SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 4 OCTOBER, 1958

CONCENTRATION OF NEGATIVE IONS IN THE PLASMA OF A POSITIVE COLUMN

M. V. KONIUKOV

Tul' skii Pedagogical Institute

Submitted to JETP editor, July 20, 1957; revised version received December 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 908-911 (April, 1958)

The concentration of negative ions in a positive column is considered. The effect of recombination of positive ions and negative ions on the concentration of negative ions close to the axis of the discharge is investigated.

THE negative component of the plasma of a positive column in electro-negative gases consists of electrons and negative ions. Although the nature of the discharge depends, to a great extent, on the concentration of negative ions, in the work reported in Refs. 1 – 4 this concentration was not considered and no attempt was made to investigate the effects associated with charged-particle kinetics in the column. It is the purpose of the present paper to consider these questions.

1. In the case of quasi-neutral ambipolar diffusion, when there is surface recombination of electrons and ions at the walls, the relative concentration of negative ions is obtained from the following charged-particle balance equations:

$$\begin{aligned}\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-D_e\frac{\partial N_e}{\partial r}-b_eN_eE_r\right]\right\}&=ZN_e-\beta N_e,\\ \frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-D_n\frac{\partial N_n}{\partial r}-b_nN_nE_r\right]\right\}&=\beta N_e,\\ \frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-D_p\frac{\partial N_p}{\partial r}+b_pN_pE_r\right]\right\}&=ZN_e,\end{aligned}\qquad(1)$$

where N, D and b are the concentration, diffusion coefficient, and mobility of the electrons (subscript "e"), negative ions (subscript "n") and positive ions (subscript "p"), Z is the ionization coefficient, βN_e is the number of negative ions produced in unit volume per unit time and E_r is the transverse field.

Taking

$$N_n=\varkappa N_e,\qquad(2)$$

where κ is a constant, after some simple transformations* we obtain

$$\frac{\varkappa-2\varkappa(1+\varkappa)D_e/D_p}{1+2\varkappa}\,\frac{Z-\beta}{\beta}+1=0.\qquad(3)$$

Assuming that

$$(Z-\beta)\beta(Z+\beta)^{-2}D_p/D_e\ll 1$$

the solution of Eq. (3) is of the form*

$$\varkappa=(D_e/2D_p)(Z+\beta)/(Z-\beta).\qquad(4)$$

According to Eq. (4), under the conditions which usually apply in a positive column (non-isothermal) the concentration of negative ions is two orders of magnitude larger than the concentration of electrons. This sharp difference in concentrations is a result of the assumption that surface recombination is the only mechanism for the removal of negative ions. Actually, to have a net transport of negative ions to the walls, it is necessary that the diffusion flow to the walls $\kappa D_n(\partial N_e/\partial r)$ be larger than the flux toward the axis which exists by virtue of the radial electric field

$$\frac{\varkappa D_eb_n}{b_e+b_p+2\varkappa b_n}\,\frac{\partial N_e}{\partial r},$$

---

*In carrying out the above simplification the following obvious relations have been used: $D_eb_p\gg D_pb_e$, $D_eb_n\gg D_nb_e$, $b_p\approx b_n$ and $D_n\approx D_p$.

*V. L. Granovskii has called our attention to the fact that the quantity $\varkappa$ as given in Eq. (4) does not approach 0 as $\beta\to 0$. In this limiting process the solution $\varkappa=0$ should result instead of $\varkappa=D_e/2D_p$ as follows from Eq. (4). It might seem that we are dealing with ambiguous solutions of Eq. (1). However, such is not the case. The only solution of the system for $\beta\to 0$ is $\varkappa=D_e/2D_p$ which corresponds to the stationary state in which the diffusion of negative ions is compensated by motion to the axis by virtue of the existing radial electric field. The absence of the explicit solution $\varkappa=0$ is due to the approximate nature of Eq. (1), in which no account is taken of volume recombination, the disappearance of negative ions by virtue of decay, and ion transfer to the anode region. As long as β is large, this approximation remains valid; however, when β becomes small, the equations must be supplemented by appropriate additional terms. Taking account of the latter leads to the appearance of the single stationary solution $\varkappa=0$. From this point of view the limiting transition indicated above is not really valid.

and this situation is possible only when $\kappa > D_e/2D_p$.

2. The existence of high concentrations of negative ions implies small transverse and longitudinal gradients, a situation which is not very probable. Since this is a result of the fact that removal of negative ions has been attributed only to surface recombination at the walls, to obtain the next approximations it is necessary to take account of volume recombination.*

In this case the system of charged-particle balance equations (1) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-D_n\frac{\partial N_n}{\partial r}-b_nN_nE_r\right]=\beta N_e-\alpha N_nN_p,\right.$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-D_p\frac{\partial N_p}{\partial r}+b_pN_pE_r\right]\right\}=ZN_e-\alpha N_nN_p, \tag{1a}$$

where α is the volume recombination coefficient. The electron balance, however, remains unchanged.

It is difficult to find a solution for this system of non-linear equation over the entire cross section of the column. Hence we shall limit ourselves to a calculation of the negative-ion concentration only in the region of the discharge axis, where it can be assumed that κ is constant.† If furthermore, ambipolar quasi-neutral diffusion is assumed, after a series of simple transformations, we obtain a system of equations which determines the concentration of negative ions and the distribution of electrons close to the discharge axis:

$$\frac{(1+\varkappa)(D_eb_p+D_pb_e)+\varkappa(D_eb_n-D_nb_e)}{b_e+\varkappa b_n+(1+\varkappa)b_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N_e}{\partial r}\right)+(Z-\beta)N_e=0, \tag{5}$$

$$\frac{\varkappa(D_eb_n-D_nb_e)-\varkappa(1+\varkappa)(D_pb_n+D_nb_p)}{b_e+\varkappa b_n+(1+\varkappa)b_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N_e}{\partial r}\right)=[\beta-\alpha\varkappa(1+\varkappa)N_e^0]N_e.$$

Eliminating N_e from Eq. (5), we obtain an equation for determining the relative concentration of negative ions at the axis

$$\frac{\varkappa(D_eb_n-D_nb_e)-\varkappa(1+\varkappa)(D_pb_n+D_nb_p)}{(1+\varkappa)(D_eb_p+D_pb_e)+\varkappa(D_eb_n-D_nb_e)}(Z-\beta)+\beta=\alpha N_e^0\varkappa(1+\varkappa), \tag{6}$$

which, since $D_eb_n \gg D_nb_e$, $D_eb_p \gg D_pb_e$, $b_p \approx b_n$, and $D_p \approx D_n$, results in the expression

$$\frac{\varkappa-2\varkappa(1+\varkappa)D_p/D_e}{1+2\varkappa}\frac{Z-\beta}{\beta}+1=\frac{\alpha N_e^0}{\beta}\varkappa(1+\varkappa). \tag{6a}$$

Rather than carry out a general investigation of Eq. (6a) we consider two particular cases:

(a) $\alpha = 0$ and there is no volume recombination. Eq. (6a) becomes Eq. (3) which has already been studied in the first section.

(b) $Z = \beta$ and each electron which appears becomes a negative ion. Equation (6a) assumes the form

$$(\alpha N_e^0/\beta)\varkappa(1+\varkappa)-1=0. \tag{7}$$

Here we are considering the case in which all negative ions are removed by volume recombination. The negative-ion concentration, which is constant over the entire space, is determined by the expression*

$$\varkappa=-{}^1/_2+{}^1/_2\sqrt{1+4\beta/\alpha N_e^0}, \tag{8}$$

which, when $4\beta/\alpha N_e^0 \ll 1$, becomes

$$\varkappa\approx\beta/\alpha N_e^0. \tag{8a}$$

A general investigation of Eq. (6a) is best carried out by graphical methods.† Assuming that

$$D_p/D_e\approx 10^{-2}, \qquad (Z-\beta)/\beta\approx 1$$

(the second condition means that half of all the electrons which are produced lead to the formation of negative ions), we obtain the following table of values of the concentration as a function of the parameter $\alpha N_e^0/\beta$:

$\alpha N_e^0/\beta =$	0	0.001	0.01	0.1	0.5	1
$\varkappa =$	150	34	10.9	3.3	1.15	0.6.

Thus, when volume recombination is taken into account the relative concentration of negative ions at the axis is reduced.‡

3. A direct measurement of the concentration of negative ions is difficult; hence it is easier to examine the variation in the transverse electric

---

*The small cross section for recombination of electrons and positive ions as compared with the cross section for ion recombination[5] means that the former can be neglected.

†If the entire cross section of the column is considered, $\varkappa = \varkappa(r)$. However, over a small region close to the discharge axis it may be assumed that $\varkappa$ is constant. This assumption is strictly justified in the absence of either volume recombination or surface recombination. Moreover, the assumption concerning the proportionality of the charged components used Refs. 3, 4, 6, leads to reasonable results.

*When $\beta \to 0$, the concentration given by Eq. (8) becomes 0.

†The solutions of Eq. (6) lie between two limiting values: $\varkappa = (D_e/2D_p)[(Z+\beta)/(Z-\beta)]$ corresponding to the case in which only surface recombination is considered and $\varkappa = \beta/\alpha N_e^0$ which applies when only volume recombination is considered.

‡The reduction in the concentration of negative ions is due to the possibility of removal even when the resultant negative-ion flux is toward the discharge axis.

field due to these ions.* Let E_r be the transverse field in the column when there are no negative ions and E_r^0 the transverse field in the case in which only surface recombination is assumed for the negative ions. Using the Schottky boundary condition and computing this ratio we find

$$E_r^0 / E_r = (1 + 2\varkappa b_n / b_e)^{-1}. \qquad (9)$$

A measurement of the transverse fields makes it possible to estimate the concentration of charged particles and thus to obtain information on the relative importance of surface recombination and volume recombination.

The destruction of negative ions in collisions should also play a decisive role in negative-ion column kinetics. We propose to examine this question in the future.

In conclusion we wish to express our gratitude to Professor V. L. Granovskii and Docent A. A. Zaitsev with whom we had a number of illuminating discussions and to E. V. Korotkov for help in carrying out the work.

---

*A. A. Zaitsev has indicated a possible method of estimating the transverse field. Since the curvature of the stratum is determined by the magnitude of the transverse gradient, by studying its behavior when electro-negative gases are added it is possible to estimate the relative change in the transverse gradient.

[1] R. Holm, Z. Physik **75**, 171 (1932).

[2] R. Seeliger and R. Kruschke, Physik Z. **34**, 883 (1933).

[3] A. von Engel and M. Steenbeck, Elektrische gasentladungen, ihre physik und technik, Vol. I, (Berlin, 1934).

[4] R. Seeliger, Ann. Physik **6**, 93 (1949).

[5] V. L. Granovskii, Электрический разряд в газах (Electrical Discharges in Gases) Vol. I (1952).

[6] G. Ecker, Proc. Phys. Soc. (London) **B67**, 485 (1954).

Translated by H. Lashinsky
181

SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 4 OCTOBER, 1958

MONTE CARLO CALCULATION OF AN ELECTRON-PHOTON CASCADE IN LEAD

V. V. CHAVCHANIDZE, R. S. SHADURI, and V. A. KUMSISHVILI

Institute of Physics, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor September 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 912-915 (1958)

A statistical-probability method, based on random trials, is given for using a model to calculate the effects in an electron-photon cascade initiated in a layer of lead by a γ-ray quantum. This is a modification of the Monte Carlo method. Some typical curves calculated in this way are presented. Two methods are indicated for carrying out this sort of calculations by the use of high-speed electronic calculating machines.

WE have used a method of random trials, which is a modification of the well known Monte Carlo method,[1-3] for the calculation of the electron-photon cascade shower produced in a lead plate of thickness 0.5 cm by a γ-ray quantum of energy 500 Mev. The problem reduces to a stepwise reproduction of the picture of the natural process of the production of a cascade shower by a single primary particle. The "feeding in" of a large number of primary particles gives the distributions in energy and angle of the electrons, positrons, and γ-ray quanta emerging from the plate.

We have followed essentially the method of "drawing" given in the papers of Chavchanidze.[4,5] In the drawing we used tables of random numbers,[6] which make it unnecessary to use machines to ob-