_e}{d\Gamma_v} \frac{v}{v_v}. \quad (1.10)$$

In Eq. (1.9) the factor $(1 + \bar{N}_v)$, where \bar{N}_v is the average number of photons with the frequency v , includes the effects of both spontaneous and stimulated emission.

Taking account of the effects of dispersion, we have for the number of photon levels

$$d\Gamma_v = 8\pi K^2 dK = \frac{8\pi v^2}{c^3} \zeta^3 \frac{d \ln \zeta_v}{d \ln v} dv,$$

where the photon wave vector $K = \zeta v/c$, and ζ is the index of refraction. The speed of a photon in material with the index of refraction ζ is

$$v_v = dv/dK = (c/\zeta) d \ln v / d \ln \zeta_v.$$

In a similar way we find for the decrease of the number of electrons with the energy ϵ owing to the inverse processes $S^{2,1}$:

$$\begin{aligned} Z'(\varepsilon) = & (N_a + n) n^2 \left\{ \int_0^\infty v(\varepsilon) \frac{\bar{n}(\varepsilon)}{n} v(\varepsilon_1) \frac{\bar{n}(\varepsilon_1)}{n} S^{2,1}(\varepsilon, \varepsilon_1) N_e(\varepsilon_1) d\varepsilon_1 + \int_0^\infty v(\varepsilon) \frac{\bar{n}(\varepsilon)}{n} v(\varepsilon_1) \frac{\bar{n}(\varepsilon_1)}{n} S^{2,1}(\varepsilon_1, \varepsilon) N_e(\varepsilon_1) d\varepsilon_1 \right. \\ & \left. - \int_0^{\varepsilon - \varepsilon_0} v(\varepsilon) \frac{\bar{n}(\varepsilon)}{n} v(\varepsilon_1) \frac{\bar{n}(\varepsilon_1)}{n} S^{2,1}(\varepsilon_1, \varepsilon - \varepsilon_1 - \varepsilon_0) d\varepsilon_1 \right\}. \end{aligned} \quad (1.13)$$

Furthermore, as follows from the principle of detailed balance, or from the equation $Z = Z'$, for equilibrium [cf. Eq. (2.1)] we have the following relation between the cross-sections of the direct and inverse processes

$$S^{2,1}(\varepsilon_1, \varepsilon_2) = \frac{v(\varepsilon_1 + \varepsilon_2 + \varepsilon_0)}{v(\varepsilon_1)v(\varepsilon_2)} \frac{N_e(\varepsilon_1 + \varepsilon_2 + \varepsilon_0)}{N_e(\varepsilon_1)N_e(\varepsilon_2)} S^{1,2}(\varepsilon_1 + \varepsilon_2 + \varepsilon_0, \varepsilon_2). \quad (1.14)$$

The cross-section $q(\epsilon)$ for ionization by collision is defined in terms of $S^{1,2}$ in the following way:

The quantity $v_v d\Gamma_v$ itself has the simple form

$$v_v d\Gamma_v = (8\pi v^2/c^2) \zeta^2 dv = v_v 8\pi v^2 dv / v_v^3, \quad (1.11)$$

just as if there were no dependence of ζ on v . For frequencies $h\nu \sim \epsilon_0$ in the majority of semiconductors the relation $\epsilon_d \approx \zeta^2$ between the index of refraction and the dielectric constant holds with good accuracy.

To describe the processes of ionization by collision and triple recombination we introduce the cross-section $S^{1,2}(\epsilon_1, \epsilon_2)$ for the process in which an electron with energy ϵ_1 produces ionization by collision with the ejected electron going into an energy interval at ϵ_2 and the original electron left still in the conduction band (the indices 1, 2 on S mean that at the start of the process there was one electron in the conduction band, and at the end there are two). The cross-section $S^{2,1}(\epsilon_1, \epsilon_2)$ describes the inverse process, in which one of the two electrons recombines (namely the first one) while they are both interacting with the impurity. The change of the number of electrons with energy ϵ owing to ionization processes $S^{1,2}$ consists of three parts: (1) the electron ionized from the impurity goes into the interval $d\epsilon$, (2) the electron producing the ionization goes into the interval $d\epsilon$, and (3) an electron that itself has the energy ϵ produces ionization and leaves the interval $d\epsilon$. Corresponding to these three processes we have:

$$\begin{aligned} Z(\varepsilon) = & (N_d - n) n \left\{ \int_{\varepsilon + \epsilon_0}^\infty v(\varepsilon_1) \frac{\bar{n}(\varepsilon_1)}{n} \frac{S^{1,2}(\varepsilon_1, \varepsilon)}{N_e(\varepsilon)} N_e(\varepsilon_1) d\varepsilon_1 \right. \\ & + \int_{\varepsilon + \epsilon_0}^\infty v(\varepsilon_1) \frac{\bar{n}(\varepsilon_1)}{n} \frac{S^{1,2}(\varepsilon_1, \varepsilon_1 - \varepsilon - \varepsilon_0)}{N_e(\varepsilon)} N_e(\varepsilon_1) d\varepsilon_1 \quad (1.12) \\ & \left. - \int_0^{\varepsilon - \varepsilon_0} v(\varepsilon) \frac{\bar{n}(\varepsilon)}{n} \frac{S^{1,2}(\varepsilon, \varepsilon_1)}{N_e(\varepsilon)} N_e(\varepsilon_1) d\varepsilon_1 \right\}. \end{aligned}$$

$$q(\varepsilon) = \int_0^{\varepsilon - \varepsilon_0} S^{1,2}(\varepsilon, \varepsilon_1) d\varepsilon_1, \quad (1.15)$$

and the concrete expression for $S^{1,2}$, for example as given by the classical Thomson formula, is

$$S^{1,2}(\varepsilon, \varepsilon_1) = \pi e^4 / \varepsilon (\varepsilon_1 + \varepsilon_0).$$

As will be seen from what follows, it is necessary to take into account all three possibilities for ionization and recombination. Usually only the first term is used in calculating the ionization. This would, however, lead to an incorrect result in calculating the change of the total energy of the free electrons.

Equations (1.1) to (1.3), with the $\varphi(\epsilon)$ determined by Eq. (1.7) and the equations following it, describe the change of the distribution function n of the electrons in various sorts of processes, on condition that the distribution \bar{N}_ν of the photons at each point of coordinate space is a known function. \bar{N}_ν is determined by the equation of diffusion of radiation and the appropriate boundary conditions. This equation clearly has the form

$$\frac{\partial N_\nu(\vartheta, \varphi)}{\partial t} + v_\nu \nabla N_\nu(\vartheta, \varphi) + (N_d - n) \kappa_\nu v_\nu N_\nu(\vartheta, \varphi) \quad (1.16)$$

$$- (N_a + n) \kappa_e v \left(\frac{1}{4\pi} + N_\nu(\vartheta, \varphi) \right) \bar{n} \frac{d\Gamma_\nu}{d\Omega_\nu} = 0,$$

where $N_\nu(\vartheta, \varphi)$ is the number of photons of frequency ν in the direction ϑ, φ , so that $\int N_\nu(\vartheta, \varphi) \sin \vartheta d\vartheta d\varphi = \bar{N}_\nu$.

Using Eqs. (1.12) to (1.14), and also (in the intermediate steps) the concrete expression for $S^{1,2}$, we calculate the integral of Eq. (1) times $N_e d\epsilon$. As the result we get the equation of conservation of charge in the form

$$\frac{\partial n}{\partial t} - (N_d - n) \tilde{\beta} + (N_a + n) n \tilde{\beta}' - (N_d - n) n \beta \quad (1.17)$$

$$+ (N_a + n) n^2 \beta' + \text{div } \mathbf{j}/e = 0,$$

where the ionization and recombination coefficients are

$$\tilde{\beta} = \int_{\nu_0=\varepsilon_0/h}^{\infty} \kappa_\nu \bar{N}_\nu v_\nu d\Gamma_\nu; \quad (1.18)$$

$$\tilde{\beta}' = \frac{1}{n} \int_0^{\infty} \kappa_e v [1 + \bar{N}_\nu] \bar{n} d\Gamma_e = \frac{1}{n} \int_{\nu_0}^{\infty} \kappa_\nu (1 + \bar{N}_\nu) \bar{n} v_\nu d\Gamma_\nu; \quad (1.19)$$

$$\beta = \frac{1}{n} \int_{\varepsilon_0}^{\infty} v(\varepsilon) \bar{n}(\varepsilon) N_e(\varepsilon) d\varepsilon \int_0^{\varepsilon-\varepsilon_0} S^{1,2}(\varepsilon, \varepsilon_1) d\varepsilon_1, \quad (1.20)$$

$$\beta' = \frac{1}{n^2} \int_{\varepsilon_0}^{\infty} v(\varepsilon) N_e(\varepsilon) d\varepsilon \int_0^{\varepsilon-\varepsilon_0} \bar{n}(\varepsilon_1) \bar{n}(\varepsilon - \varepsilon_1 - \varepsilon_0) S^{1,2}(\varepsilon, \varepsilon_1) d\varepsilon_1, \quad (1.21)$$

and the electric current density is

$$\mathbf{j} = \frac{e}{3} \int_0^{\infty} \varepsilon v \bar{n}_1 N_e d\varepsilon. \quad (1.22)$$

By integrating the product of Eq. (1) times $\epsilon N_e(\epsilon) d\epsilon$ we can obtain the equation of conservation of energy. Here the integration of the expressions (1.12) and (1.13) for Z and Z' gives a negative quantity, unlike the case of integration of \tilde{Z} and \tilde{Z}' . This corresponds to the physics of the situation: when recombination occurs the electronic system receives the liberated binding energy of the electron and impurity. On performing the integration we get the equation of conservation of energy

$$\partial \varepsilon_e / \partial t + \text{div } \mathbf{W}_e - \mathbf{E} \cdot \mathbf{j} + Q_{el} + Q_{ep} + Q_{ei} = 0. \quad (1.23)$$

Here the average energy of the n electrons is

$$\varepsilon_e = \bar{n} \bar{\varepsilon} = \int \varepsilon \bar{n} N_e d\varepsilon; \quad (1.24)$$

the electronic thermal flux is

$$\mathbf{W}_e = \frac{1}{3} \int \varepsilon v \bar{n}_1 N_e d\varepsilon; \quad (1.25)$$

the energy transferred from the electrons to the lattice on account of emission of phonons is

$$Q_{el} = \int S N_e d\varepsilon, \quad (1.26)$$

where S is to be taken from Eq. (1.3); the energy lost by the electrons in interaction with the photons is

$$Q_{ep} = (N_a + n) n (\tilde{\gamma}' - \varepsilon_0 \tilde{\beta}') - (N_d - n) (\tilde{\gamma} - \varepsilon_0 \tilde{\beta}) \quad (1.27)$$

and that lost in processes of ionization by collision and of recombination is

$$Q_{ei} = -\varepsilon_0 (N_a + n) n^2 \beta' + \varepsilon_0 (N_d - n) n \beta, \quad (1.28)$$

where the quantities

$$\tilde{\gamma} = \int_{\nu_0}^{\infty} \kappa_\nu \bar{N}_\nu h v_\nu d\Gamma_\nu, \quad (1.29)$$

$$\tilde{\gamma}' = \frac{1}{n} \int_{\nu_0}^{\infty} \kappa_\nu (1 + \bar{N}_\nu) \bar{n} h v_\nu d\Gamma_\nu, \quad (1.30)$$

differ from $\tilde{\beta}$ and $\tilde{\beta}'$ by the factor $h\nu$ in the integrands.

The first term in Q_{ep} is the energy lost by the electrons in the conduction band in photorecombination; the second term in Q_{ep} is the energy received by the electrons in the absorption of radiation. Here $(N_a + n) n \tilde{\gamma}'$ is the radiated energy, and $(N_d - n) \tilde{\gamma}$ is the absorbed energy. The terms $\varepsilon_0 \tilde{\beta}'$ and $\varepsilon_0 \tilde{\beta}$ allow for the change of the binding energy.

The conservation equations (1.17) and (1.23) are not a complete system unless supplemented by the

equation of conservation of energy for the photons, which is obtained by integrating the product of Eq. (1.16) by $h\nu \sin \vartheta d\vartheta d\varphi$:

$$\frac{\partial \varepsilon_p}{\partial t} + \nabla \cdot \mathbf{W}_p + (N_d - n) \tilde{\gamma} - (N_a + n) n \tilde{\gamma}' = 0, \quad (1.31)$$

where the energy of the photons producing ionization is

$$\varepsilon_p = \int_{\nu_0}^{\infty} h\nu \bar{N}_\nu d\Gamma_\nu, \quad (1.32)$$

the radiative energy flux is

$$\mathbf{W}_p = \int v_\nu \nabla N_\nu (\vartheta, \varphi) \sin \vartheta d\vartheta d\varphi d\Gamma_\nu, \quad (1.33)$$

and the last two terms in Eq. (1.31) represent the energy going into ionization.

The energy transferred by the electrons to the lattice either goes to increase the lattice energy, or else is carried away owing to the thermal conductivity of the lattice, which is described by the equation

$$\frac{\partial \varepsilon_l}{\partial t} + \text{div } \mathbf{W}_l - Q_{el} = 0, \quad (1.34)$$

where ε_l is the average lattice energy and \mathbf{W}_l is the thermal flux transferred by the phonons.

We shall not write out the differential equation corresponding to (1.34) for the distribution function of the phonons since, owing to the strong interaction between phonons at temperatures above the Debye temperature, the equilibrium distribution for the temperature T , determined from Eq. (1.34) and the boundary conditions, is not upset by the action of the external field and illumination. Thus the problem is the following one. For concrete physical conditions we have to determine the distribution functions of the electrons and photons, Eqs. (1.1), (1.2), and (1.16), and, knowing these distribution functions, we must use the conservation equations (1.17) and (1.23) and the transport equations (1.22) and (1.25) to describe the various nonequilibrium processes.

It is convenient, particularly for thin specimens, to separate the radiation into an internal part, which will be denoted by \bar{N}_ν as before, and an external part \bar{N}_{ext}

Then, introducing the notations

$$\beta_{ext} = \int_{\nu_0}^{\infty} x_\nu \bar{N}_{ext}^\nu v_\nu d\Gamma_\nu, \quad (1.35)$$

$$\gamma_{ext} = \int_{\nu_0}^{\infty} x_\nu \bar{N}_{ext}^\nu h\nu v_\nu d\Gamma_\nu, \quad (1.36)$$

we have instead of Eqs. (2.16) and (2.22)

$$\begin{aligned} \frac{\partial n}{\partial t} - (N_d - n) (\tilde{\beta} + \beta_{ext}) + (N_a + n) n \tilde{\beta}' - (N_d - n) n \beta \\ + (N_a + n) n^2 \tilde{\beta}' + \text{div } \frac{j}{e} = 0, \end{aligned} \quad (1.37)$$

$\frac{\partial \varepsilon_e}{\partial t} + \text{div } \mathbf{W}_e - \mathbf{E} \cdot \mathbf{j} + Q_{el} + Q_{ep} + Q_{ei} = Q_{ep}^{ext}, \quad (1.38)$
where the energy received by the electrons as a result of the irradiation is

$$Q_{ep}^{ext} = (N_d - n) (\gamma_{ext} - \varepsilon_0 \beta_{ext}). \quad (1.39)$$

Here we do not necessarily mean by β_{ext} irradiation by light; it may also be any other agency that produces ionization of the electrons, for example an electron beam, etc.

2. THE CONSERVATION EQUATIONS FOR AN EQUILIBRIUM DISTRIBUTION FUNCTION OF THE ELECTRONS WITH AN EFFECTIVE TEMPERATURE

In the presence of external fields and irradiation of the semiconductor the distribution functions of the electrons and photons will be of a nonequilibrium nature. Owing to the slowness of transfer of energy from the electrons to the lattice, as compared with the transfer of momentum, in strong electric fields the electron gas gets heated up.* The energy distribution of the electrons of this heated gas will be mainly determined by the interaction with the phonons, since the characteristic times for exchange of energy between electrons and photons are considerably longer than the time for exchange of energy with the phonons.

The first manifestation of the nonequilibrium nature of the electron distribution is a shift of the maximum of the distribution function toward higher energies. The nonequilibrium distribution function can be approximated by an equilibrium function with a certain effective electron temperature, which is determined from the equations. The kinetic coefficients found by means of this function differ very little from the coefficients found from the exact distribution function. Moreover, such an approximation facilitates the calculations and makes the meanings of the effects clearer. Therefore we shall assume that the distribution function of the electrons is an equilibrium function with the temperature T_e .

*Generally speaking, the electron gas will get heated up in a field also in the case in which the energy is transferred to the lattice in the same time as the momentum (as, for example, in ionic semiconductors, in which the mean free path l_e against energy transfer and the free path l_p against momentum transfer are of the same order). And this heating up will also manifest itself in effects for which it is of importance. For this case, however, the very method of solving the kinetic equation is, strictly speaking, not valid for arbitrary fields, because in sufficiently strong fields (for which the heating up is appreciable) the asymmetrical part of the distribution function becomes comparable with the symmetrical part.

The energy distribution of the photons is determined on one hand by the electron distribution, and on the other hand by the penetration of the external irradiation into the interior of the specimen. It is clear that far inside a massive specimen the distribution function \bar{N}_ν of the photons will correspond to the electron distribution; i.e., if the electrons have the temperature T_e , then \bar{N}_ν is the equilibrium function with this same temperature.

In a thin specimen, where the additional radiation that appears because of the recombination of the heated-up electrons gets out of the specimen without being absorbed, the photon distribution is the equilibrium distribution with the temperature T . Just this case, which is the least favorable one for the increase of the total number of free electrons under the action of a field, is the one we shall now consider. Corresponding to the above statements, our assumptions are

$$\bar{n} = \frac{n}{N_{\text{eff}}(T_e)} e^{-\varepsilon/kT_e}, \quad N_{\text{eff}}(T_e) = 2(2\pi mkT_e/h^2)^{1/2}, \quad (2.1)$$

$$\bar{N}_\nu = [e^{h\nu/kT} - 1]^{-1}. \quad (2.2)$$

In the calculation of the quantities β , Eqs. (1.18) to (1.21), we assume that the density of the electron levels is the same as for free electrons with the effective mass m , and that the cross-section $q(\varepsilon)$ for ionization by collision is given by the Thomson formula

$$q(\varepsilon) = (\pi e_1^4/\varepsilon^2)(\varepsilon/\varepsilon_0 - 1),$$

where e_1 is the effective charge of the impurity.

Then, substituting Eqs. (2.1) and (2.2) into Eqs. (1.18) to (1.21), we get for the case $kT/\varepsilon_0 \ll 1$, $kT_e/\varepsilon_0 \ll 1$:

$$\tilde{\beta} = \tilde{B} e^{-\varepsilon_0/kT}, \quad (2.3)$$

$$\tilde{\beta}' = (\tilde{B}/N_{\text{eff}}(T))(T/T_e)^{1/2}, \quad (2.4)$$

$$\beta = B(T_e/T)^{1/2} e^{-\varepsilon/kT_e}, \quad (2.5)$$

$$\beta' = (B/N_{\text{eff}}(T)) T/T_e, \quad (2.6)$$

where

$$\tilde{B} = 8\pi x_0^2 \varepsilon_0^2 \varepsilon_d kT/c^2 h^3, \quad (2.7)$$

$$B = (4\pi e_1^4/\varepsilon_0^2) \sqrt{kT/2\pi m}. \quad (2.8)$$

Here κ_ν^0 is the absorption coefficient of light of frequency ν_0 by a single atom, and ε_d is the dielectric constant ($\varepsilon_d \approx \xi^2$). To first approximation with respect to kT/ε_0 , kT_e/ε_0 the expressions (1.29), (1.30) for $\tilde{\gamma}$ give

$$\tilde{\gamma} = \varepsilon_0 \tilde{\beta} \quad \tilde{\gamma}' = \varepsilon_0 \tilde{\beta}', \quad (2.9)$$

so that according to Eq. (1.27) the energy Q_{ep} transferred to the photons is zero. In the next approximation we get

$$\tilde{\gamma} - \varepsilon_0 \tilde{\beta} = kT \tilde{\beta}, \quad \tilde{\gamma}' - \varepsilon_0 \tilde{\beta}' = kT_e \tilde{\beta}', \quad (2.10)$$

$$Q_{ep} = (N_a + n) nkT_e \tilde{\beta}' - (N_d - n) kT \tilde{\beta}. \quad (2.11)$$

The concrete form of the expression (1.39) for the energy received by the electrons in ionization by an external source depends on the spectral composition of the irradiation. For monochromatic light of frequency $\nu_{\text{ext}} = \epsilon_{\text{ext}}/h$, or for a monochromatic electron beam with energy $\epsilon_{\text{ext}} = eV_{\text{ext}}$

$$Q_{\text{ext}} = (N_d - n)(\epsilon_{\text{ext}} - \varepsilon_0) \beta_{\text{ext}}, \quad (2.12)$$

where for light

$$\beta_{\text{ext}} = (I \Delta \nu x_\nu / h \nu)_{\nu=\nu_{\text{ext}}} = n_{\text{qu}} x_{\nu_{\text{ext}}}, \quad (2.13)$$

and for an electron beam

$$\beta_{\text{ext}} = n_{\text{el}} q(\epsilon_{\text{ext}}) = (j_{\text{ext}}/e) q(\epsilon_{\text{ext}}). \quad (2.14)$$

Here n_{qu} and n_{el} are the numbers of quanta and electrons of energy ϵ_{ext} incident per unit time on one square centimeter of surface; I is the intensity of the incident light.

The total energies incident and radiated by the specimen in the form of photons are obviously given by the expressions:

$$Q'_{\text{ext}} = (N_d - n) \varepsilon_{\text{ext}} \beta_{\text{ext}}, \quad (2.15)$$

$$Q'_{ep} = (N_a + n) n \varepsilon_0 \tilde{\beta}'. \quad (2.16)$$

To evaluate the quantity B by Eq. (2.7) we must know the experimental values of the absorption coefficient and the index of refraction of the light (or the value of the dielectric constant) for the frequency ν_0 . On the other hand, these quantities can be evaluated by means of the hydrogen-like model of the impurity, if one knows the depth ϵ_0 of the local level, the effective mass of the electron, and the resonance absorption cross-section of hydrogen.

If we regard the impurity as a Coulomb center placed in a medium of dielectric constant ε_d , then the depth ϵ_0 of the impurity level and the "radius" r_0 of the impurity are determined from the relations

$$\varepsilon_R/\varepsilon_0 = \varepsilon_d^2 m_e/m, \quad r_0 = r_H \varepsilon_d m_e/m, \quad (2.17)$$

where ε_R is the Rydberg energy, r_H is the Bohr radius, and m_e and m are the true and effective masses of the electron. The actual depth of the impurity level does not, however, correspond to the dielectric constant ε_d in the way shown in Eq. (2.17) for all semiconductors. Therefore in estimating the impurity radius r_0 it is better to use the expression

$$r_0 = r_H (\varepsilon_R m_e / \varepsilon_0 m)^{1/2}, \quad (2.18)$$

which is obtained by eliminating ε_d from Eq. (2.17).

The photoionization cross-section κ_ν is proportional both to r_0^2 and to the electromagnetic interaction constant in the medium, i.e., $\kappa_{\nu_0} \sim e_1^2/h\nu_\nu = e^2/hc\epsilon_d^{1/2}$, where $e_1 = ee_d^{1/2}$ is the effective charge. Eliminating ϵ_d , we get*

$$\kappa_{\nu_0} = \kappa_H (\varepsilon_R/\varepsilon_0)^{3/4} (m_e/m)^{5/4}. \quad (2.19)$$

In just the same way, eliminating the effective charge e_1 from the expression (2.8) for B , we get:

$$B = (4\pi e^4/\varepsilon_0 \varepsilon_R) \sqrt{kT/2\pi m_e} (m_e/m)^{5/4}. \quad (2.20)$$

If we introduce the maximum value of the collision ionization for hydrogen, which according to the Thomson formula is reached at energy $2\varepsilon_R$ and is given by $q_{\max}^H = e^4/4\varepsilon_R^2 \sim 10^{-16} \text{ cm}^{-2}$, (which agrees with the experimental value), then we get the following expressions for B and \tilde{B} :

$$\tilde{B} = (8\pi \kappa_H k T \varepsilon_d \varepsilon_0^2/c^2 h^3) (\varepsilon_R/\varepsilon_0)^{3/4} (m_e/m)^{5/4}, \quad (2.21)$$

$$B = 16 q_{\max}^H (\varepsilon_R/\varepsilon_0) \sqrt{kT/2\pi m_e} (m_e/m)^{5/4}. \quad (2.22)$$

If for example, we set $\varepsilon_0 = 0.15 \text{ ev}$, $m = m_e$ and use the data for hydrogen, $\varepsilon_R = 13.6$, $q_{\max}^H = 10^{-16} \text{ cm}^{-2}$, $\kappa_H = 6.3 \times 10^{-18} \text{ cm}^2$, we find that for $T = 300^\circ$ we have $B \approx 4 \times 10^{-9}$, $\tilde{B} \approx 5 \times 10^5$. The value $\varepsilon_0 = 0.15 \text{ ev}$ corresponds to a dielectric constant $\epsilon_d = (\varepsilon_R/\varepsilon_0)^{1/2} \sim 9.5$.

The characteristic times for photoelectric and triple-collision recombination — lifetime of the electron in the free state — are connected with the quantities $\tilde{\tau}$ and τ in the following way:

$$\tilde{\tau} = 1/(N_a + n_0) \tilde{B} = N_{\text{eff}}(T)/(N_a + n_0) \tilde{B}, \quad (2.23)$$

$$\tau' = 1/(N_a + n_0) n_0 \beta' = N_{\text{eff}}(T)/(N_a + n_0) n_0 B, \quad (2.24)$$

where n_0 is the equilibrium number of free electrons.

For the distribution function (2.1) the expression (1.25) for the energy transferred to the lattice by the electrons takes the form

$$Q_{el} = An \frac{T_e - T}{T} \left(\frac{T_e}{T}\right)^{5/2}, \quad (2.25)$$

where we have used the notation

$$A = - \left(\frac{T}{T_e}\right)^{1/2} \frac{2mv_s^2}{n} \int_0^\infty \frac{evN_e}{l} \frac{\partial n}{\partial e} d\varepsilon. \quad (2.26)$$

If we assume that l does not depend on the energy, which is correct for atomic semiconductors, then

$$A = (16mv_s^2/l) (kT/2\pi m)^{5/2}. \quad (2.27)$$

For example, with $m = m_e$, $v_s \sim 10^5 \text{ cm/sec}$,

*The same result is also obtained on the basis of the Kramers formula for the photoionization cross-section of hydrogen.

$T = 300^\circ$, and $l \sim 10^{-6} \text{ cm}$, we have the numerical value $A \sim 5 \times 10^{-4}$.

Let us now assume that there exists a type of nonradiative recombination in which the energy is in some way transferred directly to the lattice. This means that there are in the lattice excitations other than the photons of blackbody radiation that are capable of ionizing the impurity. Of whatever type these excitations may be — boson or fermion — for $kT/\epsilon_0 \ll 1$ the probabilities of ionization through the action of these excitations and of recombination with appearance of excitation will be of the forms (2.3) and (2.4), but with an unknown coefficient B , which we denote by B_b . The value of B_b can be estimated if we know the experimental value of the lifetime for radiationless recombination, by the relation (2.23):

$$\beta'_b = N_{\text{eff}}(T)/(N_a + n_0) B_b. \quad (2.28)$$

The next question is that of the rate at which equilibrium is established for the excitations that produce the nonradiative transitions, i.e., the question of the rate at which the transfer of the excitation energy by the phonons occurs. If the time for transport of the excitation energy by the phonons is smaller than the time for recombination with production of these excitations, then the temperature of the excitations will be the same as the lattice temperature. This natural assumption corresponds to the most favorable case for the increase of the number of free electrons under the action of the external factors. Then, just as for the interaction with the photons that get away without being absorbed, we have

$$\beta_b = B_b e^{-\varepsilon_0/kT}, \quad \beta'_b = (B_b/N_{\text{eff}}(T)) (T/T_e)^{5/2}. \quad (2.29)$$

In Eq. (1.37) one adds the term $-(N_d - n)\beta_b + (N_a + n)n\beta'_b$. The energy Q_{el} transferred in unit time from the electrons to the lattice, which we have to put into Eq. (1.38) instead of the expression (2.25), is given, in analogy with Eq. (2.11), by

$$Q_{el} = An \frac{T_e - T}{T} \left(\frac{T_e}{T}\right)^{5/2} \quad (2.30)$$

$$+ (N_a + n) nkT_e \beta'_b - (N_d - n) kT \beta_b.$$

In the equation (1.34) for the transfer of energy by phonons, however, we must use instead of this value the total energy received by the lattice

$$Q'_{el} = An \frac{T_e - T}{T} \left(\frac{T_e}{T}\right)^{5/2} + (N_a + n) n \varepsilon_0 \beta'_b - (N_d - n) \varepsilon_0 \beta_b \quad (2.31)$$

[cf. Eqs. (1.31) and (2.16)].

Thus the conservation equations for the electrons have the form:

$$\frac{\partial n}{\partial t} + \text{div } \frac{\mathbf{j}}{e} - (N_d - n)(\tilde{\beta} + \beta_b + n\beta + \beta_{\text{ext}}) \\ + (N_a + n)(\tilde{\beta}' + \beta'_b + n\beta') = 0, \quad (2.32)$$

$$\frac{\partial \epsilon_e}{\partial t} + \text{div } \mathbf{W}_e - \mathbf{E} \cdot \mathbf{j} + An \frac{T_e - T}{T} \left(\frac{T_e}{T} \right)^{1/2} \\ - (N_d - n)[kT\tilde{\beta} + kT\beta_b - n\epsilon_0\beta + (\epsilon_{\text{ext}} - \epsilon_0)\beta_{\text{ext}}] \quad (2.33) \\ + (N_a + n)[kT_e\tilde{\beta}' + kT_e\beta'_b - n_0\beta'] = 0.$$

By solving this system of two equations together with the transport equations one can uniquely de-

termine T_e and the non-equilibrium values of n for various processes.

¹ V. P. Shabanskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 657 (1956), Soviet Phys. JETP **4**, 497 (1957).

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20

PHOTOPRODUCTION OF ELECTRON AND μ -MESON PAIRS ON NUCLEONS

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Production of electron or μ -meson pairs on nucleons by high-energy gamma-quanta is examined. It is shown under what conditions the cross sections for these processes can be expressed in terms of the electromagnetic form factors of the free nucleon.

1. An investigation of the cross sections of the radiation processes involving production of electrons and μ mesons from nucleons at large angles, makes it possible to judge the electromagnetic properties of the nucleon (form factors)¹ or, if these are known from other experiments, to determine the limits of validity of modern quantum electrodynamics.

It was indicated in reference 2 that the electromagnetic form factor of the free nucleon can be written in the form ($\hbar = c = 1$):

$$\Gamma_\mu(q^2) = a(q^2)\gamma_\mu + i \frac{b(q^2)}{2M} \frac{1}{2} (\gamma_u \hat{q} - \hat{q} \gamma_u),$$

where q is the momentum transferred by the electromagnetic field to the nucleon ($q^2 = q^2 - q_0^2$; $\hat{q} = q_\nu \gamma_\nu$), M is the nucleon mass, and $a(q^2)$ and $b(q^2)$ are real functions. When q^2 goes to zero, $a(q^2)$ goes to 1 or 0 for protons and neutrons respectively, and $b(q^2)$ goes to the anomalous magnetic moment μ_0 (in nuclear magnetons). Substantial deviations from these limiting values of $a(q^2)$ and $b(q^2)$ are expected when $q \gtrsim \mu$, where μ is the mass of the pion. We shall therefore be interested in recoils $q \gtrsim \mu$, i.e., as will be seen later on, in sufficiently large angles. Furthermore, to determine the form factor $b(q^2)$ it is necessary

to consider large recoils, since $b(q^2)$ enters into the formula together with the factor $q(M)$ (1).

2. We consider the production of electron or muon pairs on nucleons by gamma quanta. Graphs corresponding to this process are divided into two groups. In the first group (1a) only one photon line goes to the nucleon line. Along this photon line the nucleon acquires a recoil momentum $q = k - p_+ - p_-$ (p_+ and p_- are the momenta of the pair components and k is the momentum of the incident quantum). These are graphs having the electromagnetic vertex part of the free nucleon, and can be expressed in terms of functions a and b of formula (1) by inserting Γ_μ from formula (1) when writing the matrix elements corresponding to graphs (1a). In the second group, two photon lines go to the nucleon portion of the graph (Figs. 1b and 1c indicate the general form of such a graph and two simplest graphs for a nucleon interacting only with the electromagnetic field). It is impossible to account for the meson interactions for these by introducing simple form factors of type (1).

As to the matrix elements of such graphs, it can be assumed that they are of the same order of magnitude (or less) as the matrix elements of the simplest graphs (1c), corresponding to the