# INVESTIGATION OF MULTIPLE SCATTERING OF PROTONS

F. R. ARUTIUNIAN

Physics Institute, Academy of Sciences, Armenian S.S.R.

Submitted to JETP editor September 17, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 800-806 (April, 1958)

Multiple scattering of protons with energies from 90 to 200 Mev and with energies from 40 to 60 Mev was investigated respectively in lead and copper plates of various thicknesses. The experimental data are compared with the multiple Coulomb scattering curves for a point and extended nucleus. A consequence of diffraction scattering is that the experimental results do not agree with the theoretical values for a finite sized nucleus and exceed them. The effect of nuclear scattering depends on the relation between the angle $\theta = \lambda\!\!\!^{-}/R$ and the mean square angle of multiple Coulomb scattering. The integral cross section for nuclear scattering of protons $\sigma = (0.0755 \pm 0.0375)\sigma_{geom}$ has been obtained in the angle region in which Coulomb scattering is small compared with nuclear scattering. This value is in good agreement with the optical model of an "absolutely black" nucleus.

## 1. INTRODUCTION

WE have investigated the scattering of protons with energies of 90 to 200 Mev in lead plates 7 and 4 mm thick, and from 40 to 60 Mev in 5 and 2 mm copper plates, located in a Wilson chamber. Cosmic ray protons, stopped by ionization in the Wilson chamber plates, were selected, and their momentum prior to entering the chamber was measured with a magnetic mass spectrometer. A detailed description of the experimental setup is given in Ref. 1.

The conditions for selecting particles for the scattering investigation were such that it was possible to observe elastic scattering only. The mass of an inelastically-scattered proton, determined from its momentum and range, is not the true proton mass, and such cases are not investigated. However, there is a spread in the proton masses, because of the errors in measuring the momentum and range. If one assumes that a particle corresponds to the selected upper limit of the interval for the proton masses, equal to 2300 $m_e$, not because of errors but on account of inelastic scattering, then the value of the transferred momentum will be $\Delta p_{inel} = 0.8 \times 10^8$ ev/c, which constitutes 15% of the particle momentum itself. The proton mass spectrum is well approximated by a Gaussian distribution which shows that the spread of masses is due to the errors of measuring momentum and range and not to some inelastic processes. Besides, the relatively small value of $\Delta p_{inel}$ gives reason to believe that the investigated scattering is elastic.

The projections of scattering angles in plates were investigated from zero to the maximum observable. The angles were measured independently by two observers, from photographs, by means of a protractor. A large number of measurements were averaged. The mean-square error in measuring the angle was 0.6°.

It is to be noted that only those scattering angles were investigated which corresponded to the residual tracks, equal to two or more plates; thus, if the plate in which the particle has stopped is assigned the number $k = 0$ and the plates are numbered consecutively upward from this plate, then the investigated scattering pertains only to plates with numbers $k \geq 2$. The scattering was not investigated in the plate $k = 1$, since in this case the large uncertainty in the track lowered considerably the precision of the momentum determination.

The protons were divided according to their residual tracks into four groups (I, II, III, and IV). Groups I, II, and III correspond to plates with numbers $k = 2$, 3, and 4. Group IV corresponds to plates with numbers $k = 5$ and 6.

Inasmuch as the curves of multiple Coulomb scattering are dependent very strongly on the momentum p (more accurately, on the quantity $\pi = p\beta c$), it was measured by two methods for a more precise determination. The momentum was determined, on one hand, from the residual track and on the other hand, from the momentum measured in a magnetic field and from the amount of matter traversed by the particle up to the scattering point. For each group, the average momenta calculated by both methods agree well with each other (with a precision to 2%). The momenta and energies of investigated protons for each group are shown in Table I.

TABLE I

| Group | Momentum p, $10^8$ ev/c | Energy E, Mev |
|---|---|---|
| I | 4.6±0.25 | 104±13 |
| II | 5.15±0.24 | 132±11 |
| III | 5.6±0.21 | 152±10 |
| IV | 6.17±0.16 | 186±9 |

The average momentum for each individual group has an uncertainty $\Delta p$, determined by the finite plate thickness in which the particle stops. Corrections were made for the multiple Coulomb-scattering curves, plotted with the average value of $\pi$, which take into account the existing uncertainty in the momentum determination.

The elastic proton scattering in plates, in the region of small angles, is determined primarily by multiple Coulomb scattering. Only for large angles ($\theta \gg \lambda\!\!\!^{-}/R$) does the scattering have in practice a purely nuclear aspect. The presence of nuclear scattering for protons leads to the conclusion that the experimental curves differ from the curve of Coulomb scattering.

## 2. ON THE THEORY OF MULTIPLE SCATTERING OF NUCLEAR-ACTIVE PARTICLES

The distribution function of multiple Coulomb scattering for a point nucleus was calculated by Moliere.[2,3] The nuclear dimensions begin to show up when the momentum change of the scattered particle is of the order of the "nuclear momentum," $\Delta p = p\theta \sim \hbar/R$, i.e., for angles $\theta \gtrsim \lambda\!\!\!^{-}/R$.

Scattering curves that take into account the finite nuclear dimensions, applicable to the scattering of $\mu$ mesons with momenta $p = (1.0 - 2.0) \times 10^8$ ev/c, were obtained and communicated to us by M. L. Ter-Mikaelian. The differential Coulomb scattering cross section (neglecting the atomic form factor) has the form

$$\sigma_C(\theta)\, d\Omega = \sigma_R(\theta)\, F(y) d\Omega, \quad y = \theta R / \lambda\!\!\!^{-}, \tag{1}$$

where $\sigma_R(\theta) = 4\alpha^2\lambda\!\!\!^{-2}/\theta^4$ — the Rutherford scattering cross section, $\alpha = Ze^2/\hbar v$, and $F$ is a nuclear form factor which drops off rapidly with $\theta R/\lambda\!\!\!^{-} \gg 1$ and which equals 1 for $y \ll 1$. Its values for intermediate values of $y$, according to Hofstadter's data,[4] are: $y = 0.5$; $F(y) = 0.91$; $y = 1.0$; $F(y) = 0.64$; $y = 2.0$; $F(y) = 0.294$.

From the comparison of the multiple Coulomb scattering curves for the point and extended nucleus it can be seen that the smaller the angle $\theta = \lambda\!\!\!^{-}/R$ in relation to the mean square of Coulomb scattering $\chi_C\sqrt{B}$ (Ref. 2, 3), i.e., the larger the atomic number $Z$ and the thickness of the scattering substance, the greater the influence of the nuclear dimensions. However, if there is also a nuclear interaction, then for the same momentum change $\Delta p = \hbar/R$ one must also take into account the nuclear forces; therefore the calculation of the finite nuclear dimensions in Coulomb scattering and of the nuclear field must be done simultaneously. It is clear that the nuclear scattering must be taken into account if its cross section is (for some scattering angles) comparable to the Coulomb scattering cross section. For the differential nuclear scattering cross section we shall use the data on neutron scattering for an "absolutely black nucleus."[5]

$$\sigma_{nuc}(\theta)\, d\Omega = R^2 \,|\, J_1(y)/\theta \,|^2\, d\Omega. \tag{2}$$

The nuclear and Coulomb cross sections become equal at angles $\theta_1 = 2\alpha\lambda\!\!\!^{-}/R$ for $\alpha \gtrsim 1$ and $\theta_2 = 2\sqrt{\alpha\lambda\!\!\!^{-}}/R$ for $\alpha \ll 1$. If angles $\theta_1$ and $\theta_2$ exceed significantly the mean-square angle of multiple scattering ($\alpha \gg 1$), then the Coulomb interaction predominates over the nuclear one in the angle region of interest to us. It is then valid to take into account the nuclear dimensions which manifest themselves at deviation angles $\lambda\!\!\!^{-}/R$, and disregard nuclear scattering which becomes of the order of Coulomb scattering for angles much larger than $\lambda\!\!\!^{-}/R$. When $\alpha \approx 1$, the effects of nuclear dimensions and nuclear scattering are of the same order, but when $\alpha \ll 1$ the nuclear scattering shows up considerably earlier than the nuclear form factor in Coulomb scattering. The thicker the scattering substance and the higher its $Z$, the larger the mean-square angle of multiple Coulomb scattering $\chi_C\sqrt{B}$ and the greater the effect of nuclear scattering.

The total differential scattering cross section is

$$\sigma_{total}(\theta)\, d\Omega = \sigma_R(\theta)\, N(y, \alpha)\, d\Omega, \tag{3}$$

where $N(y, \alpha)$ is the coefficient which accounts simultaneously for the effects of the finite nucleus dimensions and of nuclear scattering.

Following the calculations of Ref. 5, we can simply use the coefficient $N(y, \alpha)$, which must be multiplied by the Rutherford scattering cross section in order to obtain the correct formula for the single scattering cross section, which in turn yields the curve of multiple scattering. This coefficient, for $\alpha \lesssim 1$, is everywhere larger than unity and only for $\alpha \gg 1$ is it smaller than unity. This is well illustrated by a series of experimental curves in Ref. 6, which give the dependence of $N(y, \alpha) = \sigma_{total}/\sigma_R$ on the scattering angle for 22-Mev protons for a large number of elements. For light elements $\alpha < 1$ and $N(y, \alpha) > 1$, and $N(y, \alpha) < 1$ only for very heavy elements. We shall use these experimental curves for comparison with our experimental data.

As follows from calculations in Ref. 5, the experimental curves for the dependence of $N(y, \alpha)$ on the scattering angle, cited in Ref. 6, depend only on the magnitude of $\alpha$. Having the interval of velocities for protons under consideration ($\beta = 0.44 - 0.55$ for lead and $\beta = 0.3 - 0.35$ for copper), we determine the corresponding values of $\alpha$ for our conditions:

$$\alpha_{Pb} = 1.37 - 1.1 \text{ and } \alpha_{Cu} = 0.723 - 0.625.$$

We next determine the equivalent elements which correspond to our values of $\alpha_{Pb}$ and $\alpha_{Cu}$ for 22-Mev protons.

TABLE II

| $\theta°$ | $y = \theta R/\lambda\!\!\!^{-}$ | $N(y)$ | | |
|---|---|---|---|---|
| | | Ref. 5 $\alpha = 0.2$ | Ref. 5 $\alpha = 1.73$ | Ref. 6 |
| 0 | 0 | 1 | 1 | 1 |
| 2 | 0.705 | 3.8 | 1.6 | 1 |
| 4 | 1.41 | 7.10 | 3.5 | 1.1—1.4 |
| 6 | 2.11 | 12.5 | 6.7 | 1.1—1.3 |
| 8 | 2.82 | 12.1 | 6.4 | 1.4 |
| 10 | 3.53 | 6.0 | 4.0 | 1.3 |
| 12 | 4.23 | 5.0 | 2.60 | 1.2 |
| 14 | 4.92 | | | 1.05 |
| 16 | 5.63 | | | 1.0 |
| 18 | 6.34 | | | 0.95 |

TABLE III

| $\theta°$ | $y = \theta R/\lambda\!\!\!^{-}$ | $N(y)$ | | |
|---|---|---|---|---|
| | | Ref. 5 $\alpha = 0.2$ | Ref. 5 $\alpha = 1.73$ | Ref. 6 |
| 0 | 0 | 1 | 1 | 1 |
| 2 | 0.208 | 1 | 1 | 1 |
| 4 | 0.416 | 1 | 1 | 1 |
| 6 | 0.624 | 3.4 | 1.6 | 1 |
| 8 | 0.832 | 4.0 | 1.4 | 1.05 |
| 10 | 1.04 | 4.2 | 1.5 | 1.15 |
| 12 | 1.248 | 5.6 | 2.0 | 1.20 |
| 14 | 1.456 | 6.5 | 3.4 | 1.17 |

Tables II and III show the values of $N(y, \alpha)$ for lead (from the graph for Nb) and copper (from the graph for Fe), respectively, in the region of angles investigated by us. The same tables show the coefficients $N(y, \alpha)$ as calculated in Ref. 5 for $\alpha = 0.2$ and $\alpha = 1.73$. These coefficients, in the region of scattering angles investigated by us, are always greater than unity, whether obtained from the experimental data[6] or from the theoretical calculations.[5] Note, however, that the use of the experimental data of Ref. 6 for $N(y, \alpha)$ is based on Ref. 5 and that the coefficients $N(y, \alpha)$ according to theoretical calculations[5] are considerably higher than the same coefficients according to the experimental data of Ref. 6. It is possible that the coefficients $N(y, \alpha)$ as calculated in Ref. 5 have higher values because no account was taken of the diffuseness of the nuclear edge or of its transparency. However, there are no experimental and theoretical papers covering the energies and substances of interest to us.

As far as the degree of blackness is concerned, since the range in nuclear matter, for neutrons and consequently for protons with energies of 20 to 140 Mev, is approximately constant, (it varies from $3 \times 10^{-13}$ to $4 \times 10^{-13}$ cm in the interval of proton energies investigated by us, i.e., 90 to 200 Mev for lead and 40 to 60 Mev for copper), then, to a first approximation, one can consider the nuclei of lead and copper as "absolutely black."

We can thus conclude that allowance for the nuclear interaction and nuclear dimensions leads to an increase in the Rutherford scattering cross-section for the point nucleus, in the angle region of interest to us, and not to a decrease due to finite nuclear dimensions, as occurs for particles which undergo only Coulomb interactions. The experimental data for protons must lie above the Molière curve, or at least coincide with it when $N(y, \alpha)$ is close to unity.

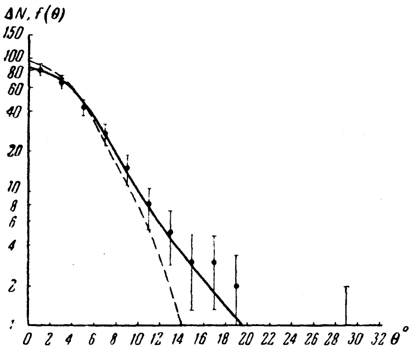


FIG. 1. Differential distribution of scattering angles for protons in 7 mm lead plates for group I. Solid line – curve of multiple Coulomb scattering for a point nucleus, dotted line – for extended nucleus; n = 254.

## 3. EXPERIMENTAL RESULTS AND THEIR EVALUATION

In Fig. 1 are shown the experimental distribution of scattering angles for one of the groups (group I) for scattering by 7 mm Pb, and the corresponding theoretical curves of multiple Coulomb scattering for a point and an extended nucleus. The scattering-angle distributions for the remaining groups are similar in nature and are therefore not shown. Since the projections of scattering angles are under study, all theoretical relationships and curves, used for comparison with experimental data, pertain to scattering-angles projections.

The "$\chi^2$ criterion" can be used to check the agreement between the experimental data and the theoretical curves. For group I $(P_{\chi^2})_{extend} = 10^{-5}$ and $(P_{\chi^2})_{point} = 0.94$. It follows from all these curves that the experimental data agree with the scattering curve for the point nucleus, but differ markedly from the curve for an extended nucleus.

In Fig. 2 are shown the combined scattering-angle distributions for all the groups (I, II, III, IV). The scattering angles are given in units of $\varphi$, where $\varphi$ is the relative scattering angle ($\varphi = \theta/\chi_c\sqrt{B}$). The superiority of such a construction lies in the fact that we can combine all of the groups, by previously constructing the distributions according to $\varphi$ for each group separately, and then summing them up.

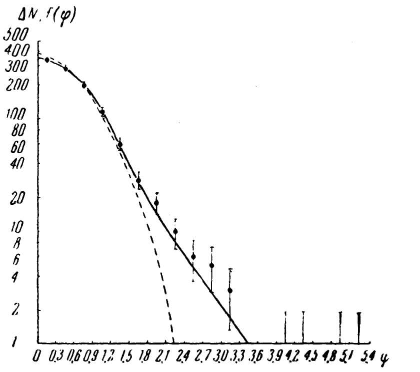


FIG. 2. Differential distribution of scattering angles for protons in 7 mm lead plates for all the groups combined. Solid line – curve of multiple Coulomb scattering for a point nucleus, dotted line – for extended nucleus; n = 1060.

The experimental curves for angles $\varphi > 1.8$ (Fig. 2) are less steep than the scattering curve for the point nucleus. This is explained by the fact that with large angles, the diffraction scattering cross-section is on the average inversely proportional to the cube of the scattering angle [Eq. (2)], whereas the pure Coulomb scattering cross section to its fourth power [Eq. (1)].

There are relatively few experimental data on proton scattering in 4 mm lead plates. They are therefore not subdivided into groups and the combined graph for all the groups is shown (Fig. 3), although the construction of the experimental and theoretical distributions was made by intervals, and the data were summed afterwards. It is seen from Fig. 3 that the experimental data agree equally well with the point nucleus and with the curve of the extended nucleus $[(P_{\chi^2})_{point} = 0.94]$ and $[(P_{\chi^2})_{extend} = 0.93]$.

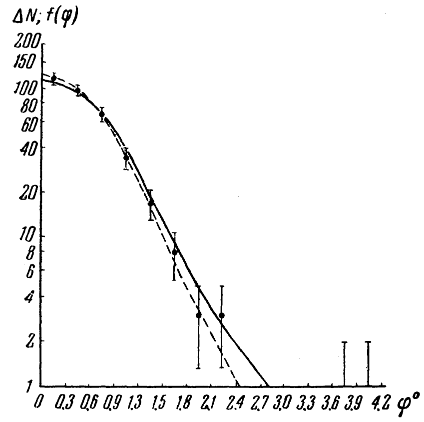


FIG. 3. Differential distribution of scattering angles for protons in 4 mm lead plates. Solid line – curve of multiple Coulomb scattering for a point nucleus, dotted line – for extended nucleus; n = 349.

Comparing scattering curves for two different thicknesses of scattering plates (7 mm Pb and 4 mm Pb), we see that the effect of nuclear scattering increases with the increase in plate thickness. In the region of large scattering angles, where the Coulomb scattering is very small, and consequently the interference of Coulomb and nuclear scattering plays no role, all the scattering cases can be considered to be purely nuclear. The experimental scattering cross-section for angles $\varphi > 3.3$ for 7 mm Pb plates (Fig. 2) was found to be

$$\sigma = (0.0755 \pm 0.0375)\,\sigma_{geom}.$$

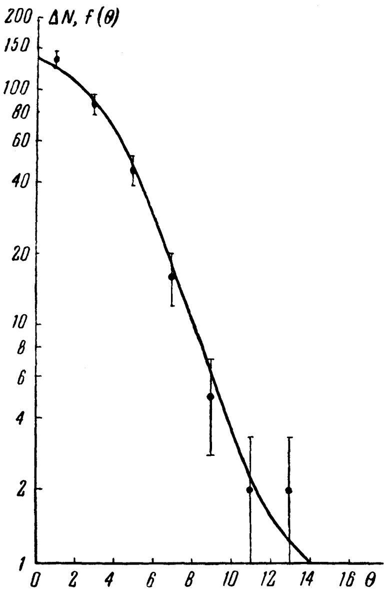


FIG. 4. Differential distribution of scattering angles for protons in 5 mm copper plates; n = 290.

From the asymptotic expression for the diffraction scattering formula[2] for large angles one can obtain the integral diffraction scattering cross section for projections of scattering angles larger than $\theta_0$:

$$\sigma = (4\lambda / \pi^2 R\theta_0)\sigma_{geom}, \quad (4)$$

which gives for angles $\varphi > 3.3$ a cross section $\sigma = 0.06\ \sigma_{geom}$, in good agreement with its experimental value.

The experimental data on proton scattering in

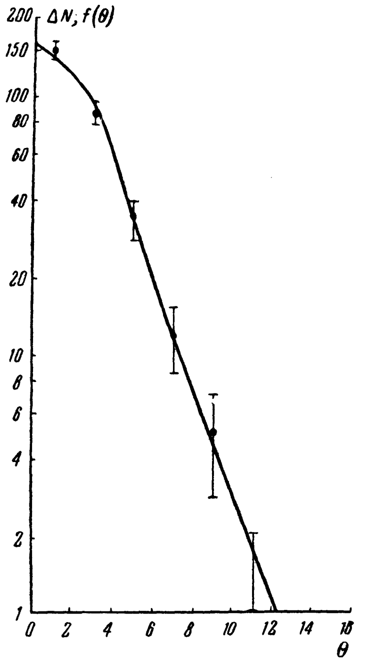


FIG. 5. Differential distribution of scattering angles for protons in 2 mm copper plates; n = 290.

copper, with scattering plates 5 and 2 mm thick, are shown in Figs. 4 and 5 respectively. These data are compared only with the curve of multiple-scattering by a point nucleus because the effect of finite nucleus dimensions is negligible for the copper nucleus and the investigated momentum range.

While the mean-square angles $\chi_c\sqrt{B}$ (averaged over all groups) for lead and copper are almost identical ($\sim 5°$), the value of $\theta = \lambda/R$ for copper ($\sim 7°$) is 2.7 times larger than the same value for lead (2.6°). One concludes from this that the effect of diffraction scattering is also small, and the experimental points must agree well with the curve of multiple Coulomb scattering for the point nucleus. This is clearly seen in the curves shown in Figs. 4 and 5 and Table III, where the coefficients $N(y, \alpha)$ are very close to unity.

It follows from the above that, depending on the energies of the protons, the plate thickness, and substance in which scattering is being investigated, the nuclear scattering takes place in a diverse manner. If $\lambda/R < \chi_c\sqrt{B}$, then the nuclear scattering shows up in the region of multiple scattering; if, however, $\lambda/R \gtrsim \chi_c\sqrt{B}$, then the nuclear scattering is apparent only at angles considerably larger than the mean-square angle of multiple scattering. The larger the effect of finite nuclear dimensions, the larger is the effect of the nuclear scattering, and vice versa. This, in the final analysis, leads us to the conclusion that the elastic proton scattering in the region of multiple scattering can be approximated artificially by the curve of Coulomb scattering for a point source.*

The author expresses his thanks to Prof. A. I. Alikhanian for his interest in the problem and for his participation in its discussion, and to W. L. Ter-Mikaelian for developing the scattering theory given in this paper and for his participation in the discussion of the data.


[1] Daion, Fedorov, Merzon, and Shostakovich, Приборы и техника эксперимента (Instruments and Meas. Engg.) 1, 3 (1957).

[2] G. Z. Molière, Naturforsch, 3A, 78 (1948).

[3] H. A. Bethe, Phys. Rev. 89, 1256 (1953).

[4] R. Hofstadter, Revs. Mod. Phys. 28, 214 (1956).

[5] A. I. Akhiezer and I. Ia. Pomeranchuk, Некоторые вопросы теории ядра (Certain Problems in Nuclear Theory), Moscow, 1950.

[6] D. C. Peaslee, Annual Rev. Nucl. Sci. 5, 126 (1955).


Translated by H. Kruglak
164

*Note added in proof (March 31, 1958). We have established recently that in a scattering investigation by experimental setups of this type it is necessary to take into account the geometry of the apparatus. For mesons this calculation would lead to a substantial change in the scattering curve; for protons the change is several percent.