The successful conduct of the investigation was greatly assisted by advice and active participation in discussions on the part of E. I. Zababakhin, S. B. Kormer, E. A. Negin and G. I. Gandel' man. In the initial stages some very valuable experimental information was obtained by D. M. Tarasov and A. A. Bakanova. The numerous complicated experiments were performed with the aid of the technicians A. A. Zhiriakov, S. P. Pokrovskii and A. N. Kolesnikova. The authors are extremely gratetul to all of these colleagues.

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SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 4 OCTOBER, 1958

DYNAMIC COMPRESSIBILITY OF METALS UNDER PRESSURES FROM 400,000 TO 4,000,000 ATMOSPHERES

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A method for the determination of pressures and densities of shock compressions is proposed which is based on the measurement of the velocities of propagation of strong shock waves. The dynamic compressibility of copper, zinc, silver, cadmium, tin, gold, lead and bismuth were measured by this method in the pressure range from 400,000 to 4,000,000 atm. The highest degrees of compression (by factors 2.26 and 2.28) were observed in lead and bismuth, which possess the largest atomic volumes. The highest absolute density (32.7 g/cm^3) was recorded for gold.

INTRODUCTION

DYNAMIC methods of investigation in high pressure physics are based on the compression of matter by means of strong shock waves. Experimentally measurable parameters of shock waves are D, the velocity of propagation of a wave front in an undisturbed medium, and U, the velocity of matter behind the wave front. Having determined these parameters, from mass and momentum conservation we obtain the density

$$\rho = \rho_0 D / (D - U) \tag{1}$$

and the pressure of a shock compression

$$P = \rho_0 U D. \tag{2}$$

For the complex determination of shock wave parameters we[1] have developed two methods of investigation, the "splitting-off" method and the "deceleration" method.

In the "splitting-off" method measurements are obtained of the velocity of a shock wave in an obstacle and of the velocity W of the free surface of the obstacle after the shock wave passes through it. W is approximately twice the velocity of matter behind the wave front. The "deceleration" method, which involves very exact initial premises, is based on the recording of the velocity of a "shock driver" and of the shock wave velocity in the target following the shock. These methods were used[1] to obtain shock compression curves for iron of varying initial density. On the basis of the experimental findings, an equation of state of iron was derived which is valid under pressures of hundreds of thousands or millions of atmospheres.

Knowledge of the shock adiabat of any one material such as iron greatly simplifies measurements of the dynamic compressibility of any other materials. As will be shown below, to obtain points of shock adiabats it is now sufficient to record a single parameter, the velocity of propagation of the shock wave. Measurements of mass velocities, which are considerably more complicated from an experimental point of view, become unnecessary. This method, which was developed by the present authors in collaboration with G. M. Gandel'man, is used to study the decay of a strong discontinuity when a shock wave is reflected from the boundary between two media. This will be designated briefly as the "reflection" method.

The present paper describes the essentials of the method and gives the results obtained for the dynamic compressibilities of copper, zinc, cadmium, tin, silver, gold, lead and bismuth under pressures from 400,000 to 4,000,000 atmospheres.

For each of these substances, the knowledge of only one dynamic adiabat is insufficient to establish the equation of state which relates pressure, temperature and density. Nevertheless, information regarding shock compressibility under pressures of hundreds of thousands and millions of atmospheres is very valuable for the testing of theoretical ideas concerning the behavior of matter under these conditions. Such information also provides a necessary link in the chain of experimental studies which lead to the establishment of empirical equations of state at high pressures.

EXPERIMENTAL METHOD

We shall consider the passage of a shock wave of known amplitude from medium A with a known equation of state into the investigated substance B (see the path-time diagram in Fig. 1). Reflection

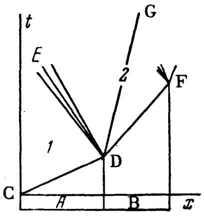

FIG. 1. x – t diagram of the reflection method. CD is the shock wave in medium A; DF is the shock wave in medium B; DE is the reflected wave in medium A; DG is the trajectory of the boundary.

of the shock wave from the interface is accompanied by decay of the discontinuity and the formation of both a transmitted and a reflected wave. The transmitted wave is always a shock compression wave. The reflected wave can be a compression wave if the dynamic rigidity of medium B is greater than the dynamic rigidity of medium A, or it is a rarefaction wave in the opposite case.

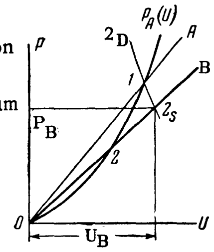

FIG. 2. P – U diagram of the reflection method. $P_A(U)$ is the shock adiabat of medium A; OA is the wave line of medium A; OB is the wave line of medium B; $1-2_s$ is the isentropic rarefaction curve from state 1; $1-2_D$ is the deceleration adiabat from state 1.

In the pressure-velocity diagram (Fig. 2), the dynamic adiabat of a single shock compression of medium A is represented by the parabolic curve $P_A(U)$, and the state in medium A before reflection from the interface by point 1 of this adiabat. After reflection, the new state 2 in medium A lies on the isentropic rarefaction curve $1-2_D$ of substance A or on its double compression shock adiabat $1-2_D$. The geometric locus of the possible states in substance B, in which the shock wave is propagated with velocity D_B, is the straight line OB, $P = \rho_0 D_B U_B$, which satisfies conservation equation (2). In Region 2 (Fig. 1) on both sides of the interface, both in the reflected wave in A and in the transmitted wave in B, the pressures and velocities are equal. In the P – U diagram (Fig. 2) this condition is represented by point 2, the intersection of the $2_s - 2_D$ curve with the "wave line" OB of medium B. The coordinates of the point of intersection determine the pressure P_B and the velocity U_B of one of the points of the dynamic adiabat of substance B. The density is found by substituting the values of U_B and D_B in Eq. (1).

For application of the reflection method, it is necessary to have available at least one substance of known dynamic adiabat and known equation of state, which is required for plotting of the rarefaction and deceleration adiabats of the barrier plate. The experimentally measured quantities in the reflection method are the parameters of state 1 and the wave velocity D_B which determines the slope of OB. For many materials, including iron, the curve $2_S - 2_D$ is very accurately approximated by the mirror image of the basic adiabat P_A. For the investigated substances whose wave lines pass close to point 1, the error resulting from this approximation is vanishingly small.

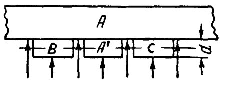

FIG. 3. Measurement of wave velocities by the reflection method. A – iron plate; A′ – specimen of iron; B, C – test specimens; d – reference measuring base.

Figure 3 is the schematic drawing of a typical experiment for determining compressibility by the reflection method. A plane shock wave produced by an explosion passes through an iron plate to which samples of iron and of the investigated substances are attached. State 1 in the transmitted wave before reflection is determined in the same experiments as the wave velocities in the test specimens, from the velocity D_{Fe} of the shock wave in the iron specimen. In the pressure-velocity diagram (Fig. 2) D_{Fe} determines the wave line OA, which forms the angle $\alpha = \arctan(\rho_{0Fe} D_{Fe})$ with the horizontal axis. State 1 is given by the point of intersection of OA with the known adiabat $P_A(U)$ of single compression of the iron plate. According to Ref. 1 the equation of the shock adiabat of iron at pressures from 400,000 to 4,000,000 atm is given by the following relation between D and U:

$$D_{Fe} = C_0' + \lambda U, \quad C_0' = 3.8 \text{ km/sec}, \quad \lambda = 1.58. \qquad (3)$$

In the coordinates pressure and velocity,

$$P_{Fe} = \rho_{0\,Fe}(C_0' + \lambda U)\, U \cdot 10^{10} \text{ bars} \qquad (4)$$

and in the coordinates pressure and relative density,

$$P_{Fe} = \frac{\rho_0 C_0'^2}{(\lambda - 1)^2} \frac{\sigma(\sigma - 1)}{[\lambda/(\lambda - 1) - \sigma]^2} \cdot 10^{10} \text{ bars}. \qquad (5)$$

The time required for passage of a shock wave through a test specimen was the difference between the time of shorting of the electrical pickup against the lower surface of the specimen and the average time of shorting of the upper pickups against the iron plate around the specimen. Signals were transmitted to the deflection plates of an oscilloscope with a driven sweep velocity of 50 km/sec. A 10-megacycle sine wave was applied to the oscilloscope beam as a scale. The accuracy of the time intervals was 5×10^{-9} sec. The measuring base, which was equal to the thickness of a specimen, was 6 – 8 mm and the thickness of the iron plate was 6 to 9 mm. The wave velocity was obtained through division of the reference base thickness by the measured time. In order to prevent reduction of the wave velocities by the unloading waves from the edges of the specimens, the specimen diameters were twice their thickness. Specimens were fastened to the plate by pieces of Wood's alloy with the necessary openings for insulated wires. Three specimens (A′, B, C in Fig. 3) were used in each trial.

EXPERIMENTAL RESULTS

The dynamic compressibility of the metals was measured in three runs which differed with regard to the shock wave pressure in the iron plate. In the first run the pressures were produced by reflecting a detonation wave from the outer surface of the plate.

In the measurements for zinc, cadmium, lead, bismuth and tin, the plate was 10 mm thick and the specimens were 8 mm thick. Thus the middle of the reference thickness was 14 mm from the first surface. At this distance the velocity of the shock wave in the iron specimens was found to be 5.30 km/sec, corresponding to a pressure of 0.40×10^{12} bars. Measurements on copper, silver and gold were made with a plate of somewhat lesser thickness. Under these conditions at the middle of the reference thickness $D_{Fe} = 5.38$ km/sec and the pressure was 0.42×10^{12} bars.

In the second run the dynamic compressibility of all of the tested metals was determined at a pressure of 1.40×10^{12} bars in the iron plate; in the third run, at 3.60×10^{12} bars. As previously, the characteristics of the shock wave in iron refer to a surface passing through the middle of the reference thickness.

The velocities of propagation of the shock waves are given in Table I, as well as the parameters of the shock waves in the iron plates, the initial densities ρ_0 of the test specimens and the velocities C_0 of the propagation of acoustic waves of volume compression. The latter were calculated by means of the formula $C_0 = 1/\sqrt{\rho_0 \kappa}$, using the initial density of the material and the volume compression coefficient κ taken from Ref. 2.

As was shown in the preceding Section, the data in Table I are sufficient for an unambiguous determination of the pressures and densities of shock compression. Table II contains the corresponding parameters of the shock adiabat points.

The experimental results can be used to determine the shock adiabats by means of Eqs. (1) and (2) with interpolated relations between the wave and mass velocities (Fig. 4). The results of this calculation are given in Table III and are repre-

TABLE I

Metal	Cu	Zn	Ag	Cd	Sn	Au	Pb	Bi	Parameters of shock wave in iron plate			
ρ_0, g/cm³	8.93	7.14	10.49	8.64	7.28	19.30	11.34	9.80				
C_0, km/sec	3.95	2.92	3.08	2.34	2.64	2.98	1.91	1.85	D, km/sec	U, km/sec	P, 10^{12} bars	ρ/ρ_0
D, km/sec I	5.36	—	4.69	—	—	4.27	—	—	5.38	1.00	0.42	1.23
	—	4.70	—	4.10	4.20	—	3.52	3.37	5.30	0.95	0.40	1.23
II	7.13	6.85	6.76	6.32	6.36	5.70	5.33	5.36	7.53	2.36	1.40	1.46
III	10.16	9.90	9.45	9.14	9.02	8.06	7.65	7.94	10.63	4.32	3.60	1.69

TABLE II

Metal	Series I			Series II			Series III		
	ρ/ρ_0	P, 10^{12} bars	U, km/sec	ρ/ρ_0	P, 10^{12} bars	U, km/sec	ρ/ρ_0	P, 10^{12} bars	U, km/sec
Cu	1.21	0.45	0.94	1.47	1.46	2.29	1.70	3.80	4.19
Zn	1.28	0.35	1.04	1.59	1.24	2.54	1.87	3.26	4.61
Ag	1.25	0.46	0.93	1.48	1.55	2.19	1.75	4.01	4.05
Cd	1.33	0.36	1.02	1.63	1.33	2.44	1.94	3.49	4.42
Sn	1.35	0.33	1.08	1.69	1.20	2.59	2.10	3.10	4.73
Au	1.20	0.59	0.71	1.45	1.95	1.78	1.69	5.13	3.30
Pb	1.38	0.39	0.97	1.78	1.41	2.34	2.26	3.70	4.26
Bi	1.45	0.35	1.05	1.86	1.30	2.47	2.28	3.45	4.45

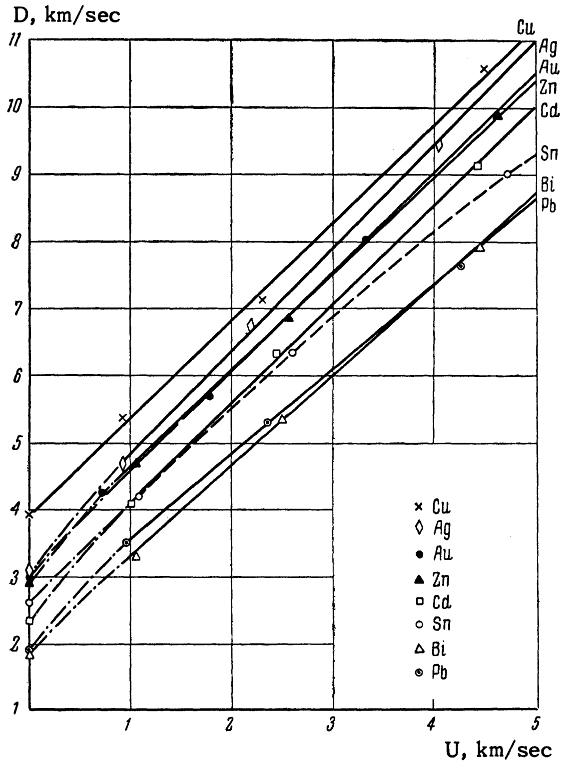

FIG. 4. D – U diagram of the tested metals. The solid lines are portions of curves which are approximately represented by the linear relation $D = C_0' + \lambda U$.

sented graphically in Fig. 5 in the coordinates pressure and relative compression $\sigma = \rho/\rho_0$.

Figure 4 shows that for all of the metals studied except tin the relation between D and U for $U > 1$ km/sec is approximated with sufficient accuracy by the linear relation $D = C_0' + \lambda U$. The dynamic adiabats can then be represented by Eq. (5). The values of C_0' and λ are given in Table IV. Since the constants C_0' and λ were selected from the requirement of the best possible approximation for the entire investigated velocity range, C_0' differs somewhat from the actual initial sound velocities given in Table I.

We note that at pressures of the order of 400,000 atm the results of the present work agree with the values of the dynamic compressibility of copper and zinc in Ref. 3.

CONCLUSION

Table V gives for zero pressure and 3,500,000 atm the relative compression σ, the gram-atomic volume V_A, the wave velocity D, the mean shock compression modulus $\rho_0 D^2$ and the increment ΔE of internal energy of all of the metals studied including iron.

This table shows a definite relation between the

TABLE III

σ	P, 10^{12} bars							
	Cu	Zn	Ag	Cd	Sn	Au	Pb	Bi
1.1	0.16	0.09	0.12	0.07	0.05	0.25	0.05	—
1.2	0.40	0.21	0.34	0.17	0.14	0.56	0.14	—
1.3	0.71	0.38	0.63	0.32	0.27	1.01	0.26	—
1.4	1.14	0.61	1.04	0.52	0.43	1.63	0.42	0.29
1.5	1.72	0.91	1.61	0.79	0.65	2.45	0.60	0.43
1.6	2.49	1.32	2.40	1.15	0.92	3.57	0.82	0.59
1.7	3.52	1.85	3.51	1.64	1.26	5.06	1.09	0.80
1.8	—	2.57	5.01	2.30	1.65	—	1.40	1.06
1.9	—	—	—	3.22	2.09	—	1.79	1.39
2.0	—	—	—	4.49	2.57	—	2.25	1.80
2.1	—	—	—	—	3.10	—	2.80	2.31
2.2	—	—	—	—	3.77	—	3.46	2.95
2.3	—	—	—	—	—	—	4.26	3.76

degree of compression and the initial atomic volume. Elements with large initial atomic volumes are compressed more strongly that those with smaller atomic volumes. Thus, at 3,500,000 atm the density of bismuth is increased by a factor of 2.28 and that of iron by only 1.67. The ratio of the atomic volumes of these elements, which at atmospheric pressure is 3, reduces to 2.2 at 3,500,000 atm.

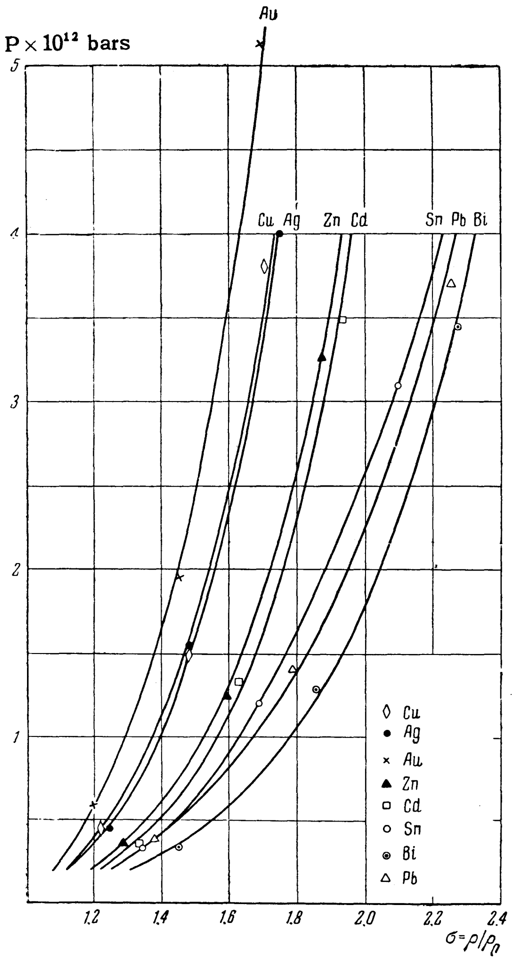

FIG. 5. Dynamic adiabats of copper, zinc, silver, cadmium, tin, gold, lead and bismuth.

TABLE IV

Metal	C_0', km/sec	λ	ρ_0, g/cm^3
Cu	3.90	1.46	8.93
Zn	3.20	1.45	7.14
Ag	3.30	1.54	10.49
Cd	2.65	1.48	8.64
Au	3.15	1.47	19.30
Pb	2.30	1.27	11.34
Bi	2.00	1.34	9.80

With higher pressure there is a many-fold increase in the wave velocity and mean shock compression modulus, which is the ratio of the applied pressure to the relative reduction of volume. The change of the modulus indicates a strong increase in the elasticity of the material with higher densities and temperatures. For copper, the shock wave velocity at 3,500,000 atm exceeds the sound velocity at atmospheric pressure by the factor 2.46, while the modulus $\rho_0 D^2$ increases by about the factor 6. For lead at 3,500,000 atm, the wave velocity increases by the factor 3.93 and the shock compression modulus by the factor 15.

At high pressures there is a large increment of the specific internal energy, which is calculated from the equation

$$E = P(v_0 - v)/2.$$

Thus the internal energy of 1 g of bismuth at 3,500,000 atm exceeds the explosive energy of 1 g of TNT by the factor 2.5. The present paper does not include the question as to what portion of this energy is thermal or the relation between the thermal and elastic pressures.

In conclusion the authors wish to thank the following for their assistance: A. N. Kolesnikova,

TABLE V

Element	σ $P = 3.5 \times 10^{12}$ bars	V_A, cm^3/g-atom		D, km/sec		$\rho_0 D^2 \cdot 10^{12}$ bars		ΔE 10^{10} erg/g
		$P = 0$	$P = 3.5 \cdot 10^{12}$ bars	$P = 0$	$P = 3.5 \cdot 10^{12}$ bars	$P = 0$	$P = 3.5 \cdot 10^{12}$ bars	$P = 3.5 \cdot 10^{12}$ bars
Fe	1.67	7.12	4.26	4.63	10.53	1.68	8.70	9.0
Cu	1.70	7.11	4.18	3.95	9.75	1.39	8.48	8.1
Zn	1.89	9.16	4.84	2.92	10.19	0.61	7.41	11.0
Ag	1.71	10.28	6.01	3.08	8.96	0.99	8.42	6.9
Cd	1.93	13.01	6.72	2.34	9.15	0.47	7.23	10.0
Sn	2.16	16.30	7.54	2.64	9.44	0.51	6.49	13.0
Au	1.59	10.22	6.43	2.98	6.99	1.71	9.43	3.3
Pb	2.21	18.27	8.25	1.91	7.5	0.41	6.38	8.0
Bi	2.27	21.32	9.39	1.85	7.99	0.33	6.25	10.0

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Note added in proof (March 10, 1958). The results obtained in the first experimental series also agree with the measurements of the dynamic compression of metals from 150,000 to 400,000 atm [Walsh, Rice, McQueen and Yarger, Phys. Rev. **108**, 196 (1957)], which were brought to the attention of the present authors after this paper was sent to press.

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EXPERIMENTAL DETERMINATION BY AN OPTICAL METHOD OF THE STRESSES IN AN ANISOTROPIC PLATE UNDER THE ACTION OF A CONCENTRATED FORCE. II

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The state of stress in an anisotropic plate made of a 60% TlBr + 40% TlI single crystal of the cubic system was investigated by an optical method. The case of a concentrated force directed along the (110) direction is considered.

1. The present paper is a supplement to Ref. 1, in which an attempt at an optical method of the investigation of stress in anisotropic media was carried out. The theoretical bases of this method was set forth in Ref. 2, while a more detailed methodological instructions were given in Ref. 3. In Ref. 1, the problem of the effect of a concentrated force on an anisotropic plate cut from a single crystal of the alloy TlI + TlBr parallel to the plane of the cube was considered (the crystal belonged to the cubic system). The direction of the force coincided with the direction of the maximum of the modulus E, i.e., with the (100) direction.

According to the method given previously, the stresses at an arbitrary point of an anisotropic plate are determined from the formulas

$$\tan 2\beta = k \tan 2\varphi, \quad (1)$$

$$\delta = C_\beta d (\sigma_1 - \sigma_2), \quad (2)$$

where β is the angle determining the direction of