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POLARIZATION OF ELECTRONS IN BETA-DECAY

A. I. ALIKHANOV, G. P. ELISEEV, V. A. LIUBIMOV and B. V. ERSHLER

Academy of Sciences, U.S.S.R.

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In connection with the revision of the law of conservation of parity, experiments to observe the longitudinal polarization of electrons in β -decay were carried out. It was found that the β -electrons were emitted with spin opposite to the direction of motion. The degree of longitudinal polarization is consistent with the value -v/c.

In a short communication¹ published in JETP, experiments were described which established the longitudinal polarization of β -electrons, predicted by the theory following from the hypothesis of non-conservation of parity in weak interactions.²

In this article we shall describe the experimental conditions and controls in detail, and introduce corrections in the calculation of the degree of longitudinal polarization of the electrons from the experiments, since, (1) in calculating the amount that the spin was turned in crossed electric and magnetic fields, which was necessary for calculation of the expected azimuthal asymmetry, an incorrect formula was used in Ref. 1, (2) in connection with this, several simplifications and neglects were made, which influence the final result substantially at the high-energy end of the experiment, and (3) several corrections were increased. These corrections in the value of the polarization of electrons of energy near 300 kev amounted to 10 - 25%.

In order to establish the longitudinal polarization of electrons in β -decay and to measure its value, we used the azimuthal asymmetry in the single scattering of electrons at an angle near 90°. Since azimuthal asymmetry occurs only for electrons with spin component perpendicular to the direction of motion, it was necessary to change the longitudinal polarization of the electrons into a transverse one. For this purpose, a beam of electrons from a radioactive source was sent through crossed electric and magnetic fields which, while not changing the direction of the electrons in the first approximation turns the spin with respect to the direction of motion of the electrons.

DESCRIPTION OF THE APPARATUS

The apparatus consisted of an arrangement for turning the spin and an arrangement for measuring the intensities of electrons scattered at a large angle for different azimuthal angles in the range $0-360^{\circ}$. A longitudinal cross section of the apparatus is shown on Fig. 1, and, on Fig. 2, a transverse cross section of the counting part of the apparatus, where two Geiger-Müller counters acting in coincidence are placed for the registration of scattered electrons.

The arrangement for turning the spin consisted of a longitudinal electrical condenser, placed in an evacuated metal tube which, in turn, was placed between poles of a permanent magnet. The condenser was formed of two plexiglass sheets of thickness 6 mm and length 30 cm with deep notches (4 mm). The notches in the plexiglass sheets, between which there remained only cross pieces of width 3 mm, acted as traps for electrons in order to diminish the number of electrons in the beam scattered from the walls of the condenser. A thin (10μ) aluminum foil glued on the inner surface served as facing for the condenser. The distance between the plates of the condenser was 12 ± 0.15 mm. The length of the condenser was 25 cm; however, the effective length of the electric field was somewhat greater, since the scattered



FIG. 1. Cross section of the apparatus in the plane perpendicular to the magnetic field. 1- source, 2- scatterer (gold), 3-Geiger counters, 4- absorbing filter, 5- aluminum electrodes of the condenser (foil of 10 μ), 6-plexiglass, 7- collodion film (0.3 mg/cm²), 8-plexiglass container, 9_{1,2,3} - brass body.

electric field extended beyond the edges of the condenser. In addition, the length of the plexiglass sheets was 50 mm greater than the length of aluminum foil, and its surface, owing to surface conduction, acted as an extension of the surface of the condenser. This effective increase in length of the condenser lay between 0.4 and 1.4 cm.

The magnetic field was produced by a permanent magnet made from an alloy of magnico with poles of armco-iron of length 25 cm and width 6 cm. The gap between poles was equal to 4 cm. The permanent magnet was furnished with coils which made it possible to magnetize and remagnetize the magnet. The topography of the magnetic field was plotted in three dimensions; in the region traversed by the electron beam it turned out to be uniform within the limits of accuracy of the measurements (1-1.5%).

In order to limit the scattered magnetic field in the upper part of the apparatus where the electron scattering took place, an iron shield was mounted at a distance of 0.8 cm from the end of the condenser. Measurement of the topography of the field showed that the scattered magnetic field was well shielded and, near the scatterer, was not more than 1.5% of its value in the gap. From the curve of fall off of the magnetic field at the upper edge, it was established that the effective length of the magnetic field here was increased by 0.5 cm. At the lower edge of the magnet the scattered field was substantially more extended, dropping to half its value at a distance of 2 cm from the edge of the pole. This scattered field increased the effective length of the magnetic field by 1.5 cm. Thus, the effective length of the whole system of fields which turned the spin exceeded the geometrical dimensions of the magnet by 2 cm, i.e., was equal to 27 cm.

In the method of crossed fields in the form we were able to employ it, the electrons are not focussed and therefore a high intensity can be reached only at the expense of resolution with respect to energy. The effect of azimuthal asymmetry falls off slowly with increasing electron energy; the degree of polarization of β -electrons, according to the theory, rises slowly with energy ($\sim v/c$), the β -spectrum of heavy elements is a slowly decreasing function of electron energy and, finally, the angle of turning of the spin is also a slowly changing function. Therefore, the measurements of polarization of β -electrons were all carried out with a wide spectrum of electron energies.

Thus, the main purpose of the crossed electric and magnetic fields in our experiments, which influenced the spectral composition of the electrons falling on the scatterer only slightly, was to turn the spin. The sharp falloff of the scattering with electron energy, $\sigma \sim (pv/c)^{-2}$ was essential in determining the form of the energy spectrum of electrons which underwent scattering.

Calculation of the curves of resolution in the crossed fields by analytic means turned out to be difficult. Therefore, they were obtained by numer-



FIG. 2. Counting part of the apparatus. Electrons move out of the paper. The combinations of fields are denoted + and -.

ical calculations. Two such curves of resolution for two different values of v_0/c and values of the field, close to those used in this work, are shown on Fig. 3.

ACCURACY OF MEASUREMENT OF THE ELEC-TRIC AND MAGNETIC FIELDS

The electric field was measured by an electrostatic apparatus to an accuracy of 1.5%. The gap in the condenser had a tolerance of ~0.15 mm, and thus the value of the voltage of the electric field was determined to an accuracy of ~2%.

The magnetic field was measured in the usual way by a ballistic galvanometer and a coil, calibrated by use of standard mutual inductances. The error in the direct measurement of the magnetic field, that in the calibration and measurement of the ratio of the mean magnetic field to the field at the point of measurement, constituted about 3%. Thus, the overall error in the measurement of the ratio of fields E/H was ~ 3.5%.

SOURCE OF BETA-ELECTRONS

The source of β -electrons was in the form of a spot of uniform thickness and diameter 1 cm on an aluminimum backing of thickness $10\,\mu$, which was attached by its edges to an aluminum ring. The ring was fixed in the container — a brass cylinder of diameter 32 mm and length 84 mm, put inside a layer of plexiglass to exclude the scattering back of electrons.

The source was Sr⁹⁰ with an admixture of Sr⁸⁹ made up from fragment solutions; in the course of



FIG. 3. Resolving power of the apparatus. F(E) is the probability of traversal of electrons of energy E through the apparatus: the continuous curve is for $v_0/c = E/H = 0.925$, the dashed one, for $v_0/c = 0.775$.

time Y^{90} was formed in the source, with an amount reaching equilibrium in almost all experiments. The mixture of Sr^{89} was 35% of that of Sr^{90} . The composition of the source was determined by the decay curve.

The electron energy spectrum of such a source is shown on Fig. 4. All of the elements making up



FIG. 4. Energy spectra of electrons in arbitrary units; $\varepsilon_{\rm K}$ is the kinetic energy of the electron in Kev, a is the original spectrum of the source, b is the spectrum of electrons incident on the counter in the experiment with energy 300 Kev, c, the same, at energy 750 Kev.

the source have unique transitions (that is, their β -transitions all have $\Delta j = 2$ and change of parity). For unique transitions, one can expect that the degree of electron polarization is close³ to v/c.

The source thickness plays an essential role in such measurements. Longitudinally polarized electrons traversing layers of the source material and undergoing multiple scattering in them, are depolarized, since the electrons change their direction of flight in scattering, whereas the spin retains its direction. The correction for depolarization was introduced into the calculation of the azimuthal symmetry through the factor $\cos \theta \approx 1 \cdot -\frac{1}{2}\theta^2$, where θ^2 is the mean square angle of multiple scattering.

In order to calculate the quantity $\overline{\theta}^2$, we employed the formula given by Bethe and Ashkin⁴

$$\bar{\theta}^2 = 0.157 \frac{Z (Z^2 + 1) t}{A (p v / c)^2} \ln [1.13 \cdot 10^4 Z^{4} A^{-1} t], *$$

where Z is the atomic number, A is the atomic weight, p is the momentum in Mev/c, and t is the thickness in g/cm^2 .

In this work sources of two thicknesses were employed, 4 and 1.5 mg/cm^2 . For both thicknesses the correction for depolarization in the source, according to the formula of Bethe and

*For calculation of the quantity $\overline{\theta^2}$ in Ref. 1, we used the formula of Rossi and Greisen $\overline{\theta^2} = \left(\frac{21.3}{pv/c}\right)^2 t'$, where p is the

momentum in Mev/c, t' the thickness in $X_0 = 10 \text{ g/cm}^2$). However, the formula of Rossi and Greisen is incorrect and gives values 5-7 times larger than those that follow from the formula given by Bethe and Ashkin.⁴ Ashkin, was small. It is to be expected that the depolarization depends not only on the mean thickness of the source, but also on the dimensions of the individual crystal aggregates and, in the limit, on the individual crystals. The second source was prepared so that the size of the crystals was minimal. For this, the sedimentation of the solution was carried out quickly. Observation of the source under a microscope showed, by visual estimate, that the crystal sizes were $\sim 10 \,\mu$.

MEASUREMENT OF THE ANISOTROPY IN ELECTRON SCATTERING

The part of the apparatus in which the electron scatterer and counters were placed was separated from the condenser by a thin film of collodion on netting.

The electron beam traversed the film and fell on the scatterer, joined to the frame at an angle of 45° to the axis of the beam. Two selfquenching counters with windows cut out were prepared on the same frame for counting electrons in coincidence and were placed at the angle of $(90 \pm 4)^{\circ}$ (and also, in one of the experiments, at 105°) to the axis of the electron beam. The vertical dimensions of the window in the counters were such that the solid angle subtended by them included only part of the scatterer and certainly did not include the frame of the scatterer. The frame of the scatterer had dimensions larger than the cross section of the beam, as was easy to ascertain by carrying out measurements with only the frame and without it at all.

The first window of the counter was covered by a film of collodion of thickness 0.5 mg/cm^2 , supported by a thin net. The window between counters was covered by aluminum foils of different thicknesses, depending on the part of the electron spectrum, but was not less than 40μ for the filtering of electrons of energy less than 50 Kev. The counters were joined through a rubber tube, which was fitted hermetically through the apparatus, to a balloon of large volume with a given mixture pressure and, thus, a constant mixture pressure was maintained in the counters, even with a small current in the vacuum.

The counters, together with the frame of the scatterer, were rotated around the axis of the beam so that the scatterer was placed in strictly the same relationship to the counters for all azimuthal angles. The scattered electrons were always counted "in traversal". As is well known, this differs from "on reflection" by the substantially fewer electrons having undergone multiple scattering.

Multiple scattering is the main obstacle to the measurement of single scattering and, correspondingly, in the measurement of azimuthal asymmetry. In the work of Alikhanian, Alkihanov and Vaisenberg⁵ studying the scattering of electrons at 90°, experimental results relating to the role of multiple scattering were obtained and in the work of Artsimovich⁶ an attempt to estimate this effect theoretically on the basis of these data was made. According to the data,⁵ in the scattering of 680-Kev electrons at 90° "on reflection" in the interval of thickness 0.34 - 0.81 mg/cm² of Au, the intensity of scattered electrons was proportioned to the thickness. That showed that the admixture of multiple scattering was small.

Starting from these data and the well known dependence $\sigma \sim (c/pv)^2$, it is possible to estimate the allowed thickness of scatterer for electrons near 300 Kev. This turned out to be close to 0.5 mg/cm². According to Artsimovich's criterion, at the scattering angle of 90°,

$$I = I_0 \left(1 + 7\overline{\theta^2} \right) t,$$

, where I is the intensity of scattered electrons, I_0 is the intensity of singly-scattered electrons per unit thickness of the target, t is the thickness of the scatterer and θ^2 is the mean square angle for multiple scattering. For 0.5 mg/cm^2 and energy 300 Kev, the admixture of multiple scattering, according to this criterion, is 30%. This estimate gives an upper limit for the proportion of multiple scattering, since (1) the criterion of Artsimovich is larger than the multiple scattering in heavy elements and (2) in the position "in traversal" the multiple scattering is several times smaller than "on reflection". A direct experimental means of evaluating the multiple scattering consists of measuring the azimuthal asymmetry for at least two different thicknesses and finding the true value by extrapolating to zero thickness of scatterer. This has the disadvantage that it requires measurements of greater accuracy than the accuracy of measurement of the actual value of the azimuthal asymmetry.

In the measurement of azimuthal asymmetry, the experimental conditions should guarantee no purely instrumental asymmetry and give a control on this. Such a control was provided by substituting for the scatterer consisting of a heavy element (gold) a light element (aluminum) for which the asymmetry for scattering through such an angle is an order of magnitude less. However, we considered this control insufficient, since it was difficult to carry it out in a continuous fashion. In order to substitute one scatterer for another, the apparatus had to be opened, the vacuum destroyed, then renewed, the condenser readjusted, etc. After such changes it would not be possible to be sure that instrumental asymmetries did not occur in the apparatus which escaped observation.

In the method employed, we arranged a second independent and uninterrupted control. It is easy to see that by reversing the field we excluded all types of asymmetry except for that coming from the sign of the field. In this last case the instrumental asymmetry should show up very distinctly in the direction perpendicular to the direction of measurement of the physical asymmetry, because it is in precisely this direction in which the field deflects the particles (i.e., has a maximum effect on the beam) and in which the physical asymmetry vanishes.

This control, which was very convenient, did not, however (in the case of asymmetries coming from the fields), determine the sign and magnitude of the necessary corrections. These could be obtained only by using the control with the scatterer of a light element.

Using both control methods, we could exclude instrumental asymmetry with complete certainty.

CALCULATION OF THE EXPECTED AZIMUTHAL ASYMMETRY EFFECT AND POLARIZATION OF ELECTRONS

In calculating the expected azimuthal asymmetry, it is necessary to know the angle through which the spin of the electron was turned in the crossed fields and the dependence of the azimuthal asymmetry on the scattering angle and energy of the polarized electrons.

For a monochromatic $(v = v_0)$ beam of electrons parallel to the axis of the apparatus, the angle through which the spin is turned is determined by the expression*

$$\varphi = \frac{300Hl}{p_0 c} \sqrt{1 - (v/c)^2},$$

where H is the magnetic field in oersteds, l is the length of the crossed fields in cm, p is the electron momentum in ev/c; $v_0/c = E/H$, where E is the strength of the electric field.

In our experimental conditions, we are dealing with an essentially non-monochromatic and nonparallel beam. The task of calculating the rotation of the spin in this case turned out to be quite complicated, and was carried out by Ter-Martirosian (see the appendix).

Usually, the quantity entering into theoretical calculations of the magnitude of azimuthal asymmetry is the spin-component in the rest system of the electron perpendicular to the direction of motion of the electron in the laboratory system. The tables of Sherman,⁷ which we used, for the value of the azimuthal asymmetry for various scattering angles and various electron energies for Z = 80were calculated in this way. To calculate the turning of the spin in crossed fields, it is convenient to go to a system of coordinates moving with velocity v_0 , in which the electric field vanishes. After this transformation, an expression for the rotation of the spin for an arbitrary value of v/c and angle α between the axis of the apparatus and the direction of motion of the electron was obtained, the transformation to the laboratory system was carried out, and then to the system of rest for the spin and the laboratory for the velocity.

As a result (see Appendix), a rather complicated expression was obtained for $\sin \dot{\varphi}$, where φ is the angle of rotation of the spin. Neglecting terms containing the small angle α , we obtain

$$\sin \varphi = \sin \varphi_0 \left\{ \frac{\left[\{ (v / v_0) - 1 \} + \{ 1 - (vv_0 / c^2) \} \cos \varphi_0 \right]^2}{\left[1 - (v^2 / c^2) \right] \left[1 - (v_0^2 / c^2) \right]} + \sin^2 \varphi_0 \right\}^{-1/2},$$

and for small φ

$$\varphi = \frac{300Hl}{pc} \frac{v_0}{v} \sqrt{1 - v^2/c^2}.$$

In the formulas given (and in Table 7) one can see the magnitude of $\sin \varphi$ depends rather strongly on the energy of the electrons, in particular, for large energies. To obtain the correct value φ for v_0/c near to 1, it is necessary, as far as possible, to calculate the form of the electron spectrum accurately.* The spectrum of scattered electrons with energy near 300 Kev was determined in the following way. For each energy interval the source spectrum (from Fig. 4) was multiplied by the resolving power (from Fig. 3) and then by the relative value of the scattering cross section. For the experiment with electron energy 750 Kev the resolving power played no role and the form of the spectrum was determined by the scattering cross section and the curve of absorption of electrons in the filter between the I and II counters, the thick-

^{*}In our first communication, the factor $\sqrt{1-(v/c)^2}$ was left out of this formula and the calculation of the expected effect and polarization were carried out with an incorrect formula. We thank Prof. L. Rosenfeld for calling our attention to this.

^{*}In the work of Ref. 1 we did not consider this necessary, since the expected effect calculated from the incorrect formula depended very weakly on energy.

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TABLE I. E = 18.3 Kev/cm; H = 80.5 Oe; t = 0.537 mg/cm² Au; T = 4 mg/cm² Sr+Y

0°	180°	90°	270°	Sign of the field
$98.5 \pm 6.2 \\98.6 \pm 4.8$	101.7 ± 6.3 108.7 ± 4.2	118.8 ± 4.0 99.1 ±2.0	101.0 ± 4.1 118.4 ± 2.4	+
100	0.1 ± 3.9 3.5 ± 3.6	99.5 118.6	5 ± 1.8 5 ± 2.1	Weighted mean

TABLE II. $t = 0.537 \text{ mg/cm}^2 \text{ Au}; T = 1.5 \text{ mg/cm}^2 \text{ Sr} + Y$

0 °	180°	90 °	270°	Sign of the field
12.8 ± 1.4 11.2 ± 0.8	$13.9\pm1.0 \\ 8.9\pm0.9$	$15.6 \pm 0.6 \\ 8.9 \pm 0.5$	10.8 ± 0.6 14.7 ±0.6	+
12.5 <u>-</u> 10.8 <u>-</u>	$\pm 0.6 \pm 0.7$	9.8 15.2	$_{\pm 0.4}^{\pm 0.4}$	Weighted mean

Scatterer $t = 5.4 \text{ mg/cm}^2 \text{ Al}$

33.0 ± 2.1	37.3 ± 2.2	36.5 ± 1.2	33.4±1.2	+
35.8 ± 2.2	32.0 ± 2.5	32.9 ± 1.4	37.8±1.5	
$36.5+32.5\pm$	1.5 1.6	33.1 37.1	$_{\pm 0.9}^{\pm 0.9}$	Weighted mean

TABLE III. E = 20 Kev/cm; H = 86 Oe; t = 0.17 mg/cm² Au; T = 1.5 mg/cm² Sr+Y

0.	180°	90°	270°	Sign of the field
120.0 ± 7.0 112.5 ± 4.0	120.2 ± 5.8 121.5 ± 4.0	135.0 ± 5.6 113.3 ± 2.3	$\begin{array}{c} 106.0 \pm 7.0 \\ 136.0 \pm 2.7 \end{array}$	+
116. 120.	3 ± 3.2 7 ±3.3	109.6 135.5	5 ± 2.2 5 ± 2.1	Weighted mean

TABLE IV. E = 20 Kev/cm; H = 71 Oe; t = 1.9 mg/cm²; T = 4 mg/cm² Sr + Y

0°	180°	9 0°	270°	Sign of the field
29.5 ± 0.7 29.4 ± 1.0	29.8 ± 0.8 28.9 ± 1.0	31.3 ± 0.4 27.6 ± 0.5	$^{28.7\pm0.5}_{29.6\pm0.5}$	+
29.6 29.2	$\pm 0.6 \\ \pm 0.6$	28.1 30.4	$\pm 0.3 \pm 0.3$	Weighted mean
28.9 <u>+</u> 1.0	29.9±0.9	29.7 <u>+</u> 0.8	29.2 <u>+</u> 0.8	Field off

$e_{xD} = 21.8\%$
asymmetry \delta
azimuthal
of the
ed value
Expecte
*.⊳
TABLE V.*

$\mathbf{r}_{\mathbf{k}}$ 2002503003004004505006007009001500130015001500 v/c 0.700.7140.7850.9410.8350.8340.8370.8350.8370.8370.8370.937 </th <th></th> <th>4</th> <th></th> <th></th> <th></th> <th></th>												4				
$ \frac{v/c}{\Delta_{1}} \begin{pmatrix} v/c}{\delta_{2}} & 0.70 \\ 0.714 \\ 0.725 \\ 0.492 \\ 0.492 \\ 0.492 \\ 0.492 \\ 0.147 \\ 22 \\ 0.105 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.063 \\ 0.063 \\ 20 \\ 0.063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\ 0.0063 \\$	εĸ	200	250	300	350	007	450	500	600	750	006	1050	1200	1350	1500	1650
p 53.0 51.6 49.8 47.4 45.6 44.5 43.2 39.0 36.4 32.6 29 27 25 23 p 31.2 27.2 22.7 18.9 16.3 14.4 12.1 8.9 6.3 4.45 3.4 2.6 2.0 1.5 p 31.2 27.2 22.7 18.9 16.3 14.4 12.1 8.9 6.3 4.45 3.4 2.6 2.0 1.5 p 1028 1060 590 284 147 85 48 5 4 2 2 1 1 p 1028 1060 590 284 147 85 48 7 32 48 7 5 2 1 1 p 1028 1060 590 284 147 85 48 7 32 48 7 5 2 1 1 1 1	υ/c sin φ	0.70 0.840			$\begin{array}{c} 0.81 \\ 0.492 \end{array}$	0.83 0.433		0.86 0.327						$0.96 \\ 0.082$		$\begin{array}{c} 0.97 \\ 0.056 \end{array}$
$\Delta \frac{v}{c}$ sin φ 31.227.227.222.718.916.314.112.18.96.34.453.42.62.01.51.20 v 3339261596485422111 v 3339261596485422111 v 102810605902841478548713218752110* $v_{\rm K}$ is the kinetic energy of the electrons in Kev, φ is the angle between the direction of the velocity of the electron and the direction of the velocity of the electron and the direction of the velocity of the electron in the laboratory system after traversal of the crossed fields, Δ is the azimuthal asymmetry for lectrons 100% polarized in the transverse direction, T N is the number of electrons incident on the counter, in relative units; N is the rough the absorber between the counters times the cross section for scattering of the electrons through the angle considered in the scattering of the electrons through the angle considered in the scatter function.	% `∕	53.0			47.4	45.6		43.2					27	25		22
$\frac{1}{c}N$ $\frac{1}{c}Sin_{\varphi}N$ $\frac{33}{1028}$ $\frac{26}{1060}$ $\frac{15}{590}$ $\frac{9}{284}$ $\frac{6}{147}$ $\frac{4}{85}$ $\frac{8}{71}$ $\frac{8}{71}$ $\frac{1}{32}$ $\frac{4}{18}$ $\frac{2}{7}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{1}{1}$ $\frac{1}{0}$ $\frac{1}{1000}$ $\frac{1}{50}$ $\frac{1}{200}$ $\frac{1}{284}$ $\frac{1}{147}$ $\frac{1}{85}$ $\frac{4}{48}$ $\frac{1}{71}$ $\frac{1}{32}$ $\frac{1}{18}$ $\frac{1}{7}$ $\frac{1}{5}$ $\frac{1}{2}$ $\frac{1}{1}$ $\frac{1}{1000}$	$\Delta \frac{v}{c} \sin \varphi$	31.2	27.2	22.7	18.9	16.3	14.1	12.1	8.9	6.3	4.45	3.4	2.6	2.0	1.5	1.20
$\frac{v}{c} \sin \varphi \cdot N = 1028 = 1060 = 590 = 284 = 147 = 85 = 48 = 71 = 32 = 18 = 7 = 5 = 2 = 1 = 0$ $\frac{*}{2} \sin \varphi \cdot N = 100\% polaric energy of the electrons in Kev, \varphi is the angle between the direction of the spin in the rest system of the electron and the direction of the velocity of the electron in the laboratory system after traversal of the crossed fields, \Delta is the azimuthal asymmetry for lectrons 100% polarized in the transverse direction, 7 = 10\% is the number of electrons incident on the counter, in relative units; N is the rouduct of the curve of resolution of the apparatus with respect to energy times the probability of traversal of an electron of given energy nough the absorber between the counters times the cross section for scattering of the electrons through the angle considered in the scatter times the energy spectrum of the source.$	N	33	39	26	15	6	9	4	8	ഹ	4	2	7	1	÷.	0
* ε_{K} is the kinetic energy of the electrons in Kev, φ is the angle between the direction of the spin in the rest system of the electron and ne direction of the velocity of the electron in the laboratory system after traversal of the crossed fields, Δ is the azimuthal asymmetry for lectrons 100% polarized in the transverse direction, ⁷ N is the number of electrons incident on the counter, in relative units; N is the roduct of the curve of resolution of the apparatus with respect to energy times the probability of traversal of an electron of given energy trough the absorber between the counters times the cross section for scattering of the electrons through the angle considered in the scat- screr times the energy spectrum of the source.	$\frac{v}{c} \operatorname{sin}_{\phi \cdot N}$	1028	1060	590	284	147	85	48	71	32	18	2	ഹ	7	Ŧ	0
	ϵ_{K} is the set of the set times the set times the set of the	ne kinetic of the vel)% polariz e curve of bsorber be te energy	energy of t locity of th ed in the tr resolution etween the spectrum o	the electron e electron ransverse t of the ap counters f f the sour	i in Kev i in the lab direction, ⁷ paratus wi times the c	, φ is the oratory s N is th th respec	angle be ystem al le numbe le numbe tion for tion for	etween t ther trave or of elec rgy time scatterin	he direc ersal of ctrons ir s the pr ng of the	ttion of t the cros ncident obability electro	the spin ssed fiel on the c y of trav ns throu	in the r lds, Δ is ounter, i ersal of igh the i	est syst s the azi in relati an elec angle co	:em of th imuthal ve units vron of nnsidere	ne electr asymmet ; N is given en d in the	on and rry for tthe ergy scat-

$\delta_{\rm exp}=31.7\%$
asymmetry
azimuthal
of the
value
Expected
VI.
TABLE VI.

1800	$\begin{array}{c} 0.975\\ 0.060\\ 24.0\\ 1.40\\ 0\\ 0\\ 0\\ \end{array}$
1650	$ \left[\begin{array}{c ccccccccccccccccccccccccccccccccccc$
1500	$\begin{array}{c} 0.97\\ 0.071\\ 27.4\\ 1.88\\ 2\\ 2\\ 4\\ 4\end{array}$
1350	0.96 0.091 34.6 3.02 2 6
1200	0.95 0.114 41.2 4.46 3 13
1050	0.94 0.140 46.0 6.05 5 30
	0.93 0.165 49.0 7 52
750	0.91 0.219 53.6 10.7 10.7 107
600	$\begin{array}{c} 0.89\\ 0.281\\ 57.0\\ 14.3\\ 15\\ 214\end{array}$
200	0.86 0.364 61.6 19.2 8 154
450	0.84 0.420 63.8 22.5 12 270
400	0.83 0.473 64.5 25.3 21 532
350	$\begin{array}{c} 0.81\\ 0.547\\ 67.2\\ 29.8\\ 31\\ 924 \end{array}$
300	$\begin{array}{c} 0.78\\ 0.643\\ 69.1\\ 34.7\\ 52\\ 1804\\ \end{array}$
250	0.74 0.776 71.3 40.9 72 2942
200	0.70 0.892 73.0 45.6 42 1914
м _э	$\begin{array}{c c} & v/c & 0 \\ & v/c & 0 \\ \Delta & \phi & 73 \\ \Delta & c & sin \phi \\ N & 42 \\ \Delta & c \\ \Delta & sin \phi N \end{array} $

TABLE VII. Expected value of the azimuthal asymmetry $\delta_{exp} = 6.8\%$

2000	$\begin{array}{c} 0.98 \\ 0.045 \\ 16 \end{array}$	0.71	0.3	0
1900	0.98 0.046 0.110 110 110 110 110 110 110 110 110 1	2.11 1.50 1.43 1.34 1.24 0.72 0.71	0.6	-
1800	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	1.24	1.0	4
1700	$\left \begin{array}{c} 0.97 \\ 0.063 \\ 22 \end{array} \right $	1.34	1.6	5
1600	$\begin{smallmatrix}0.97\\0.067\\22\end{smallmatrix}$	1.43	2.5	4
1500	$\left {\begin{array}{*{20}c} 0.97 \\ 0.070 \\ 22 \end{array} } \right $	1.50	3.3	ß
1400	$[0.96]{0.088}$	2.11	4.6	10
1300	$\begin{array}{c} 0.96\\ 0.095\\ 25\end{array}$	2.28		14
1200	$\begin{array}{c} 0.95\\ 0.115\\ 27\end{array}$	2.95	8.9	26
1100	$\begin{smallmatrix}0.95\\0.122\\27\end{smallmatrix}$	3.14	11.6	36
1000	$\begin{array}{c} 0.94 \\ 0.153 \\ 29 \end{array}$	4.17	15.0	62
006	$\begin{array}{c} 0.93 \\ 0.170 \\ 32.6 \end{array}$	5.15	19.4	100
800	$\begin{array}{c} 0.92 \\ 0.195 \\ 34.5 \end{array}$	6.19	24.5	152
700	$\begin{array}{c} 0.91 \\ 0.222 \\ 36.4 \end{array}$	7.35	29.0	213
600	$\begin{array}{c} 0.89\\ 0.281\\ 39.0 \end{array}$	9.75	30.6	298
500	$\begin{array}{c} 0.86 \\ 0.359 \\ 43.2 \end{array}$	13.3		319
ж _а	v/c sin φ Δ, %	$\Delta \frac{v}{c} \sin \varphi$	N	$\Delta \frac{v}{c} \sin \varphi N$

ness of which was $0.08 \text{ g/cm}^2 \text{ Al} + 0.08 \text{ g/cm}^2 \text{ Cu}$. The absorption curves were taken for the case of bad geometry.*

The dependence of the scattering cross section on energy in the range 600 - 2500 Kev was taken from the experimental data of Ref. 5. Since the table of Sherman finishes at v/c = 0.9, for large v/c the necessary theoretical values for the magnitude of the asymmetry for scattering at 90° were obtained from the curve, also given by Sherman.

EXPERIMENTAL RESULTS

The experimental results for three series of measurements at energies near to 300 Kev are given in Tables 1, 2 and 3, and in Table 4, for an energy of 750 Kev.[†] The measurements corresponding to Tables 1-4 were carried out with a source of thickness $T = 4 \text{ mg/cm}^2$, and the others with a thickness of 1.5 mg/cm^2 . In experiment II, the control experiment with an aluminum scatterer was carried out, showing the absence of instrumental asymmetry to an accuracy of $\pm 3.5\%$.

We consider first the data of the tables for the experiments I and II, since these measurements differ only by the target thickness.

From the tables it is evident that, in the plane going through the spin direction and the electron velocity, the asymmetry is almost absent for both signs of fields. At the same time, in the plane perpendicular to the spin, the asymmetry significantly exceeds the errors of measurement and changes sign with change of sign of the field.

In experiment I the value of the azimuthal asymmetry from measurements with both signs of the field was equal to $\delta_1 = (17.4 \pm 2.6)\%$ and in experiment II $\delta_2 = (21.0 \pm 2.5)\%$. As noted above, these two experiments differed only by the thickness of the source. Corrections for depolarization in the first source were ~3.5%, in the second, ~1%, i.e., very small. Therefore the results of the two experiments can be put together, where-upon

$$\delta_{\text{mean}} = (19.2 \pm 1.8)\%.$$

Including in the error the inaccuracy in determining the instrumental asymmetry $\pm 3.5\%$, we obtain

$$\delta_{\text{mean}} = (19.2 \pm 3.8)\%$$

All intermediate numbers for the calculation of

*In the work of Ref. 1 we determined the lower limit of the spectrum by the extrapolated mean free path and, in addition, did not take into account the presence of Sr^{89} , which greatly increases the number of electrons in the energy range 500-800 Kev.

[†]In Ref. 1, Tables 1-4 were given.

the expected values of azimuthal asymmetry and the corresponding electron polarization are given in Table 5. (The expected azimuthal asymmetry is, from Table 5:

$$\delta_{\text{exp.}} = \sum_{i} \Delta_i \left(v_i / c \right) \sin \varphi_i N_i \left| \sum_{i} N_i = 21.8 \%. \right)$$

As a result we obtain

$$P_{\text{me}an} = (0.88 \pm 0.18) (v/c).$$

We will return to the correction that must be introduced into these results as a result of multiple scattering after consideration of experiment III.

The conditions of experiment III were different in that (1) the scattering angle was equal to $105 \pm 4^{\circ}$; (2) the thickness of the scatterer was equal to 0.17 mg/cm^2 ; (3) the background was diminished; however, since the intensity of scattering fell by a factor of 3, it constituted $\frac{1}{2}$ of the intensity with the scatterer present.

From Table 3 it can be seen that there is an asymmetry in the directions $0 - 180^{\circ}$ which changes sign with change in the sign of the fields. Its mean value is (14.5 ± 8.5) %. In the directions $90 - 270^{\circ}$, the asymmetry - both physical and instrumental was (42.8 ± 4.8) %. Results of the measurements with the aluminum scatterer, which are given in the same table, make is possible to determine the magnitude and sign of the correction which must be introduced to obtain the value of the physical asymmetry. The instrumental asymmetry has the same sign as the physical, and is equal to $(11.3 \pm 3.7)\%$. However, in the scattering with the aluminum foil of thickness 20μ , a 2% physical asymmetry should be present. With this in view, we obtain from experiment III

$$\delta_3 = (33.6 \pm 6.0) \%$$
.

From Table 6, the expected $\delta_{exp} = 31.7\%$, and thus

$$P = (1.06 \pm 0.19) (v/c).$$

The data obtained for the polarization are necessary for the correction due to multiple scattering in the scattering foils. As we said above, this correction can be obtained by extrapolating to zero scatterer thickness the inverse of the azimuthal asymmetry obtained from the measurements with different thicknesses of scatterer. However, since this correction is not larger in magnitude than the statistical errors, it cannot be determined in this way from two experimental values of the polarization obtained by us with two different thicknesses of scatterer — 0.17 and 0.537 mg/cm². Therefore, we used experimental data obtained in our laboratory with different apparatus, on which Vishnevskii et al. also measured the polarization of electrons in scattering at 90° from gold foils of different thicknesses and for different electron energies from 100 to 200 Kev. According to these data, for electrons in the energy interval 160 - 200 Kev and scattering "in transmission" in gold, the correction to the azimuthal asymmetry because of multiple scattering is determined by the expression

$$\delta_{0} = \delta(t) \left[1 + \frac{(0.022 \pm 0.004)}{(pv/c)^{2}} t \right],$$

which in our case at a mean energy of 300 Kev and foil thickness 0.17 mg/cm^2 gives the magnitude of the correction as 4%, and for a foil 0.537 mg/cm^2 , 13.5%.

Introducing these corrections, we have:

 $P := (1.00 \pm 0.20) (v/c)$ for the first two experiments; $P := (1.10 \pm 0.19) (v/c)$ for the third experiment.

The mean of these is

$$P = (1.05 \pm 0.14) (v/c).$$

We note one circumstance common to all three experiments. In the directions $0 - 180^{\circ}$ the sum of counts is somewhat less than the sum of counts at 90 and 270°. This difference scarcely exceeds the limits of error, and is even sometimes less than these; however, it has a systematic character and, in the mean, is 5%. The origin of this difference is easy to understand. The deflecting field acts in the direction $0 - 180^{\circ}$. Thus, the dimensions of the beam in these directions are somewhat larger than in the direction $90 - 270^{\circ}$, owing to the fact that part of the scattered electrons in the direction $0 - 180^{\circ}$ does not fall into the angle of acceptance of the counters in the vertical direction.

In experiment IV, measurements of the electron polarization were carried out at a higher energy – 750 Kev. In this case the electrons emitted by Sr^{90} are almost all filtered out and only the electrons emitted by Sr^{89} and Y^{90} remained. The limit of the Sr^{89} spectrum is 1460 Kev and, of the Y^{90} one, 2260 Kev. Since the number of electrons of high energy after scattering drops sharply according to the law $\sim \sigma = \mathrm{B} (\mathrm{pv/c})^{-2}$, the electrons with energy > 1500 Kev hardly contribute to the azimuthal asymmetry. Thus, the decay electrons of Sr^{89} , having a softer spectrum than those of Y^{90} , play a large part in this measurement.

The experimental conditions are indicated in Table 4. Data of the control experiment with fields turned off are presented there in the same way. As noted above, the resolving power of the apparatus in this energy region is low. The field does not have a deflecting action on the electrons, but only turns their spin. The spectrum of electrons is determined by the scattering law and the filter between the counters. In fact, it is evident from the table, that, with the fields turned off, the number of counts in the $0-180^{\circ}$ plane and the mean number of counts in the $90-270^{\circ}$ plane do not change within the limits of error of the measurements (~2%). Therefore it is impossible to conceive of any asymmetry connected with the switching of the fields, larger than this quantity. This can be seen from the equality of intensities in the directions $0-180^{\circ}$ with the fields turned off.

Geometrical asymmetry is excluded by the switching of the fields and, besides, from the identities shown for all four angles with the field absent, it is clear that it is practically absent.

From Table 4 we obtain

$$\delta_4 = (7.85 \pm 1.4) \%.$$

Assuming that the absence of instrumental asymmetry is known to us to an accuracy of 2%, we obtain

$$\hat{o}_4 = (7.85 \pm 2.5)\%$$
.

From Table 7, the expected value of the asymmetry is

$$\delta_{\text{exp.}} = (6.8 \pm 0.8) \%$$
.

The error in the expected value arises from its great sensitivity to the mean energy of the electron spectrum, which cannot be accurately determined. Thus,

$$P = (1.16 \pm 0.4) (v/c).$$

CONCLUSIONS

In the present work the polarization of electrons of energy 200 – 1500 Kev emitted in β -decay was observed by the method of electron scattering. It was shown that the degree of polarization was near to v/c to an accuracy of 15% for mean energy 300 Kev and 40% for mean energy 750 Kev. Similar experiments with results near to ours have been communicated simultaneously in a series of papers.

Frauenfelder et al.⁸ observed the polarization for electrons near 70 Kev emitted by Co^{60} using the deflection of electrons in an electric field and electron scattering. Nikitin et al.,⁹ using the same method, showed the same effect for electrons of energy 120 Kev emitted by Cu^{64} . Cavanagh et al.¹⁰ using the same method as we of crossed fields to turn the spin, observed the polarization of β electrons of energy 120 Kev emitted by Co^{60} . Later, the polarization of β -electrons has been observed by other means than measurement of the azimuthal asymmetry in the scattering.

The establishment of the polarization of β electron was, together with the experiments of Wu et al.¹¹ and Lederman et al.,¹² a strong experimental proof of the violation of parity in weak interactions.

In conclusion, we should express our gratitude to K. A. Ter-Martirosian for deriving the formula for the turning of the spin in the crossed fields, to L. Ia. Suvorov, M. P. Anikina and V. D. Laptev for separating and preparing the source of Sr, to A. S. Kronrod for calculating the intensity and to M. E. Vishnevskii for helpful information on the role of multiple scattering.

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APPENDIX

On the Turning of the Spin in Crossed Electric and Magnetic Fields

Below we present considerations derived by K. A. Ter-Martirosian on the motion of the spin upon the traversal of the electron through crossed electric and magnetic fields.

Let an electron, the spin of which is parallel to

its initial velocity* β , go into constant and uniform fields E and H (E parallel to the y-axis, and H, to the z-axis). The vector β lies in the x,yplane† and makes angle α with the x-axis. It is not assumed that $\beta = \beta_0$, where $\beta_0 = E/H$. The fields act in the region between two parallel planes (x = 0 and x = l), the distance between which is l.

It is required to find the angle φ through which the spin of the electron is turned after it has traversed the region in which the field acts, that is, the angle between the direction $n(n^2 = 1)$ of the spin of the electron after traversing the field in the system of reference K^0 , in which the electron is at rest, and the direction $\nu = \beta_1/\beta$ of its final velocity.[‡]

For a covariant description of the spin, it is convenient to introduce a 4-vector (σ, σ_0) defined in such a way** that in the rest system K^0 its spatial part coincides with **n**, and its time-like part is equal to zero. According to definition, $\sigma^2 = \sigma^2 - \sigma_0^2 \equiv \mathbf{n}^2 = 1$ in an arbitrary system. Under a Lorentz transformation from the laboratory system to the system K^0 , the perpendicular component $\sigma_{\perp} = [\nu(\sigma\nu)]$ of the spin does not change, i.e., $\sigma_{\perp} = \mathbf{n}_{\perp}$. Therefore $\sin \varphi = |\mathbf{n}_{\perp}|/|\mathbf{n}| \equiv |\sigma_{\perp}| = \sigma_{\perp} = \mathbf{\nu} \times [\sigma \times \mathbf{\nu}], |\sigma \times \mathbf{\nu}|$. Since σ and ν lie in the x,y plane, then

$$\sin \varphi = \left| \frac{\sigma_x \beta_y - \beta_x \sigma_y}{\sqrt{\beta_x^2 + \beta_y^2}} \right|_{t'=\tau},$$
 (1)

where it is noted that all quantities relate to the instant $t' = \tau$ of exodus of the electron from the region of the field.

For calculation of the components of the vectors σ and β it is convenient to go to the system of reference K' which moves along the x axis with velocity $\beta_0 = E/H$ (it is assumed that E < H, i.e., $\beta_0 < 1$) with respect to the laboratory system. The quantities relating to the system K' will be denoted by primes.

In the system K' the electric field vanishes

$$\mathbf{E}' = 0, \ \mathbf{H}' = \mathbf{H} \sqrt{1 - \beta_0},$$

[†]The generalization to the case in which β_z is different from zero can be carried out in an elementary fashion.

[‡]The magnitude of this angle determines the magnitude of left-right asymmetry of the subsequent scattering of the electron in the Coulomb field.

******To do this, as is well known, one should set $i\sigma_{\mu} = \varepsilon_{\mu\nu}\lambda_{\rho}\sigma_{\nu}\lambda u_{\rho}$, where $\varepsilon_{\mu\nu}\lambda_{\rho}$ is the completely antisymmetric unit matrix ($\varepsilon_{1234} = 1$), $\sigma_{\nu\lambda}$ is the antisymmetrical spin tensor (in the rest system K°, $\sigma_{12} = s_3$, $\sigma_{31} = s_2$, $\sigma_{23} = s_1$, with s the spin-vector), and u_{ρ} is the 4-vector electron velocity.

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^{*}It will be assumed everywhere that the spin is measured in units of the light velocity, i.e., $\beta = v/c$.

Therefore, in it the spin $\sigma'(t')$ and velocity $\beta'(t')$ of the electron rotates uniformly in the x', y' plane:

$$\omega' = ecH' / \epsilon' = (eH / m_e c) \sqrt{(1 - \beta^2) (1 - \beta_0^2)}, \quad (3)$$

and β'_X , β'_y and σ'_X , σ'_y are the velocity and spin of the electron at the moment of exodus from the region of the fields.

Using the Lorentz transformation from the laboratory system to the system K', it is clearly possible to express $\sin \varphi$ in Eq. (1) in terms of the spin components and velocities in the system K':

$$\sin \varphi = \frac{\sigma'_{y}(\tau) \left[\beta'_{x}(\tau) + \beta_{0}\right] - \left[\sigma'_{x}(\tau) + \beta_{0}\sigma_{0}\right]\beta'_{y}(\tau)}{\sqrt{\left[\beta'_{x}(\tau) + \beta_{0}\right]^{2} + \beta'^{2}_{y}(\tau)\left(1 - \beta^{2}_{0}\right)}}, \quad (4)$$

and the projections β'_X , β'_Y and σ'_X , σ'_Y of the velocity and spin at t' = 0 through α , β and β_0 . A simple calculation and substitution in Eq. (2) give

$$\sigma'_{x}(\tau) = \frac{(\cos \alpha - \beta \beta_{0}) \cos \varphi_{0} - \sqrt{1 - \beta_{0}^{2} \sin \alpha \sin \varphi_{0}}}{\sqrt{(1 - \beta^{2})(1 - \beta_{0}^{2})}},$$

$$\sigma'_{y}(\tau) = \frac{(\cos \alpha - \beta \beta_{0}) \sin \varphi_{0} + \sqrt{1 - \beta_{0}^{2}} \sin \alpha \cos \varphi_{0}}{\sqrt{(1 - \beta^{2})(1 - \beta_{0}^{2})}},$$

$$\beta'_{x}(\tau) = \frac{(\beta \cos \alpha - \beta_{0}) \cos \varphi_{0} - \beta \sqrt{1 - \beta_{0}^{2}} \sin \alpha \sin \varphi_{0}}{1 - \beta \beta_{0} \cos \alpha},$$

$$\beta'_{y}(\tau) = \frac{(\beta \cos \alpha - \beta_{0}) \sin \varphi_{0} + \beta \sqrt{1 - \beta_{0}^{2}} \sin \alpha \cos \varphi_{0}}{1 - \beta \beta_{0} \cos \alpha}.$$
(5)

The substitution of Eq. (5) into Eq. (4) gives a very long expression, which we won't reproduce here. The formulae (5) and (4) solve the required problem, if the quantity $\varphi_0 = \omega' \tau$ is found, i.e., the time τ in the system K' of movement of the electron through the region of the fields. In this system, the length of the projection l' of the path of the electron on the x' axis will be $l = l\sqrt{1-\beta_0^2}$, therefore, τ is determined by the condition

$$l' = \tau \beta_0 + \int_0^{\tau} \beta'_x(t') \, dt'.$$
 (5')

Using Eqs. (3) and (5), this condition leads to the

following equation for φ_0 :

$$\varphi_{0} - \frac{1 - (\beta_{0} / \beta \cos \alpha)}{1 - \beta_{0}^{2}} (\varphi_{0} - \sin \varphi_{0}) - \frac{(1 - \cos \varphi_{0}) \tan \alpha}{\sqrt{1 - \beta_{0}^{3}}}$$
$$= \frac{eHl}{cp \cos \alpha} \sqrt{1 - \beta_{0}^{2}}, \qquad (6)$$

where $p = m_e \beta c / \sqrt{1 - \beta^2}$ is the initial momentum of the electron. In practice to a high accuracy one can consider that φ_0 is equal to the right-hand side, since the second and third terms on the lefthand side are very small* in comparison with φ_0 .

In the case $\alpha < 1$ and $\varphi_0 < 1$, neglecting terms of order α^2 , φ_0^2 and $\alpha \varphi_0$, the simple formula

$$\varphi \approx \frac{eHl}{cp} \frac{\beta_0}{\beta} \sqrt{1 - \beta^2}.$$
 (7)

follows from Eqs. (4) - (6).

A relatively simple formula can be obtained also in the important case of small α and arbitrary φ_0 . For this, expansion of Eq. (5) in powers of α and substitution into Eq. (4) gives

$$\sin \varphi = \sin \varphi_0 \left[\sin^2 \varphi_0 + \frac{[(\beta / \beta_0) - 1 + (1 - \beta \beta_0) \cos \varphi_0]^2}{(1 - \beta^2)(1 - \beta_0^2)} \right]^{-1/2} \times [1 + \alpha \Delta_1 + \alpha^2 \Delta_2 + \dots],$$
(8)

where Δ_1, Δ_2 , etc. — of the order of magnitude unity — are some functions of β_0 , β and φ_0 which are easy to calculate.

For $\alpha = 0$ and $\beta = \beta_0$ the electron trajectory is rectilinear and Eq. (7) or Eq. (6) and Eq. (8) lead to the well-known equation:

$$\varphi = (eHl / cp) \sqrt{1 - \beta^2}.$$

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^{*}These terms determine the correction to φ_0 connected with the curvilinear nature of the trajectory of the electron. For $\alpha = 0$ and $\beta = \beta_0$, the trajectory is rectilinear; as can be seen, the terms vanish in this case.