THE NATURE OF PARTICLES THAT CARRY AWAY MOST OF THE ENERGY IN NUCLEAR COLLISIONS AT MODERATE ENERGIES

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The electron-nuclear shower transition curve in a dense absorber is computed by assuming the existence of an energetically-preferred particle in each step of the nuclear-cascade process. The fraction of disintegrating particles in the generating component is estimated by comparing this curve with absorption in air.

1. INTRODUCTION

RECENTLY obtained experimental data indicate that in high-energy nuclear collisions that lead to multiple meson production a considerable portion of the energy of the incident particle is apparently concentrated in one of the emerging particles. This follows both from the individual observations (mostly the Schein stars[1] and the HBD star[2]) and from the behavior of the nuclear-active component of cosmic rays in the atmosphere[3,4] and the structure of extensive atmospheric showers.[5,6]

It is clear that the existence of such a preferred particle is responsible for the most important features of the nuclear-cascade process and affects substantially the comparison of experimental data with various theories concerning the elementary act.

An important problem, which can still be considered unsolved, is connected with the very nature of this particle. Thus, for example, it follows from the Heisenberg theory[7] that, at least at large impact parameters, the nucleon transfers only an insignificant portion of its energy to the meson field. Consequently, within the framework of this theory, the energetically preferred particle (abbreviated e.p.p.) must of necessity be a nucleon.

On the contrary, the Landau theory[8] leads only to a concentration of a large fraction of the system energy in a certain region that contains a small number of particles (on the order of unity). But in principle nothing contradicts the possibility of the being a meson in some cases and a nucleon in others. Calculations made by Belen'kii[9] lead only to the conclusion that the energy is concentrated somewhat more on the nucleons than on the pions.

On the other hand, in a previous work by one of us[10] it was shown that if the existence of a certain nuclear structure is admitted in the sense of the Bhabha theory,[11] a sharp difference in the distribution of the mass density in the nucleon and in the pion can cause the appearance of an e.p.p.

If such a collision model is analyzed on the basis of the Fermi theory,[12] one obtains a finite probability that the e.p.p. is a pion. If, the Heisenberg theory is applied to this model, this probability turns out to be even greater.

From the experimental point of view the situation is still unclear, for at large energies there is no direct and reliable method for distinguishing mesons from nucleons and hyperons. The presently-available information on the nature of the e.p.p. has been obtained indirectly and is in part contradictory.

Thus, starting with a comparison of the energy flux of the primary protons with the energy flux of the hard component in the atmosphere[13] and underground,[14] the authors of the works cited have concluded that the probability of concentration of a large fraction of the energy on the pion (or on some other particle, which disintegrates directly or indirectly into a pion or muon) is quite small (at any rate, less than follows, for example, from the Fermi-Landau theory). We shall deal with this problem later.

On the other hand, a study of the nuclear-active particles of moderate energies (i.e., particles that generate local electron-nuclear showers, usually recorded in experiments with counters) has disclosed[15-19] a clearly pronounced compensation effect. If the transition effect of the density[20-21] is excluded, this indicates directly the spontaneous disintegration of a certain fraction of the generated particles into nuclearly-passive ones. Consequently, these particles cannot be hyperons, for all known hyperons decay into nuclearly-active particles (p, π, Λ^0). By comparing the ranges for absorption of the generated particles in air and in dense substances, the authors of references 15 to 17 estimated the fraction of such decaying particles at approximately $\frac{1}{3}$.

In the interpretation of experiments of this type, however, it is necessary to bear in mind that there is a principal difference between absorption measurements in air and in dense substances. In air we measure the absorption proper of individual nuclear-active particles, whereas in a dense substance all the penetrating secondary particles, including the nuclearly-passive ones, make a certain contribution to the operation of the detector. Thus, the "absorption" of the observed effect will depend also on the multiplicity of the secondary particles, on their effective cross sections, etc.

It is indeed the purpose of this article to explain the influence of these factors on the course of the transition curve of electron-nuclear showers in dense substances. The results obtained are compared with the absorption in air and can contribute to the solution of the problem concerning the existence of unstable particle along the e.p.p.

2. TRANSITION CURVE

Let us consider a detector sensitive to a penetrating component of electron-nuclear showers, placed under a dense absorber of thickness l.*

We assume for the sake of simplicity that the detector responds with equal efficiency to any shower capable of producing at least Q_0 penetrating particles at the counter level.†

We shall schematize the elementary interaction act and the cascade process in the absorber in the following manner:

(a) When a nuclear-active particle of energy E interacts with the nucleus of the absorber, a relatively small portion of the energy is transferred to the meson field. This portion goes into production of pions, whose number depends on the energy in accordance with the following power law‡

$$n = AE^{g}. \tag{1}$$

(b) The major fraction β of the energy E is carried away by a single particle, capable of experiencing further interaction in the same absorber. We shall consider β constant in the first approximation, without losing sight of the fact that actually β can have a certain spread, which we intend to take into account in a further refinement of the model. Owing to the small dimensions of the dense absorber, the nature of the e.p.p. is of no importance. It is merely important that its effective cross section be constant and equal to the geometric cross section.

(c) The energy $(1-\beta)E$ transferred to the mesons is sufficiently small and the multiplicity of the mesons is sufficiently large, so that none of them is capable of generating new penetrating particles. In this sense they are nuclearly passive, although their absorption and geometric cross sections are the same. It must borne in mind, however, that it is these particles that cause detector operation. Let Q be the number of such particles, reaching the counter level. Assume that an "event" is registered if and only if $Q \geq Q_0$.

(d) Further generation of particles under the influence of the e.p.p. ceases as soon as the energy of the latter drops below a certain threshold value E_c.

(e) Ionization losses of the charged particles can be neglected.

(f) The problem is solved in the unidimensional approximation.

Consider now an incident particle of energy E, which experiences i collisions in the absorber. The number of events of this kind is given,

$$d\nu(l, E, i) = F(E)\,dE \cdot e^{-l}\,l^{i}/i!, \tag{2}$$

where $F(E)dE$ is the energy spectrum of the incident nuclear-active particle; for $E > E_c$ we assume that this spectrum obeys the power law form

$$F(E)dE \sim E^{-s}dE/E. \tag{3}$$

The total number of coincidences $\nu(l)$ registered per unit time at a depth l is obtained from (2) by summation and integration over all values of i and E, compatible with conditions (c) and (d). The corresponding limits of summation and integration can be derived in the following manner.

Let E_i be the energy of the e.p.p. after the i-th collision. Condition (d) makes it possible to determine a certain critical number of collisions i_c, beyond which generation of nuclear showers ceases (i.e., the e.p.p. is absorbed). This critical number is obtained from

$$\beta^{i_c}E = E_c. \tag{4}$$

On the other hand, it follows from (a), (b), and (c) that $Q = Q(E, l, i)$. This equation admits in principle of a solution $i = i(Q, l, E)$. For a given energy E it is possible to determine the

*The unit of thickness is taken to be the interaction length.

†The generalization to the case of another form of detector-operation probability presents no difficulties in principle.

‡A system of units is used in which $\hbar = c = M = 1$, M being the mass of the nucleon.

minimum number of collisions i_0 necessary to produce, at a depth l, those Q_0 particles needed to trigger the equipment:

$$i_0 = i(Q_0, l, E). \tag{5}$$

It follows hence that the incident nuclear-active particle is registered by the instrument only if

$$i_0(Q_0, l, E) \leqslant i_c(\beta, E). \tag{6}$$

Owing to the discrete character of the quantities i_0 and i_c, the value of i' satisfying the equation

$$i_0(Q_0, l, E) = i_c(\beta, E), \tag{7}$$

corresponds to a certain finite interval of values of E.

Let E′ be the minimum value of E in that interval [i.e., the minimum value of E commensurate with the solution of Eq. (7)]. It follows then from (2), (5), and (6) that

$$\nu(l) = \int_{E'}^{\infty} F(E)\,dE \sum_{i_0}^{\infty} e^{-l} l^i / i!. \tag{8}$$

This indeed is the sought equation for the transition curve.

For a specific calculation of this curve we need merely write down the explicit dependence of Q_0 on E and i. We shall use for this purpose the Rozental' cascade equations,[22] simplified for the conditions of our special model.

Let $P_i(l)$ be the number of nuclear active particles of the i-th generation at a depth l and let $M_i(l)$ be the number of "passive" mesons [in the sense of condition (c)] of the same generation. The cascade equations assume under these conditions the following simple form

$$\begin{aligned} dP_i(l)/dl &= -P_i(l) + P_{i-1}(l), \quad P_0(l) = e^{-l}, \\ dM_i(l)/dl &= -M_i(l) + AE_{i-1}^{g} P_{i-1}(l). \end{aligned} \tag{9}$$

This system can be readily integrated and yields

$$\begin{aligned} P_i(l) &= e^{-l} l^i / i!, \\ M_i(l) &= A(E/\beta)^g e^{-l} (l\beta^g)^i / i!, \qquad i \geqslant 1. \end{aligned} \tag{10}$$

From this it is easy to obtain the summation over i

$$Q(i, E, l) = A\left(\frac{E}{\beta}\right)^g e^{-l} \sum_{r=1}^{i} (l\beta^g)^r / r!. \tag{11}$$

For further calculation, it is convenient to introduce a new variable z,

$$z = (E_c/E)^g. \tag{12}$$

From (11) and (5) we also obtain

$$z = \frac{A}{Q_0}\left(\frac{E_c}{\beta}\right)^g e^{-l} \sum_{r=1}^{i_0} (l\beta^g)^r / r!. \tag{13}$$

Using the spectrum (3), it is easy to transform (8) into

$$\nu(l) = \int_0^{z'(l)} \theta z^\theta \frac{dz}{z} - \int_0^{z'(l)} \theta z^\theta \frac{dz}{z} \sum_{i=0}^{i_0-1} \frac{l^i e^{-l}}{i!}, \tag{14}$$

where $\theta = s/g$, and z' is obtained from (4) by replacing E with E′.

Taking into account the discrete (jump-like) change of i_0, the second integral in (14) can be expanded into a sum of integrals; after elementary calculations we obtain

$$\nu(l) = z'^\theta - e^{-l} \sum_{k=1}^{k=i'} (z_k^\theta - z_{k-1}^\theta) \sum_{i=1}^{k-1} \frac{l^i}{i!}, \tag{15}$$

where

$$z_0 = 0, \quad z_{i'} = z'.$$

Using (15), we have calculated the curves $\nu(l)$ for certain pairs of sensible values of β and g. It turned out that $\nu(l)$ has a sharply pronounced maximum near $l = 1$ or 2. The position of this maximum and the course of the $\nu(l)$ curve before the maximum depend little on β, but depend strongly on g. Beyond the maximum, the transition curve can be approximated by an exponential curve, the exponent is practically independent of g, and depends only on β. This can be seen also from the asymptotic behavior of (15), which for $l \gg 1$ reduces practically to

$$\nu(l) \approx [z'(l)]^\theta. \tag{16}$$

On the other hand, for $l \gg 1$ we have $z' \approx \beta^{i'g}$ and since under these conditions $i' \approx l$, then

$$\nu(l) \approx \exp\{-ls\ln(1/\beta)\} \tag{17}$$

and is independent of g.

3. COMPARISON WITH ABSORPTION IN THE ATMOSPHERE

Bearing in mind the positive results of the compensation experiments,[15-19] we shall assume that there exists a finite probability q that the e.p.p. is a charged pion (or any other meson that yields only nuclearly-passive particles upon disintegration). The cascade equations can be written in this case separately for the nucleons and for the nuclear-active mesons

$$\begin{aligned} dN_i^*(l)/dl &= -N_i^*(l) + (1-q)[N_{i-1}^*(l) + M_{i-1}^*(l)], \\ N_0(l) &= e^{-l}, \\ dM_i^*(l)/dl &= -M_i^*(l)(1 + k/l\beta^i) + q[N_{i-1}^*(l) + M_{i-1}^*(l)], \\ M_0(l) &= 0, \end{aligned} \tag{18}$$

where $k = 460/E$ for pions.

This system is readily integrated and, for energies that are not too high (i.e., in the approximation $k \gg 1$), yields the total number of particles of the i-th generation*

$$P_i(l) = [l(1-q)]^i e^{-l}/i!. \tag{19}$$

The total number of generating particles $P(l)$ is given by an expression similar to (8):

$$P(l) = \int_{E_c}^{\infty} F(E)\,dE \sum_{i=0}^{i=i_c(E)} [l(1-q)]^i e^{-l}/i!, \tag{20}$$

where $i_c(E)$ is determined from (4).

Equation (20), like Eq. (8), can be integrated by intervals. We get

$$P(l) \sim e^{-l(1-\Delta)}, \quad \Delta = (1-q)\beta^s. \tag{21}$$

As was to be expected, this result agrees with the known relation of Zatsepin,[23] with accuracy to a factor $(1-q)$. The same result can be derived also from simpler considerations, assuming only that all pions have a chance to disintegrate prior to interaction with the nucleus. From the considerations given above (see remark*) it follows that not more than 10% of the mesons still have a chance to interact with the nuclei in the considered range.

We can now proceed to compare the two obtained results with the experimental data. From the large number of experiments it is known (see, for example, reference 24) that $\Delta \approx 0.5$ in air. On the other hand, reference 18 reports the same value $\lambda_A \approx 3.5$ for the absorption ranges in lead, iron, and carbon. Reference 16 reports $\lambda_a \approx 2.6$ for water. Using the value $s = 1.5$ (reference 25), we obtain, with the aid of (17), $\beta \sim 0.7$ to 0.8. Inserting this value into (21), we get $q \approx 0.25$ to 0.30.

4. CONCLUSIONS

If the schematized model of interaction at moderate energies employed in this work is not too different from actuality, then the bulk of experimental data indicates apparently that, although most e.p.p. are nucleons, there is nevertheless a noticeable probability of the e.p.p. being an unstable particle other than a hyperon. Such particles can be $\pi^{\pm}$, K_{μ}, and K_e mesons, but not θ or τ mesons. In connection with this, it is interesting to note recent observation[26] of an electron-nuclear shower in which both the incident and emerging e.p.p. were pions.

In the present state of the problem, it is quite desirable both to refine the theoretical model and to obtain experimental data that refine the details of the phenomenon. It is important, for example, to establish whether the cited compensation experiments contain effects that are similar to the transition effect of density, and to establish the role of these effects, which can increase the apparent value of β, and consequently also of q. On the other hand it must be emphasized, that, within the framework of our model, comparison of the flux of high-energy μ mesons with the flux of primary protons does not give a negligible value of q. Using the relation, introduced by Zatsepin et al.,[14] for the flux of μ mesons with $E > 10^{13}$ arising from the first collision of the primary protons, but using merely the spectrum of the primary protons multiplied by q for the creation spectrum of the pions, we obtain $q \sim 0.14$.

Although the independence of q of the energy does not follow of necessity from the different theories of the elementary act, let us note merely that the discrepancy between the two values (0.14 and 0.25) obtained for q can be ascribed, at least partially, to the copious production, at high energies, of mesons that do not disintegrate directly or indirectly into muons or pions.

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*The solution of the system of equations (18) actually has the form

$$P_i(l) = e^{-l}\, l^i\, \Phi(i) \prod_{j=1}^{i-1} \Phi(j)/(j+1),$$

where

$$\Phi(j) = 1 - q + qj\beta^j/(j\beta^j + k).$$

$\Phi(j)$ has a maximum at $j \ln\beta = -1$; for $\beta \approx 0.8$ this corresponds to values of $j \approx 4$ and $j\beta^j \approx 1.6$. Introducing for E its value averaged over the spectrum (assuming $E_c \approx 10$, i.e., $\bar{E} \approx 30$), we find that the value of q, approximately calculated from (19), is ~ 10% below the actual value.

[1] Glasser, Haskin, Schein, and Lord, Phys. Rev. **99**, 1555 (1955).

[2] Hopper, Bismas, and Darby, Phys. Rev. **80**, 970 (1950).

[3] N. L. Grigorov and N. V. Murzin, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 21 (1953).

[4] N. L. Grigorov, Dokl. Akad. Nauk SSSR **94**, 835 (1954).

[5] Dobrotin, Zatsepin, Nikol'skii, Sarycheva, and Khristiansen, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 666 (1955) [Columbia Techn. Transl. **19**, 605 (1955)].

[6] N. A. Dobrotin, Paper delivered at International Cosmic-Rays Conference, Budapest, 1956.

[7] W. Heisenberg, Z. Physik, **126**, 569 (1949).
[8] L. D. Landau, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 51 (1953).
[9] S. Z. Belen'skii, Dokl. Akad. Nauk SSSR **99**, 523 (1954).
[10] E. Friedländer, Hung. Phys. Acta **5**, 237 (1956).
[11] H. Bhabha, Proc. Roy. Soc. **A219**, 293 (1953).
[12] E. Fermi, Progr. Theoret. Phys. **5**, 570 (1950).
[13] Vernov, Grigorov, Zatsepin, and Chudakov, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 493 (1955) [Columbia Techn. Transl. **19**, 445 (1955)].
[14] Guzhavina, Guzhavin, and Zatsepin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 819 (1956), Soviet Phys. JETP **4**, 690 (1957).
[15] Veksler, Kurnosova, and Liubimov, J. Exptl. Theoret. Phys. (U.S.S.R.) **17**, 1026 (1947).
[16] Azimov, Vishnevskii, and Khilko, Dokl. Akad. Nauk SSSR, **76**, 231 (1951).
[17] S. Azimov and V. F. Vishnevskii, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 88 (1953).
[18] Azimov, Dobrotin, Liubimov, and Ryzhkova, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 80 (1953).
[19] Friedländer, Jussim, and Mayer, Studii si Cer. de Fizica, **6**, 229 (1955).
[20] M. I. Podgoretskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 1090 (1951).
[21] Podgoretskii, Barchikov, and Rakitin, Dokl. Akad. Nauk SSSR **73**, 685 (1950).
[22] I. L. Rozental', J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 440 (1952).
[23] G. T. Zatsepin, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 1104 (1949).
[24] G. B. Zhdanov, Usp. Fiz. Nauk **54**, 435 (1954).
[25] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[26] Birger, Guseva, Kotel'nikova, Maksimenko, Riabikov, Slavatinskii, and Stashkov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 982 (1956), Soviet Phys. JETP **4**, 836 (1957).

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POSSIBILITY OF DETERMINING THE INTERACTION CONSTANTS FROM EXPERIMENTS ON $K_{\mu 3}$ DECAYS

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Expressions are derived for the spectra and polarization of μ mesons created in $K_{\mu 3}$ decays for fixed pion energies and arbitrary complex interaction functions. It is shown that, accurate to within an unimportant phase shift, it is possible to determine all interaction functions from measurements of the spectra and three polarizations, with the exception of the second vector function. The latter cannot be separated in the above experiments from the first-vector and scalar interaction functions. The presence of tensor interaction can be ascertained from measurements of the spectrum and polarizations of μ mesons at an energy close to maximum.

EXPRESSIONS for the spectrum of μ mesons, as well as for their polarization, were obtained by various authors.[1-11] However, no calculations were made for the case of arbitrary complex constants,* nor were they analyzed from the point of view of the possibility of determining all the constants experimentally. If the interaction is assumed to be local, the most general expression for the transition matrix element is of the form

$$\langle \bar{\psi}_\mu \Big\{ (g_S + i\gamma_5 g_S') + (g_V + i\gamma_5 g_V')\gamma_4 + {}^1/{}_2 M^{-1} (g_T + i\gamma_5 g_T') \times (\gamma_4 \hat{p}_\pi - \hat{p}_\pi \gamma_4) \Big\} \psi_\nu \rangle (2M^{3/2} E_\pi^{1/2})^{-1}. \quad (1)$$

Here M is the mass of the K meson, and p_π

*In fact, the interaction constants may depend on the pion energy, and shall henceforth be referred to as "functions."