If we assume the constants to be independent of the π-meson energy, the equations (11) are equivalent to the following:

$$|g_T|^2 + |g'_T|^2 = 2\mathrm{Re}(g_T g'^*_T); \qquad (12)$$

$$\mathrm{Re}(g_S g^*_T + g'_S g'^*_T) = \mathrm{Re}(g'_S g^*_T + g_S g'^*_T). \qquad (13)$$

We note that it follows unambiguously from Eq. (12) that $g_T = g'_T$. Equation (13), however, is a mere consequence of Eq. (12), i.e., a second independent check on the equality of the two tensor constants. For the case in which the equations (11) hold except that the signs of the left members are changed, we also have as the unambiguous result that the tensor constants are equal in magnitude and opposite in phase, which corresponds to the case of the "two-component antineutrino for the tensor-type interaction". Here we have used an expression for the degree of polarization which is the analogue of the equation obtained by Okun',[1] namely:

$$\overline{P} = \frac{-\overline{\Phi}^{SS}_{S'S'}J_3 + \overline{\Phi}^{VV}_{V'V'}J_3 - \overline{\Phi}^{TT}_{T'T'}(\overline{A}-\overline{B}) - \overline{\Phi}^{ST}_{S'T'}(\overline{C}-\overline{D})}{\overline{F}_{SS'}J_3 + \overline{F}_{VV'}J_3 + \overline{F}_{TT'}(\overline{A}-\overline{B}) + \overline{F}^{S'T}_{ST'}(\overline{C}-\overline{D})}.$$

The values of J_3, $\overline{C}$, $\overline{A}$, $\overline{B}$, $\overline{D}$ as functions of the electron energy are given in Ref. 1.

In conclusion we write down the probability for the emission of an electron with given energy and polarization in a given direction and the emission of a π meson with a given energy:

$$W(\mathbf{J}, E_\pi, E_e)\,dE_\pi\,dE_e = \frac{1}{2}\{\cos(\widehat{\mathbf{J}\mathbf{J}_3})\,\delta W_4 + \cos(\widehat{\mathbf{J}\mathbf{J}_2})\,\delta W_3 + \cos(\widehat{\mathbf{J}\mathbf{J}_1})\,\delta W_2 + 2W_1\}.$$

The values of δW_4, δW_3, δW_2, add W_1 and the definitions of the three directions $\mathbf{J}_{1,2,3}$ have been given in the text of the present paper.

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INELASTIC SCATTERING OF DEUTERONS

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The inelastic scattering of deuterons by nuclei is assumed to occur in some cases through direct interaction. The incident deuteron is merely scattered and forms the outgoing particle. An expression is derived for the angular distribution of the scattered deuterons. This expression agrees with the experimental data on the scattering of 14.4 Mev deuterons from the 4.61 Mev level of Li^7.

THE experiments of Holt and Young[1] on inelastic scattering of deuterons show that the angular distribution of the scattered deuterons usually has a maximum in the forward direction. This is a characteristic feature of the stripping and direct-interaction theories. Huby and Newns[2] have explained such a behavior by assuming that only one of the component parts of the deuteron interacts with the nucleus in the scattering. The other part remains outside the effective radius of the nuclear forces. The interacting nucleon excites the nucleus, and the deuteron then continues its motion as a whole. The resulting formula is similar to those derived in the stripping theory, and agreement with experiment has been obtained in many cases.

Certain discrepancies were observed recently[3] between this formula and the experimental data.

Fairbairn[4] has derived very recently another formula, in which it is also assumed that scattering proceeds through an intermediate state, composed of the bombarding nucleus and one of the deuteron nucleons, i.e., the reaction proceeds through the single-particle mechanism.

In this communication we shall represent the inelastic scattering of the deuteron as a direct process, analogous in certain respects to that examined by Austern, Butler and McManus and more recently by Butler[5] for (p, p) reactions. We assume that the incident deuteron, colliding at a certain instant of time with the proton and neutron on the surface of the nucleus, merely experiences scattering and is itself the outgoing particle.

Let $\mathbf{k}$ and $\mathbf{k}'$ be the wave vectors of the incident and scattered deuterons $\Phi_{l_1 m_1, l_2 m_2}(\mathbf{r}_1, \mathbf{r}_2)$ and $\Phi_{l'_1 m'_1, l'_2 m'_2}(\mathbf{r}_1, \mathbf{r}_2)$ the wave functions representing the states of the proton on the surface of the nucleus prior to and after the collision ($|\mathbf{r}_1| \approx |\mathbf{r}_2| \equiv |\mathbf{r}|$). To simplify the calculations we shall not introduce the spin functions explicitly and will take spins into account only inasmuch as they determine the selection rules. We shall also assume that the target nucleus has an infinite mass. Then, the differential cross section for the direct (d, d′) reaction will be given by the expression

$$\sigma(\mathbf{k}_d, \mathbf{k}_{d'}) = \frac{M_d^2}{(2\pi\hbar^2)^2}\frac{k'}{k}\sum |I|^2,$$

where

$$I = \langle l'_1 m'_1,\ l'_2 m'_2,\ \mathbf{k}' \,|\, V \,|\, l_1 m_1,\ l_2 m_2,\ \mathbf{k}\rangle$$

$$= \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}'_1\, d\mathbf{r}'_2\, \Phi^*_{l'_1 m'_1,\ l'_2 m'_2}(\mathbf{r}_1,\ \mathbf{r}_2)$$

$$\times \exp\{-\alpha r'^2_{12} - i\mathbf{k}'(\mathbf{r}'_1 + \mathbf{r}'_2)/2\}$$

$$\times V\Phi_{l_1 m_1, l_2 m_2}(\mathbf{r}_1,\ \mathbf{r}_2)\exp\{-\alpha r'^2_{12} + i\mathbf{k}(\mathbf{r}'_1 + \mathbf{r}'_2)/2\}, \qquad (1)$$

where the Born approximation is used, and a Gaussian expression with $\alpha = 0.23 \times 10^{13}\,\mathrm{cm}^{-1}$ is used for the internal wave function of the deuteron. V is the potential of the interaction between the incident deuteron and the proton-neutron system on the surface of the nucleus. For V we take the expression

$$V = V_0\delta(\mathbf{r}_1 - \mathbf{r}'_1)\,\delta(\mathbf{r}_2 - \mathbf{r}'_2). \qquad (2)$$

Inserting (2) into (1) we get

$$I = V_0\int \Phi^*_{l'_1 m'_1,\ l'_2 m'_2}(\mathbf{r}_1,\ \mathbf{r}_2)\, e^{-2\alpha r^2_{12}}\, e^{i\mathbf{Q}(\mathbf{r}_1+\mathbf{r}_2)}$$

$$\times \Phi_{l_1 m_1,\ l_2 m_2}(\mathbf{r}_1,\ \mathbf{r}_2)\, d\mathbf{r}_1\, d\mathbf{r}_2, \qquad (3)$$

$$\mathbf{Q} = (\mathbf{k} - \mathbf{k}')/2.$$

We also use the expression

$$\Phi_{l_1 m_1,\ l_2 m_2}(\mathbf{r}_1,\ \mathbf{r}_2) = f_{l_1}(\mathbf{r}_1) f_{l_2}(\mathbf{r}_2) Y^{m_1}_{l_1}(\theta_1,\ \varphi_1)\, Y^{m_2}_{l_2}(\theta_2,\ \varphi_2) \qquad (4)$$

and put $|\mathbf{r}_1| = |\mathbf{r}_2| = |\mathbf{r}|$, so that we can write

$$\exp\{-2\alpha r^2_{12}\} = \exp\{-4\alpha r^2 + 4\alpha r^2\cos\theta_{12}\}$$

$$= e^{-4\alpha r^2}\sqrt{\frac{\pi}{8\alpha r^2}}\sum_{n=0}^{\infty}(2n+1)\, I_{n+1/2}(4\alpha r^2)\, P_n(\cos\theta_{12}), \qquad (5)$$

where $I_{n+\frac{1}{2}}(4\alpha r^2)$ is the modified Bessel function of half-integral order.[6] Inserting the formula

$$P_n(\cos\theta_{12}) = \frac{4\pi}{2n+1}\sum_{m=-n}^{n} Y^{m*}_n(\theta_1,\ \varphi_1)\, Y^m_n(\theta_2,\ \varphi_2) \qquad (6)$$

together with (4) and (5) into (3), inverting the spherical harmonics with respect to (θ_1, φ_1) and (θ_2, φ_2) with the aid of the vector-addition coefficients,[7] and finally integrating over the solid angles $d\Omega_1$ and $d\Omega_2$, we obtain (summation over L_1, L_2, n, K_1, K_2, M_1, M_2, m)

$$I \approx \int 4\pi dr e^{-4\alpha r^2}\sqrt{\frac{\pi}{8\alpha r^2}}\, f_{l_1}(r) f_{l_2}(r) f_{l'_1}(r) f_{l'_2}(r)$$

$$\times\sqrt{(2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)}$$

$$\times \sum (-1)^{m+m'_1+m'_2}\, i^{K_1+K_2}$$

$$\times (2n+1) I_{n+1/2}(4\alpha r^2)\, C_{l_1 l'_1}(L_1 000)$$

$$\times\ C_{l_1 l'_1}(L_1 M_1 m_1 m'_1)\, C_{l_2 l'_2}(L_2 000)$$

$$\times C_{l_2 l'_2}(L_2 M_2 m_2 m'_2)\, C_{L_1 n}(K_1 000)$$

$$\times\ C_{L_1 n}(K_1 0 M_1 - m)\, C_{L_2 n}(K_2 000)\, C_{L_2 n}(K_2 0 M_2 m)$$

$$\times J_{K_1}(Qr)\, I_{K_2}(Qr). \qquad (7)$$

J_{K_1} and J_{K_2} are spherical Bessel functions, and C are the vector-addition coefficients. As an approximation, we take the spherical Bessel functions outside the integral sign, using their values on the nuclear boundary $r = R$. We then find that the transition amplitude is expressed in the form of a sum of terms, in which the angular dependence is contained in $J_{K_1}(QR)\,J_{K_2}(QR)$, with

$$Q^2 = (k^2 + k'^2 - 2kk' \cos\theta)/4, \qquad (8)$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$.

Squaring (7), summing over the final states, averaging over the initial states, and using the Racah formalism,[8] we obtain an expression for the inelastic scattering differential cross section of the deuterons.

$$\sigma \backsim |F(R)|^2 e^{-8\alpha R^2} [(2l_1+1)(2l_1'+1)(2l_2+1)(2l_2'+1)]$$

$$\times \sum_{L_1L_2} [(2L_1+1)(2L_2+1)]^{-1/2} |C_{l_1l_1'}(L_1 000)\, C_{l_2l_2'}(L_2 000)|^2$$

$$\times \sum_f (-1)^f \Big| \sum_{nK_1K_2} (2n+1)\, I_{n+1/2}(4\alpha R^2)$$

$$\times C_{L_1n}(K_1 000)\, C_{L_2n}(K_2 000)$$

$$\times Z(K_1L_1K_2L_2,\ nf)\, j_{K_1}(QR)\, j_{K_2}(QR)\Big|^2 \qquad (9)$$

Here L_1 takes on values from $|l_1 - l_1'|$ to $|l_1 + l_1'|$, and K_1 ranges from $|L_1 - n|$ to $|L_1 + n|$, with L_2 and K_2 behaving analogously. Summation over f extends from $L_1 - L_2$ to $L_1 + L_2$. The coefficients Z (Ref. 8) are connected with a Racah coefficient by the relation

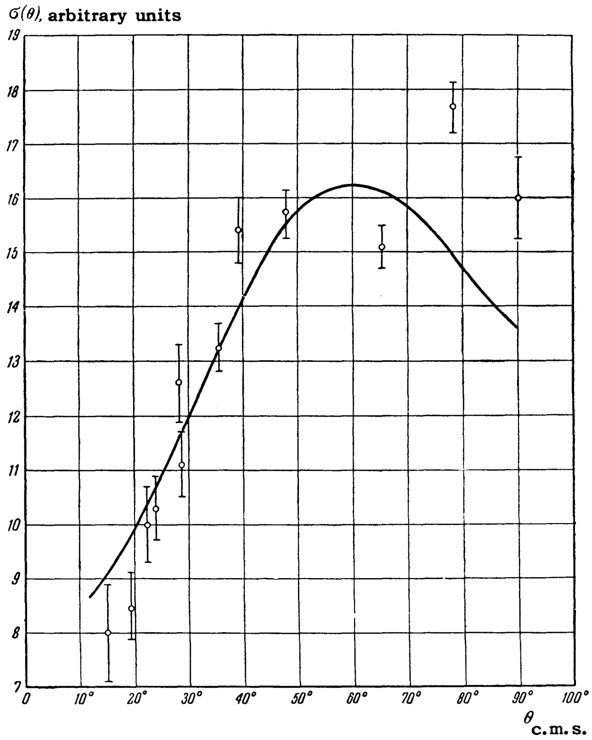
$$Z(K_1L_1K_2L_2,\ nf) = i^{f-K_1+K_2} [(2K_1+1)$$

$$\times (2K_2+1)(2L_1+1)(2L_2+1)]^{1/2}$$

$$\times W(K_1L_1K_2L_2,\ nf)\, C_{K_1K_2}(f\,000).$$

The angular distribution of the scattered deuterons, given in (9), depends on the spherical Bessel functions j_{K_1} and j_{K_2}. In addition, as can be seen from (9), there is no interference between the different values of L_1, L_2, and f. Given L_1 and L_2, which can be obtained from the values of the angular momenta of the levels under examination, it is possible to choose for f an allowed value that gives the best agreement with experiment.

For comparison with experiment, we made use of the measurements of Levine, Bender, and McGruer,[3] who obtained an angular distribution for 14.4-Mev deuterons, inelastically scattered by the 4.61-Mev level of the Li^7 nucleus.



The observed distribution (c.m.s.) increases in the interval from 17° to ~ 70°. This distribution is not in agreement with the Huby and Newns theory.

In the ground state of Li^7, $J = (3/2)^-$, while for the 4.6 Mev level $J = (5/2)^-$. According to the selection rules

$$\mathbf{J}_f - \mathbf{J}_i = (\mathbf{L}_1 + \mathbf{L}_2) + \mathbf{I} + \mathbf{I},$$

and since the parity does not change in this transition, we can take $L_1 = L_2 = 0$, and then $f = 0$. In this case, that portion of (9) which determines the angular distribution will be of the form

$$\sigma \backsim | j_0^2 + 4.614\, j_1^2 + 7.848\, j_2^2 + 9.17\, j_3^2$$

$$+ 8.37\, j_4^2 + 6.604\, j_5^2 + 5.016\, j_6^2 |^2. \qquad (10)$$

The angular distribution that follows from (10) is shown in the diagram together with the experimental values obtained from Ref. 3. A satisfactory agreement is seen. The nuclear radius of Li^7 was taken to be 6.5×10^{-13} cm, the value used in Ref. 3 when discussing the $Li^7(d, d')Li^7$ reactions at the 0.478 Mev level; $\alpha = 0.23 \times 10^{-13}\,cm^{-1}$ (Ref. 9).

The mechanism considered can play a substantial role in the case of inelastic scattering of deuterons, for example, by N^{14} and C^{12} nuclei. Experiments carried out with these two nuclei (and possibly also with others) can be reconciled with the formulas of the stripping and direct-interaction theories. The contribution of each of these processes will depend on the structures of the nuclei studied. One can hope information on the structures of various nuclei can be obtained in this manner.

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SEMI-PHENOMENOLOGICAL THEORY OF NUCLEON-NUCLEON INTERACTION

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Results of the calculation of nucleon-nucleon interaction potentials are presented. The calculations were made within the framework of the semi-phenomenological isobaric theory. The computed deuteron parameters and scattering of low-energy nucleons agree satisfactorily with experiment. An unsuccessful attempt is made to employ the computed potentials for a description of the scattering of high energy nucleons (~ 100 Mev).

1. INTRODUCTION

TAMM, Gol'fand, and Fainberg[1] have proposed a semi-phenomenological theory of nucleon-meson interaction where, in addition to the ordinary nucleon state with mechanical and isotopic spins $1/2$, there is consideration of their excited isobaric state with mechanical and isotopic spins $3/2$. This isobaric state, which is introduced purely phenomenologically, permits us to describe the behavior of the cross sections for the scattering[1] and photoproduction[2] of π mesons on nucleons in a fairly large meson energy range up to 400 Mev.

The semi-phenomenological theory of Ref. 1 involves four free parameters: the nucleon excitation energy Δ, the pseudovector meson-nucleon coupling constant g/μ (where μ is the mass of the π meson), the pseudoscalar coupling constant $g' = sg$ (where s is a number) and the constant g_1 which determines the probability of a nucleonic transition from its unexcited state to the isobaric state or vice versa. The values of these parameters were chosen to provide the best possible fit of experimental data on meson-nucleon scattering and meson photoproduction. The success of this procedure induced us to apply the semi-phenomenological isobaric theory to the problem of nuclear forces and specifically to the deuteron and nucleon-nucleon scattering.

Our calculation showed that inclusion of isobaric states greatly changes the results of the ordinary theory of nuclear forces, in which isobars are neglected. For example, when isobars are included the potential energy of nucleons in ^1S and ^3S states increases proportionally to $1/r^3$ for $r \to 0$, whereas when isobars are not taken into account the potential energy in the ^1S state (unlike ^3S) has, as we know, only the simple pole $1/r$.