integration, $\overline{H}$ is the mean value of the z component of the magnetic field in a circle of radius r and the remaining notation is obvious. The third equation in (1) can also be written in the form:

$$m^2(\dot{r}^2 + \dot{z}^2 - c^2) + (e/c)^2A^2 + m_0^2c^2 = 0. \quad (2)$$

3. In the quasi-stationary approximation the following solution of the field equation applies in the region of the line $r = R$, $\zeta = z - z(t) = 0$:

$$A = r\left[\frac{H_z^0}{2} - \frac{a}{2}R^2 + \frac{ar^2}{4} - \frac{H_r^0}{R}\zeta - \frac{a\zeta^2}{1-\dot{z}^2/c^2}\right], \quad (3)$$

where H_r^0 and H_z^0 are the components of the field at $r = R$, $\zeta = 0$, $z(t)$ is an arbitrary function of the time which satisfies the condition $z = c^2 \times (\partial H_z/\partial r)_0/H_r^0$ and a is a constant. In a field of this kind, when $a = -2H_z^0/R^2$, Eq. (1) has the particular solutions $r = R$ and $z = z(t)$, i.e., the electron moves in a helix of constant radius if the function $z(t)$ is given by the equation

$$ct = \sqrt{z(t)}\sqrt{\alpha + z(t)} + \alpha\ln(\sqrt{z(t)} + \sqrt{\alpha + z(t)}),$$
$$\alpha = m_0^2c^4[1 + (eRH_z^0/m_0c^2)^2]/2e^2RH_z^0H_r^0, \quad (4)$$

where the particle energy is

$$E = mc^2 = \sqrt{2e^2H_z^0H_r^0(\alpha + z)}. \quad (5)$$

In the case of relativistic initial energies and $z \gg RH_z/2H_r$, from Eqs. (4) and (5) we have

$$z = ct, \; E = E_0\sqrt{2zH_r/RH_z}. \quad (6)$$

4. If the field is displaced along the z axis with constant velocity u, i.e., $A = A(r, z-ut)$, from Eq. (1) we find

$$m(\dot{z} - c^2/u) = -Mc^2/u, \quad M = m_0(1 - \dot{r}_0^2\dot{\varphi}_0^2/c^2)^{-1/2}, \quad (7)$$

where M is the mass of the particle at $\dot{z} = 0$ and $\dot{r} = 0$.

According to Eqs. (2) and (7), an electron which originally moves in the wide section of the "magnetic bottle," where $H_z = H_0$ and $A = A_0$, falls into the "neck" of the bottle (where $H_z = H$) and then again is forced into the wide part, acquiring the following energy in the process

$$E/E_0 = 2[(m_0/M)^2 + (eA/Mc^2)^2] - 1. \quad (8)$$

If it is assumed that the motion is such that the adiabatic invariant $H_zr^2 = \text{const}$ is conserved, with a relativistic initial energy we have $A = r_0\sqrt{H_0H}$ and Eq. (8) can be written in the form:

$$E/E_0 + 1 = 2(A/A_0)^2 \approx 2H/H_0 \approx L/2l, \quad (9)$$

where L is the length of path over which acceleration takes place, and l is the length of the segment over which the field changes from H_0 to H.

5. Equations (8) and (9) do not hold if $u = c$. In this case the field equation yields

$$A = {}^1/_2 rH(z - ct) + b/r, \quad (10)$$

and from Eqs. (1), (2) and (7), for the case $H_zr^2 = \text{const}$, we have

$$E - E_0 = (e^2/2E_0)(H_0r_0^2)[H - H_0],$$
$$z - z_0 = -\frac{e^2(Hr^2)}{2M^2c^4}\int_{z_0}^{z-ct}(H - H_0)\,d\zeta. \quad (11)$$

For relativistic initial conditions $Mc^2 = -eA_0$ and since $A_0 \approx H_0r_0$, we have

$$E/E_0 - 1 \approx \tfrac{1}{2}(H/H_0 - 1) = L/l. \quad (12)$$

6. According to Eqs. (9) and (12), in a linear accelerator in which the field moves with constant velocity u, we have: $E/E_0 \approx 2H/H_0$ or $E/E_0 \approx H/2H_0$, i.e., the situation is analogous to that in the usual betatron. However, in contrast to the betatron the strong field H can be concentrated in a very small region since $He^2 = \text{const}$.

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DETERMINATION OF THE VELOCITY OF IONIZING PARTICLES USING A HIGH-FREQUENCY ELECTRIC FIELD FOR TRACK MARKING

G. A. ASKAR'IAN

P. N. Lebedev Physical Institute, Academy of Sciences, U.S.S.R.

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An interesting method of making direct determinations of the velocities of ionizing particles in a Wilson cloud chamber has recently been proposed by Gabor and Hampton.[1] In this scheme, the tracks are "marked" by a high-frequency (rf) electric field by using the difference in intensity of electron cascades as a function of field strength at the time the free electrons are produced. This difference is due to the absorption of free electrons and the dissipation of electron energy or other fast processes the duration of which is less than the period of the rf field. In the present paper we present an elementary analysis of the modulation of the ionic density of a track, derive an expression for the track-marking interval for fast particles, and indicate methods by which the effectiveness of this method can be enhanced.

The equation which characterizes the free-electron population

$$dn_e/dt = wn_e - n_e/\tau \tag{1}$$

contains the probability $w(t)$ of an increase in the number of electrons per unit time and the lifetime τ for the free electrons. Using the solution of this equation

$$n_e = n_0 \exp\left\{\int_{t^*}^{t}\left(w - \frac{1}{\tau}\right)dt\right\}, \tag{2}$$

we obtain an expression for the number of ions which are produced

$$n_i = \int_{t^*}^{t} \frac{n_e}{\tau}\, dt = \int_{t^*}^{t} \frac{n_0}{\tau}\left\{\exp\left[\int_{t^*}^{t}\left(w - \frac{1}{\tau}\right)dt\right]\right\}dt. \tag{3}$$

Here t^* is the time at which the free electron is produced and n_0 is the initial number of free electrons per unit track length, which depends on the ionizing power of the particle. The function $(w - 1/\tau)$ is of the form $p\{f(|E(t)|/p) - 1/\tau\}$ where p is the pressure of the gas, $E(t)$ is the intensity of the electric field, where we have neglected the weak dependence of τ on the external field, while the function f is an extrapolation of some power function (cf. for example Ref. 2). Because the external field is periodic it is possible to expand the function $w - 1/\tau$ in a Fourier series:

$$w - 1/\tau = \sum_{0}^{\infty} a_k \cos 2k\omega(t - t_k) \tag{4}$$

(2ω is the modulation frequency of the rf electric field).

For the time being we shall be interested in the case in which the electron avalanches have time to decay, i.e., $(w - 1/\tau)_{\text{ave}} = a_0 < 0$; if this condition is not satisfied, extended trains of rf field pulses can lead to "breakdown." (An analysis of increasing electron avalanches is of interest only in the case of short rf field trains, for example pulse trains cut off sharply immediately after the passage of the particle.)

Substituting Eq. (4) in Eq. (3), we have

$$n_i = \int_{t^*}^{t} \frac{n_0}{\tau} \exp\{a_0(t - t^*)\} \exp\left\{\sum_{1}^{\infty} \frac{a_k}{2k\omega} \times [\sin 2k\omega(t - t_k) - \sin 2k\omega(t^* - t_k)]\right\} dt.$$

We expand the second exponential in the integrand in a power series and limit ourselves to the first power; the resulting expression is

$$n_i \simeq \frac{n_0}{\tau|a_0|} \times \left\{1 - \sum_{1}^{\infty} \frac{a_k}{\sqrt{a_0^2 + 4k^2\omega^2}} \sin\left[2k\omega(t^* - t_k) + \tan^{-1}\frac{a_0}{2k\omega}\right]\right\}.$$

Substituting the values of t^* and t_k as functions of points along the track $t^* = (l - l_0)/v$, where v is the velocity of the particle and l is the distance along the track) we obtain a function which characterizes the spatial ion distribution. An analysis of the ion distribution function makes it possible to estimate the dependence of $(w - 1/\tau)$ on the instantaneous value of the rf electric field. These results can also be used for choosing the optimum conditions for which the functions $n_i(l)$ change most rapidly, thus making possible a more accurate determination of the positions of the maxima and thus a more accurate determination of the period of the spatial distribution. In particular, the formula which has been obtained exhibits the spatial periodicity of the distribution. For example, at low particle velocities, when the wave properties of the rf field can be neglected (t_k = const), the distribution period is $\Delta l = \pi v/\omega$. For particles with arbitrary velocities, which move as quasi-plane traveling waves ($t_k = z/c' = (l - l_0) \times \cos\theta/c'$ where c' is the velocity of propagation of the wave and θ is the angle between the direction of motion of the particle and the direction of propagation of the wave), the distance between adjacent "condensations" along the track is

$$\Delta l = \frac{\pi}{\omega} \frac{v}{|1 - (v/c')\cos\theta|}$$

(c' may be different from the velocity of light). This same formula can be obtained directly from the expression for the intensity of the wave field at the instant the electrons are produced $E(t^*) = E_0 \sin \omega z(1/v \cos\theta - 1/c')$ using the requirement that conditions be reproduced at the instant the electrons are produced.

In passing, we may note the possibility of reducing the required rf power by superimposing a quasi-uniform base electric field which is constant for a period of time which does not cause significant distortion of the track (the direction of this field is then reversed, etc.). The electron production conditions are satisfied only when the vector direction of the base field coincides with that of the supplementary rf field; hence the marking intervals become twice as long.

The use of lower rf power, more stable conditions in the excitation of the field, and an investigation of the possibilities of sharp reduction of the rf field after the flight of the particle may all reduce the requirements on stability of the rf amplitude and increase the effectiveness of the marking process.

In addition to synchronous operation with a pulsed accelerator, detection of cosmic particles using triggered rf field trains, and controlled operation of a cloud chamber, it is interesting to consider the possibilities of triggering the rf field from a radiation precursor of a cosmic particle or an early particle in an avalanche of cosmic particles.

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RELAXATION OF DEUTERIUM NUCLEI IN PARAMAGNETIC SOLUTIONS

A. I. RIVKIND

Physico-Technical Institute, Kazan Branch, Academy of Sciences, U.S.S.R.

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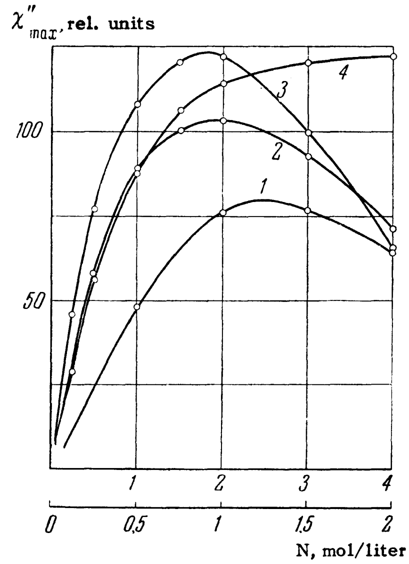
AN investigation has been made of spin relaxation of deuterium nuclei in solutions containing Cr^{+++}, Mn^{++}, Fe^{+++} and Cu^{++} in heavy water. The measurements were made by means of a modified saturation-curve method[1,2] in which the degree of nuclear saturation is varied by changing the concentration of paramagnetic ions in the solution rather than by changing the amplitude of H_1, the oscillating magnetic field.[3] All the experiments were carried out at room temperature at $\nu_0 \cong 2.6$ mc/sec. An analysis of the results indicates the following.

1. The nuclear relaxation time for the same concentration of a given paramagnetic salt in D_2O and H_2O obeys the relation

$$T_{1d} / T_{1p} = (\gamma_p^2 / \gamma_d^2)\,\alpha, \qquad (1)$$

where T_{1d} is the longitudinal relaxation time for deuterons in a D_2O solution, T_{1p} is the longitudinal relaxation time for protons in a H_2O solution, γ_p and γ_d are the nuclear gyromagnetic ratios $(\gamma_p^2/\gamma_d^2 \cong 42.4)$ and α is a numerical factor. In solutions containing Cr^{+++}, Fe^{+++} and Cu^{++} $\alpha \sim 4.2$, with sizable departures toward lower values for copper. Mn^{++} ions have much less effect on relaxation of deuterium nuclei: $\alpha \sim 6.8$ in solutions of $MnCl_2$ and $Mn(NO_3)_2$.

2. As has been indicated earlier in the case of proton relaxation,[2,4] the time T_1 for deuterons becomes much longer (up to a change of one order of magnitude) as a consequence of the production of complexes in the solution in which the water molecules in the hydrate shells of the paramagnetic ions are replaced by other diamagnetic particles. It may be noted, moreover, that the manganese complex behaves the same as the other ion complexes. Thus, for example, in any of the ion complexes* $Cr\,EDTA^-$, $Mn\,EDTA^{--}$, and Fe EDTA, in Eq. (1) $\alpha \sim 4.2$.



Curves obtained with solutions of nitrate salts: 1) Cu++; 2) Cr+++; 3) Fe+++; 4) Mn++. The upper scale on the absicca axis refers to Curve 1; the lower scale refers to Curves 2, 3 and 4.

3. In the Figure is shown the maximum intensity of the deuteron resonance line χ''_{max} as a function of the molar concentration N of the paramagnetic ion in D_2O. The amplitude of the oscillating field H_1 and the amplitude of the modulating field H_m are fixed. As the quantity N increases the relaxation time T_1 becomes shorter and the line intensity increases, reaching a value which applies in the absence of saturation. However