will depend on the structures of the nuclei studied. One can hope information on the structures of various nuclei can be obtained in this manner.

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SEMI-PHENOMENOLOGICAL THEORY OF NUCLEON-NUCLEON INTERACTION

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Results of the calculation of nucleon-nucleon interaction potentials are presented. The calculations were made within the framework of the semi-phenomenological isobaric theory. The computed deuteron parameters and scattering of low-energy nucleons agree satisfactorily with experiment. An unsuccessful attempt is made to employ the computed potentials for a description of the scattering of high energy nucleons ($\sim$ 100 Mev).

1. INTRODUCTION

TAMM, Gol'fand, and Fainberg[1] have proposed a semi-phenomenological theory of nucleon-meson interaction where, in addition to the ordinary nucleon state with mechanical and isotopic spins $^1/_2$, there is consideration of their excited isobaric state with mechanical and isotopic spins $^3/_2$. This isobaric state, which is introduced purely phenomenologically, permits us to describe the behavior of the cross sections for the scattering[1] and photoproduction[2] of π mesons on nucleons in a fairly large meson energy range up to 400 Mev.

The semi-phenomenological theory of Ref. 1 involves four free parameters: the nucleon excitation energy Δ, the pseudovector meson-nucleon coupling constant g/μ (where μ is the mass of the π meson), the pseudoscalar coupling constant $g' = sg$ (where s is a number) and the constant g_1 which determines the probability of a nucleonic transition from its unexcited state to the isobaric state or vice versa. The values of these parameters were chosen to provide the best possible fit of experimental data on meson-nucleon scattering and meson photoproduction. The success of this procedure induced us to apply the semi-phenomenological isobaric theory to the problem of nuclear forces and specifically to the deuteron and nucleon-nucleon scattering.

Our calculation showed that inclusion of isobaric states greatly changes the results of the ordinary theory of nuclear forces, in which isobars are neglected. For example, when isobars are included the potential energy of nucleons in ^1S and ^3S states increases proportionally to $1/r^3$ for $r \to 0$, whereas when isobars are not taken into account the potential energy in the ^1S state (unlike ^3S) has, as we know, only the simple pole $1/r$.

It follows that the isobaric theory leads to a nuclear force potential with a high-order singularity, so that, just as in the ordinary theory, the potential must be cut off at small distances. We shall assume that at small distances $r \le r_0$ the potential represents an infinitely great repulsion; we thus are using the potential model with an impenetrable solid nucleus. In addition to the four parameters already mentioned, the theory will thus include, as a fifth parameter, the cutoff radius r_0. The parameters Δ, g, s, and g_1 are determined from data on meson-nucleon scattering[1] and photoproduction,[2] and are assumed to be fixed. The single variable parameter r_0 can be determined provided that we obtain, on the one hand, the correct deuteron binding energy (triplet state) and, on the other hand, the correct value for the deuteron singlet level. Generally speaking, these two conditions can result, and in the ordinary theory do result (see Ref. 3, for example) in a different cutoff radius r_0 for the triplet and singlet states. A fundamental feature of the isobaric theory is that the triplet and singlet cutoff radii are very precisely identical, so that the theory actually includes only one additional parameter r_0. The present article presents the results of the application of this semi-phenomenological theory to the deuteron and to nucleon-nucleon scattering.

The general wave equation of a system of two nucleons is given in Sec. 2. Our initial equations could be written in relativistic form but would make the problem too complicated. In the present work we have used the so-called adiabatic approximation, which is essentially as follows. In first approximation the nucleons are regarded as infinitely heavy and fixed at the points $\mathbf{r}_1$ and $\mathbf{r}_2$; their static interaction potential $V(\mathbf{r})$ is then determined, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, and this potential is inserted in the Schrödinger equation. This is the simplest approximation and is fully suitable for the nonrelativistic deuteron problem of interest.

In Secs. 3 and 4 the nucleon interaction potentials are found in adiabatic approximation for states of two-nucleon systems with different isotopic and mechanical spins. Finally, Sec. 5 gives the results of a computation of deuteron parameters and nucleon scattering at low energies. Whereas these results can be regarded as in satisfactory agreement with experiment, the theory was unsuccessfully applied to a description of nucleon scattering at high energies. This is shown by the 100-Mev nucleon scattering phases which are given in Sec. 5.

2. WAVE EQUATION OF A TWO-NUCLEON SYSTEM

We take the interaction Lagrangian in the form[1] ($\hbar = c = 1$)

$$L = L_1 + L_2,$$

$$L_1 = \frac{g}{\mu}\,\bar{\psi}\gamma_5\gamma_\mu\tau\,\frac{\partial\varphi}{\partial x_\mu}\,\psi + igs\bar{\psi}\gamma_5\tau\varphi\psi,$$

$$L_2 = \frac{g_1}{\mu}\left(\bar{\psi}\mathbf{S}\,\frac{\partial\varphi}{\partial x_\mu}\,B_\mu + \bar{B}_\mu\mathbf{S}^+\,\frac{\partial\varphi}{\partial x_\mu}\,\psi\right). \tag{1}$$

Here φ is the wave function of the meson field, ψ is the nucleon field and B_μ is a spin vector which describes the excited nucleon field (the isobaric field).*

The six-row matrix $\mathbf{S}$ in isotopic space is determined by the requirement of isotopic invariance of the Lagrangian and is analogous to the ordinary isotopic matrices τ. (The explicit form of $\mathbf{S}$ is given in Ref. 1, Appendix A.4).

We shall now write the equation of motion for the wave function $v(\gamma_1\gamma_2)$ of two nucleons with positive energy. Here γ_i denotes the set of quantum numbers which characterize the state of the i-th nucleon (its momentum, total mechanical and isotopic spins and their projections on the z axis). In addition to the two-nucleon state we shall consider only states in which an additional single meson is present. The Schrödinger equation is then[4] (summing over repeated subscripts)

$$(W - E_{\gamma_1\gamma_2})\,v(\gamma_2,\gamma_2) = \frac{\langle\gamma_1\gamma_2 0\,|\,\gamma_1'\gamma_2 1\rangle\langle\gamma_1'\gamma_2 1\,|\,\gamma_1'\gamma_2'' 0\rangle}{W - E_{\gamma_1'\gamma_2}^{1}}\,v(\gamma_1',\gamma_2'')$$

$$+ \frac{\langle\gamma_1\gamma_2 0\,|\,\gamma_1\gamma_2' 1\rangle\langle\gamma_1\gamma_2' 1\,|\,\gamma_1''\gamma_2' 0\rangle}{W - E_{\gamma_1\gamma_2}^{1}}\,v(\gamma_1'',\gamma_2'). \tag{2}$$

Here W is the energy of the system, $E_{\gamma_1\gamma_2}$ is the energy of two free nucleons, $E_{\gamma_1\gamma_2 1}$ is the energy of the state consisting of two free nucleons and a single meson, the quantities $\langle\gamma_1\gamma_2 0\,|\,\gamma_1'\gamma_2' 1\rangle$ are the matrix elements of transitions from a mesonless state to a state with a meson; $\langle\gamma_1\gamma_2 1\,|\,\gamma_1'\gamma_2' 0\rangle$ represents a transition with absorption of the meson. The matrix elements are easily obtained explicitly by means of the Lagrangian (1).

We now pass in Eq. 2 to the approximation whereby the nucleon mass M and the isobar mass $M + \Delta$ are regarded as very large ($M \gg \mu$), and we also assume $W - 2M = 0$ in the denominators of the right member of the equation. The latter approximation provides considerable simplification

*Strictly speaking, the wave function of the isobar is a combination of the spin vector B_μ^σ and the bispinor D_σ. However, the interaction Lagrangian does not contain D_σ and in the limiting case of infinitely heavy nucleons and isobars which is considered below $D_\sigma = 0$.

and is fully justified for the nonrelativistic deuteron and nucleon-nucleon scattering at sufficiently low energies.

We shall use the letter α to denote the state in which both nucleons are unexcited, β to denote the state in which the first nucleon* is unexcited while the second is in the isobaric state, γ for the state in which the second nucleon is excited but not the first and δ for the state in which both nucleons are excited. We also introduce the notation

$$\Phi_n(\rho) = \frac{1}{(2\pi)^2}\int dk \frac{e^{i\mathbf{k}\rho/\mu}}{(n\Delta + \varepsilon_k)\varepsilon_k} \quad (n = 0, 1, 2);$$
$$W - 2M = V\mu; \; r = \rho/\mu; \; \varepsilon_k = \sqrt{k^2+\mu^2};$$
$$G_n = 2\rho^{-1}\Phi_n', \; H_n = 2(\Phi_n'' - \rho^{-1}\Phi_n');$$
$$N_n(\mathbf{k}, \mathbf{l}) = (\mathbf{kl})G_n + (\mathbf{k}\boldsymbol{\rho})(\mathbf{l}\boldsymbol{\rho})\rho^{-2}H_n;$$
$$N_{nm} = N_n + N_m. \tag{3}$$

Using the explicit form of the matrix elements, we rewrite Eq. (2) in the coordinate representation:

$$V\alpha = g^2(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)N_0(\sigma_1, \sigma_2)\alpha + {}^1/_2 gg_1(\boldsymbol{\tau}_1\mathbf{S}_2)N_{01}(\sigma_1, \mathbf{c}_2)\beta$$
$$+ {}^1/_2 gg(\mathbf{S}_1\boldsymbol{\tau}_2)N_{01}(\mathbf{c}_s, \sigma_2)\gamma + g_1^2(\mathbf{S}_1\mathbf{S}_2)N_1(\mathbf{c}_1, \mathbf{c}_2)\delta,$$
$$(V-\Delta)\beta = {}^1/_2 gg_1(\boldsymbol{\tau}_1\mathbf{S}_2)N_{01}(\sigma_1, \mathbf{c}_2^*)\alpha$$
$$+ {}^1/_2 g_1^2(\mathbf{S}_1\mathbf{S}_2)N_{02}(\mathbf{c}_1\mathbf{c}_2^*)\gamma,$$
$$(V-\Delta)\gamma = {}^1/_2 gg_1(\mathbf{S}_1\boldsymbol{\tau}_2)N_{01}(\mathbf{c}_1^*, \sigma_2)\alpha$$
$$+ {}^1/_2 g_1^2(\mathbf{S}_1\mathbf{S}_2)N_{02}(\mathbf{c}_1^*, \mathbf{c}_2)\beta,$$
$$(V-2\Delta)\delta = g_1^2(\mathbf{S}_1\mathbf{S}_2)N_1(\mathbf{c}_1^*, \mathbf{c}_2^*)\alpha. \tag{4}$$

The spin matrices $\mathbf{c}$ are

$$c_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1/\sqrt{3} & 0 \\ 1 & 1/\sqrt{3} & 0 & 1 \end{pmatrix};$$

$$c_y = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 0 & -i/\sqrt{3} & 0 \\ 0 & i/\sqrt{3} & 0 & -i \end{pmatrix};$$

$$c_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -2/\sqrt{3} & 0 & 0 \\ 0 & 0 & 2/\sqrt{3} & 0 \end{pmatrix}. \tag{5†}$$

*Our approximation corresponds to the case of fixed particles so that we can speak of a "first" and a "second" nucleon.

†The following relations are easily verified:

$$\sum_{\lambda=1}^{2} c_{\lambda r}^* c_{\lambda s} = \delta_{rs}; \quad \sum_{r=1}^{4} c_{\lambda r} c_{\mu r} = 2\delta_{\mu\nu}; \quad \sum_{r=1}^{4} [c_{\mu r} c_{\nu r}^*] = -\frac{2}{3} i\sigma_{\mu\nu};$$

$$\sum_{r=1}^{4} (c_{\lambda r}\mathbf{k})(c_{\mu r}^*\mathbf{l}) = \frac{2}{3}(\mathbf{kl})\delta_{\lambda\mu} - \frac{i}{3}[\mathbf{kl}]\sigma_{\lambda\mu}.$$

In Eq. (4) the following equalities were used:*

$$\bar{u}(\mathbf{k}, s)\, i\gamma_5(\hat{k} - \hat{p})\, u(\mathbf{p}, s')$$
$$= 2M\bar{u}(\mathbf{k}, s)\gamma_5 u(\mathbf{p}, s') = (\sigma, \mathbf{p} - \mathbf{k})_{ss'},$$
$$\bar{u}(\mathbf{k}, s)(\mathbf{k} - \mathbf{p})\mathbf{B}(\mathbf{p}, s') = (\mathbf{c}, \mathbf{k} - \mathbf{p})_{ss'},$$

where u represents the usual Dirac amplitudes of a particle with spin $1/2$, $\mathbf{B}$ is the spin-vector amplitude of a particle with spin $3/2$ (in our approximation ($M \to \infty$), u are two-component spinors and $B_4 = 0$), and σ are the Pauli matrices. The subscripts 1 and 2 of the operators τ, $\mathbf{s}$, σ and $\mathbf{c}$ denote the nucleon on whose variables the operations are performed. In order to go over from Eq. (4) to the ordinary non-isobaric theory of nuclear forces, it is sufficient to set $g_1 = 0$. For the potential energy of the two-nucleon interaction we then obtain the familiar expression

$$V = g^2(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)N_0(\sigma_1, \sigma_2)$$
$$= g^2(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\frac{e^{-\rho}}{3\rho}\left\{(\sigma_1\sigma_2) + \left(1 + \frac{3}{\rho} + \frac{3}{\rho^2}\right)S_{12}\right\}, \tag{6}$$

where S_{12} is the spin operator

$$S_{12} = 3(\sigma_1\boldsymbol{\rho})(\sigma_2\boldsymbol{\rho})\rho^{-2} - (\sigma_1\sigma_2). \tag{7}$$

3. THE CASE I = 0

The total isotopic spin I of a two-nucleon system can be either 0 or 1. We shall consider the first of these cases. When I = 0 it is impossible to have one unexcited and one excited nucleon since two vectors with $I = 1/2$ and $3/2$ cannot be added to give zero. Therefore in this case the wave functions β and γ are zero.

It is easily seen that in isotopic space α and δ are given by

$$\alpha = a\hat{\alpha}_0, \; \delta = d\hat{\delta}_0, \tag{8}$$

with the isotopic space matrices

$$\hat{\alpha}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; \hat{\delta}_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \tag{8'}$$

*The first of these equalities, which demonstrates the so-called equivalence of the pseudovector and pseudoscalar interactions, will cause g in Eq. (4) and subsequent equations to be replaced by g(1 + s/2M) when M is regarded as finite (see footnote† on p. 842). The last equality can easily be obtained by means of Ref. 5, where B_μ is given explicitly.

where a and d are in general matrices with the ordinary spin variable.

In the present case, (4) becomes

$$Va = -3g^2 N_0(\sigma_1, \sigma_2)a - 12g_1^2 N_1(\mathbf{c}_1, \mathbf{c}_2)d,$$
$$(V-2\Delta)d = -6g_1^2 N_1(\mathbf{c}_1^*, \mathbf{c}_2^*)a. \quad (9)$$

Eliminating d, we obtain

$$Va = -3g^2 N_0(\sigma_1, \sigma_2)a + \frac{72g_1^4}{V-2\Delta} N_1(\mathbf{c}_1, \mathbf{c}_2) N_1(\mathbf{c}_1^*, \mathbf{c}_2^*)a. \quad (10)$$

Using (3) and (7), with the notation

$$F_i = 3G_i + H_i, \quad (11)$$

we obtain the following expression for the operator $V^{I=0}$ of the potential energy of nucleons in the state I = 0:

$$V = -g^2\{(\sigma_1\sigma_2)F_0 + S_{12}H_0\} + \frac{16g_1^4}{9(V-2\Delta)}\{6F_1^2 + 12H_1^2 + (\sigma_1\sigma_2)(H_1^2 - F_1^2) + S_{12}H_1(F_1 - H_1)\}. \quad (12)$$

The first term on the right-hand side agrees (for $\tau_1\tau_2 = -3$) with the usual expression (6) for the potential energy of two nucleons, while the second term takes into account the influence of isobaric states on V. Multiplying (12) by $(V - 2\Delta)$, we obtain a quadratic equation in the interaction operator V. The solution of this equation for triplet states, with respect to the mechanical spin (S = 1), of the two-nucleon system (including the stable ^3S deuteron state) is given by

$$V^{I=0,\, S=1} = U^{0,1} + U_T^{0,1} S_{12}, \quad (13)$$

with the following notation:

$$U^{0,1} = \Delta - \frac{p}{2} - \frac{R_1}{3} - \frac{R_2}{6}, \quad U_T^{0,1} = -\frac{R_1 - R_2}{12} - \frac{q}{2},$$
$$R_1 = \sqrt{(2\Delta + p + 2q)^2 + r + s},$$
$$R_2 = \sqrt{(2\Delta + p - 4q)^2 + r - 2s},$$
$$p = g^2F_0, \; q = g^2H_0, \; r = \frac{64}{9}g_1^4(5F_1^2 + 13H_1^2),$$
$$s = \frac{128}{9}g_1^4 H_1(F_1 - H_1).$$

When S = 0 the eigenvalue of S_{12} is zero and the eigenvalue of $(\sigma_1\sigma_2)$ is -3. Substituting these values into (12), we obtain the following equation for the two-nucleon interaction potential:

$$V = 3g^2F_0 + \frac{16g_1^4}{V-2\Delta}(F_1^2 + H_1^2),$$

one of whose solutions is

$$V^{I=0,\, S=0} = \Delta + \frac{3}{2}g^2F_0 - [(\Delta + {}^3/_2 g^2F_0)^2 + 16g_1^4(F_1^2 + H_1^2) - 6g^2F\Delta]^{1/2}. \quad (14)$$

As $r \to \infty$ the solution for V with the positive radical approaches 2Δ, while the solution with the negative radical approaches zero. The last solution evidently represents the potential energy of two nucleons in the state I = 0, S = 0.

4. THE CASE I = 1

We now consider a system of two nucleons with total isotopic spin I = 1. By charge invariance the value of I_z, which is the projection of I on the z axis, is not significant, but for definiteness we shall assume $I_z = 1$. For $I_z = 1$ only the following components of the wave functions are nonvanishing:

$$\alpha_{1/2,\,1/2},\; \beta_{1/2,\,1/2},\; \beta_{-1/2,\,3/2},\; \gamma_{3/2,\,-1/2},\; \gamma_{1/2,\,1/2},\; \delta_{3/2,\,-1/2},\; \delta_{1/2,\,1/2},\; \delta_{-1/2,\,3/2},$$

where the subscripts denote the projections of the isotopic spin of the i-th nucleon on the z axis.

Using the fact that the wave function of the system $v(\gamma_1, \gamma_2)$ must satisfy the equation

$$\hat{I}^2 v = I(I+1)v,$$

where $\hat{I}$ is the operator of the total isotopic moment, we can represent $\alpha, \beta, \gamma, \delta$ by

$$\alpha = \hat{a}\hat{\alpha}_1, \; \beta = \hat{b}\hat{\beta}_1, \; \gamma = \hat{c}\hat{\gamma}_1, \; \delta = \hat{d}\hat{\delta}_1. \quad (15)$$

Here we have introduced the following matrices in isotopic space:

$$\hat{\alpha}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad \hat{\beta}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\sqrt{3} & 0 & 0 & 0 \end{pmatrix};$$
$$\hat{\gamma}_1 = \begin{pmatrix} 0 & -\sqrt{3} \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix};$$
$$\hat{\delta}_1 = \begin{pmatrix} 0 & 0 & -\sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{3}/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (16)$$

$\hat{a}, \hat{b}, \hat{c}, \hat{d}$ are independent of the isotopic coordinates but are matrices in ordinary spin space. Substituting (15) into the general system (4), we obtain

$$V\hat{a} = g^2 N_0(\sigma_1, \sigma_2)\hat{a} + 4gg_1\{N_{01}(\sigma_1, \mathbf{c}_2)\hat{b} + N_{01}(\mathbf{c}_1, \sigma_2)\hat{c}\} + 10g_1^2 N_1(\mathbf{c}_1, \mathbf{c}_2)\hat{d},$$
$$(V-\Delta)\hat{b} = gg_1 N_{01}(\sigma_1, \mathbf{c}_2^*)\hat{a} - g_1^2 N_{02}(\mathbf{c}_1, \mathbf{c}_2^*)\hat{c},$$
$$(V-\Delta)\hat{c} = gg_1 N_{01}(\mathbf{c}_1^*, \sigma_2)\hat{a} - g_1^2 N_{02}(\mathbf{c}_1^*, \mathbf{c}_2)\hat{b},$$
$$(V-2\Delta)\hat{d} = 4g_1^2 N_1(\mathbf{c}_1^*, \mathbf{c}_2^*)\hat{a}. \quad (17)$$

The last of these equations is used to eliminate $\hat{d}$ from the first equation of (17). We proceed as follows to eliminate $\hat{b}$ and $\hat{c}$.

We consider singlet states with respect to the mechanical spin (S = 0). Then the four components of $\hat{a}$ ($\hat{a} = a_{\lambda\mu}$, where λ and μ are spin indices) can be represented by

$$\hat{a} = \begin{vmatrix} 0 & a \\ -a & 0 \end{vmatrix}, \quad (18)$$

where a depends on spatial coordinates alone. We shall obtain the solutions of the second and third equations of (17) in the form

$$\hat{b} = \xi N_{01}(\sigma_1 \mathbf{c}_2^*)\hat{a}, \quad \hat{c} = \xi N_{01}(\mathbf{c}_1^*, \sigma_2)\hat{a}, \quad (19)$$

where ξ is the sought factor. Substituting (19) into (17) for S = 0, we obtain a relation for ξ:

$$\xi = gg_1[(V-\Delta) - g_1^2(G_{02} + {}^2/_3 H_{02})]^{-1} \quad (20)$$

Substituting (19) and (20) into (17) and eliminating $\hat{d}$ from the first equation of (17) by means of the fourth equation, we obtain an equation for $\hat{a}$, from which there is derived the following cubic equation for determination of the potential energy $V^{I=1,\,S=0}$ in the system with I = 1, S = 0:

$$V = -g^2 F_0 + \frac{16g^2 g_1^2 (H_0 + H_1)^2}{3(V-\Delta) - g_1^2(F_0 + F_2 + H_0 + H_2)} + \frac{80 g_1^4 (F_1^2 + H_1^2)}{9(V-2\Delta)}. \quad (21)$$

It can be shown that for all distances between the nucleons this equation has one negative and two positive roots for V, which for $\rho \to \infty$ approach 0, Δ and 2Δ, respectively. The negative root of (21) must obviously be interpreted as the potential energy of nucleons in the state I = 1, S = 0.

We shall finally consider a system of two nucleons in the state with both total isotopic and ordinary spins equal to unity. Above, in (17), equations were given describing the behavior of a two-nucleon system in state I = 1 with arbitrary mechanical spin. (17) is a set of four equations with respect to the functions $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, with $\hat{d}$ easily eliminated by means of the last equation, whereas the elimination of $\hat{b}$ and $\hat{c}$ is more complicated.

We shall obtain the solution of (17) for the triplet state S = 1 in the form

$$\hat{b} = \xi_1(\sigma\hat{a}\mathbf{c}^*) + \xi_2(\sigma\rho)\hat{a}(\mathbf{c}^*\rho)\rho^{-2} + \xi_3\hat{a}(\widetilde{\sigma\rho})(\mathbf{c}^*\rho)\rho^{-2},$$
$$\hat{c} = \hat{b}, \quad (22)$$

where ξ_i are coefficients, with the sign ~ denoting transposition. Substituting (22) into the second equation of (17), we obtain a relation for a linear combination of the matrices in (22). By equating coefficients of the different matrices we obtain the following set of equations for the determination of ξ_i:

$$(V-\Delta)\xi_1 = gg_1 G_{01} + {}^1/_3\xi_1 g_1^2 G_{02} + {}^1/_3\xi_2 g_1^2 G_{02} + {}^1/_3\xi_3 g_1^2 G_{02},$$
$$(V-\Delta)\xi_2 = gg_1 H_{01} - {}^2/_3\xi_2 g_1^2 G_{02} - \xi_3 g_1^2 G_{02} + {}^1/_3\xi_1 g_1^2 H_{02} - {}^2/_3\xi_3 g_1^2 H_{02},$$
$$(V-\Delta)\xi_3 = -{}^1/_3\xi_2 g_1^2 G_{02} - \xi_1 g_1^2 H_{02} - {}^2/_3\xi_2 g_1^2 H_{02}. \quad (23)$$

Obtaining ξ_i from (23) and substituting (22) into the first equation of (17), we obtain an equation for $\hat{a}$, which describes a system of two unexcited nucleons.

The equations for $\hat{a}$ can be written as

$$[V - E_1(V)]\hat{a} = E_2(V)(\sigma\rho)\hat{a}(\widetilde{\sigma\rho})\rho^{-2}; \quad (24)$$

where

$$E_1(V) + E_2(V) = g^2(G_0 + H_0) + \frac{16}{3}gg_1 G_{01}(4\xi_1 + \xi_2 + \xi_3) + \frac{16}{3}gg_1 H_{01}(\xi_1 + \xi_2 + \xi_3) + \frac{80}{9}\frac{g_1^4}{V-2\Delta}(5G_1^2 + 4G_1H_1 + 2H_1^2),$$
$$E_1(V) - E_2(V) = g^2(G_0 - H_0) + \left(\frac{32}{3}gg_1 G_{01} + \frac{16}{3}gg_1 H_{01}\right)(2\xi_1 + \xi_2 - \xi_3) + \frac{80}{9}\frac{g_1^4}{V-2\Delta}(5G_1^2 + 2G_1H_1 + 2H_1^2),$$

with ξ_1, ξ_2, ξ_3 obtained from (23). The operator $(\sigma_1\rho)(\sigma_2\rho)\rho^{-2}$ in (24) has in the triplet state (S = 1) the three eigenvalues 1, 1, −1. We denote by V_1 the root of the equation

$$V - [E_1(V) + E_2(V)] = 0, \quad (25)$$

which corresponds to the eigenvalue 1 of the operator, and by V_{-1} the root of the equation

$$V - [E_1(V) - E_2(V)] = 0, \quad (26)$$

TABLE I. Values of the nuclear potentials

ρ, $\hbar/\mu c$	$U^{0,1}$, Mev	$U_T^{0,1}$, Mev	V^{00}, Mev	$V^{1,0}$, Mev	$U^{1,1}$, Mev	$U_T^{1,1}$, Mev
0.46	—846.14	—219.35	—456.00	—1254.8	—1044.9	11.12
0.5	—600.57	—184.07	—294.77	— 935.00	— 768.14	11.62
0.6	—250.78	—131.16	— 89.87	— 462.36	— 365.29	12.74
0.7	—102.87	— 99.34	— 18.87	— 242.68	— 181.88	13.15
0.8	— 41.72	— 73.53	5.357	— 128.5	— 89.24	12.64
1.0	— 11.39	— 38.40	12.77	— 39.18	— 21.83	9.494
1.2	— 5.217	— 21.20	10.46	— 14.02	— 5.556	6.154
1.5	— 2.491	— 9.799	6.624	— 4.352	— 0.4264	3.134
2.0	— 1.050	— 3.346	3.077	— 1.250	0.2214	1.078
2.5	— 0.5022	— 1.339	1.499	— 0.5330	0.1517	0.4448
3.0	— 0.2522	— 0.5884	0.7564	— 0.2583	0.0800	0.1968

TABLE II

r_s, $\hbar/\mu c$	r_t; $\hbar/\mu c$	Q, $(\hbar/\mu c)^2$	p, %
2.25	1.37	0.17	8
(1.92)	(1.22)	(0.14)	See Ref. 6

which corresponds to the eigenvalue -1 of the same operator. (25) and (26) are fifth-degree equations in V, and the roots of these equations which approach 0 in the limit $r \to \infty$ obviously represent to the potential energy of two unexcited nucleons.

When the potential energy of two nucleons in the state I = 1, S = 1 is represented as

$$V^{I=1,\,S=1} = U^{1,1} + U_T^{1,1} S_{12}, \qquad (27)$$

where S_{12} is the spin operator (7), then for the functions $U^{1,1}$ and $U_T^{1,1}$ depending on the separation we have

$$U^{1,1} = {}^1/_3\,(2V_1 + V_{-1}), \quad U_T^{1,1} = {}^1/_6\,(V_1 - V_{-1}). \qquad (28)$$

5. NUMERICAL RESULTS

Numerical values for the potentials of interest were obtained from Eqs. (12), (14), (21), and (27) of the present article. The constants in the expressions for the potentials were obtained by making the theoretical formulas of Refs. 1 and 2 agree as well as possible with experiments on meson photoproduction and scattering by nucleons. The optimum values of the constants were*

$$\Delta = 2.1\mu; \quad g^2 = 0.085; \quad g_1^2 = 0.063; \quad s = 1.8, \qquad (29)$$

with the nucleonic mass taken as $M = 6.75\,\mu$.

Table I contains the values of the potentials, calculated from the set of constants in (29)†

As already stated in the introduction, these potentials possess a singularity of high order at the origin, thus leading to the difficulty associated with the absence of a stable deuteron state and zero separation of the nucleons. We have therefore replaced our potentials at distances $r \le r_0$ by an infinitely high repulsive barrier. The cutoff radius r_0 was determined separately for the singlet and triplet states of the deuterons; the identical value $r_0 = 0.46\ \hbar/\mu c$ was obtained in both instances. In this respect the isobaric theory differs from the ordinary theory of nuclear forces,[3] since, as we know, the singlet and triplet radii do not coincide in the latter.

The potentials were used to calculate the parameters which describe the deuteron and nucleon scattering at low energies; these are the effective ranges of nuclear forces in the singlet (r_s) and triplet (r_t) spin states, the quadrupole moment (Q) and the admixture of the D wave in the triplet state (p, %). These quantities and the corresponding experimental values (in parentheses) are seen in Table II. Although the theoretical values generally somewhat exceed the experimental values, on the whole the agreement of theory and experiment at low energies can be regarded as satisfactory.

As has already been indicated, our treatment is essentially nonrelativistic and applicable only to sufficiently low energies. To determine more completely the limits of applicability of the theory developed here it was of interest to use the derived nuclear potentials in a description of nucleon-nucleon scattering at energies as much higher as possible. For this purpose we calculated the phase shifts of nucleon-nucleon scattering at 100 Mev in the laboratory system. The results, which are given in Table III, show that our adiabatic approximation cannot be used to describe nucleon scattering at this energy. This would also be evident simply from the fact that the total cross sections σ_{pp} and σ_{pn} were 75 and 124 mb, respectively, compared with the experimental values ~ 34 and

*In comparing these numerical values of the constants with those given in Ref. 1, it must be remembered that in the present article we have used an isotopically invariant formalism, whereas the formalism employed in Ref. 1 is based on a classification by charge states. It is easily seen that as a result our values of g^2 and g_1^2 must be one half as large as the corresponding values in Ref. 1.

†Actually, in calculating the potentials from Eqs. (13), (14), (21) and (27) we did not use the constant g^2 equal to 0.085 but instead $g^2(1 + s/2\,M)^2 = 0.11$ (see footnote* on p. 839), but the results were very little affected.

TABLE III. Phase shifts of nucleon-nucleon scattering at 100 Mev

State	$^1S_0$	$^1P_1$	$^1D_2$	$^1F_3$	$^3P_0$	$^3P_1$	$^3D_2$	$^3F_3$
Phase shifts	δ_0^0 0.395	δ_1^0 −0.2938	δ_0^2 0.1034	δ_3^0 −0.0518	$\delta_{I=0}$ 0.969	$\delta_{I=1}^\beta$ −0.1238	$\delta_{I=2}^\beta$ 0.609	$\delta_{I=3}^\beta$ −0.0368

State	$^3S_1 + {}^3D_1$			$^3P_2 + {}^3F_2$			$^3D_3 + {}^3G_3$		
Phase shifts and parameters of the mixture*	$\delta_{J=1}^\alpha$ 0.622	$\delta_{J=1}^\gamma$ −0.344	$\eta_{J=1}^\alpha$ 0.053	$\delta_{J=2}^\alpha$ 0.3328	$\delta_{J=2}^\gamma$ −0.001	$\eta_{J=2}^\alpha$ −0.2544	$\delta_{J=3}^\alpha$ 0.112	$\delta_{J=3}^\gamma$ −0.0879	$\eta_{J=3}^\alpha$ 0.7499

*We have followed the notation used in Ref. 7.

~ 70 mb. The angular distributions were also entirely unsatisfactory. A more exact non-adiabatic approximation might to some extent correct this discrepancy, but we are inclined to believe that the semi-phenomenological isobaric theory of nuclear forces which has been developed here is limited to low energies not above a few Mev.

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