Letters to the Editor

LATERAL DISTRIBUTION FUNCTION OF PHOTONS AT CASCADE SHOWER MAXIMUM

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In an earlier publication[1] of the authors, the electron lateral distribution function (LDF) was calculated using the moment method. Knowledge of the photon LDF is necessary for analysis of cosmic-ray measurements, especially for interpretation of data on extensive air showers. Molière[2] calculated the photon LDF at the cascade electron-photon shower maximum neglecting ionization losses. Although the calculations of Molière have been often criticized, it can be stated that their result is correct for $r \ll 1$. Eyges and Fernbach[3] calculated the photon LDF by finding graphically a function such that its first few moments coincide with the moments of the required function. The function obtained is correct for $r \gtrsim 1$.

We calculated the photon LDF using the moment method. We made use of the fact that the LDF $N_\Gamma(x_r)$ of photons with energy $> E$ is proportional to $(\ln x_r)/x_r$ for $x_r \to 0$,[2] where $x_r = Er/E_s$, $E_s = 21$ Mev. The function $x_r N_\Gamma(x_r)$ can be approximated by the sum of polynomials

$$x_r N_\Gamma(x_r) = \mathrm{Ei}\,(-\alpha\sqrt{x_r}) \sum_{n=0}^{N} a_n R_n(\alpha x_r). \qquad (1)$$

where $R_n(\alpha x_r)$ are polynomials orthogonal in the interval $(0, \infty)$ with weight function $\mathrm{Ei}(-\alpha\sqrt{x_r})$. The polynomials $R_n(\alpha x_r)$ and the conjugate orthogonal polynomials $R_n^+(\alpha x_r)$ are determined from the conditions[1]

$$\int_0^\infty x_r^{2n'}\,\mathrm{Ei}\,(-\alpha\sqrt{x_r})\,R_n(\alpha x_r)\,dx_r = \begin{cases} \gamma & \text{for } n' = n \\ 0 & \text{for } n' < n, \end{cases}$$

$$\int_0^\infty \mathrm{Ei}\,(-\alpha\sqrt{x_r})\,R_n(\alpha x_r)\,R_{n'}^+(x_r)\,dx_r = \delta_{nn'}. \qquad (2)$$

The explicit expressions for R_0, R_1, R_2, are

$$R_0(\alpha x_r) = (\gamma/2L_0)\,\alpha^2;$$

$$R_1(\alpha x_r) = (\gamma/2L_1)\,\alpha^6\,[-A_1 + L_0\alpha\,\alpha x_r];$$

$$R_2(a x_r) = (\gamma/2L_2)\,\alpha^{10}\,[(A_1A_4 - A_2A_3) - (A_0A_4 - A_2A_2)\,\alpha\alpha x_r + L_1\alpha^2(\alpha x_r)^2], \qquad (3)$$

where

$$L_0 = A_0; \quad L_1 = A_0A_3 - A_1A_2;$$

$$L_2 = A_6L_1 + A_5(A_2A_2 - A_0A_4) + A_4(A_1A_4 - A_2A_3);$$

$$A_n = \Gamma(2 + 2n)/(2 + 2n).$$

Using formulae (3) we obtain the following expressions for the coefficients a_n:

$$a_0 = 1/\gamma; \quad a_1 = (1/\gamma)\,[-L_0^{-1}A_2\alpha^{-4} + \overline{x_\Gamma^2}];$$

$$a_2 = (1/\gamma)\,[L_0^{-1}(A_5A_2 - A_3A_4)\,\alpha^{-8} + L_1^{-1}(A_1A_4 - A_0A_5)\,\alpha^{-4}\overline{x_\Gamma^2} + \overline{x_\Gamma^4}],$$

where $\overline{x_\Gamma^n}$ is the n-th moment of the function $N_\Gamma(x_r)$.

The coefficient α can be found from the condition that the highest moment of the required function in the expansion of Ref. 1 is equal to the corresponding moment of the weight function $\mathrm{Ei}(-\alpha\sqrt{x_r})$.

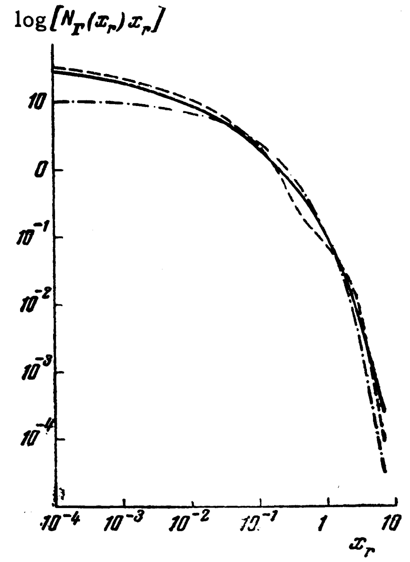

FIG. 1

Results of the calculation of the photon LDF are shown in Fig. 1 (solid curve) where the Molière curve (dashed) is also included. The curves differ by less than 20% up to $x_r \sim 0.1$. It should be noted that the curves are similar up to $x_r \sim 0.01$. For $x_r \sim 1$ the curves differ by a factor of two, but the Molière distribution is already inaccurate in that region. Our curve is shown also in Fig. 2, where the points correspond to calculations of Eyges and Fernbach. The functions differ by less than 10% for $x_r \sim 1-5$. The photon LDF calculated by us using the first three moments differs from the more accurate function by less than 10%.

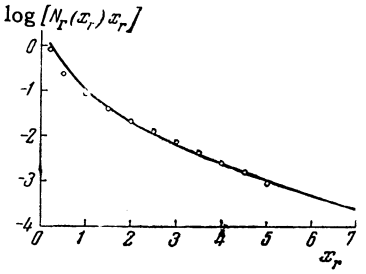

FIG. 2

The electron LDF[1] is also shown in Fig. 1 (dot-dash curve). It should be noted that, even for $x_r = 10^{-4}$ which corresponds to distances of 2×10^{-5} radiation units for particles with energy $\geq 10^8$ ev (i.e. distances < 0.5 cm in air at sea-level), the values of the photon LDF are only three times larger than those of the electron LDF, in spite of the fact that for $x_r \rightarrow 0$ the function $x_r N_\Gamma(x_r)$ diverges as $\ln x_r$, while $x_r N_p(x_r)$ remains finite.

[1] V. V. Guzhavin and I. P. Ivanenko, Dokl. Akad. Nauk SSSR **113**, 533 (1957), Soviet Phys. "Doklady" **2**, 131 (1957).
[2] G. Molière, Phys. Rev. **77**, 715 (1950).
[3] L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).

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140

MOTION OF ELECTRON ALONG SELF-INTERSECTING TRAJECTORIES

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IN Ref. 1 we derived an equation for the motion of an electron with arbitrary law of dispersion $E(k)$ in a magnetic field $(H = H_z)$ and obtained (following I. M. Lifshitz and Kosevich[2]) with the aid of the quasi-classical approximation the following equation

$$S(E, k_3) = 2\pi\alpha_0^{-2}(n + 1/2), \quad \alpha_0^2 = hc/eH,$$

for the energy levels of an electron moving in closed trajectories (the intersections of the surface $E(k) = \text{const}$, and the plane $k_3 = \text{const}$). In the present work we shall consider a case when the trajectory has a form of a closed self-intersecting curve ("figure eight"). However, the consideration given below pertains also to the case when the "figure eight" has a narrow neck and when it breaks up into two closed regions.

Near the point of self-intersection it is impossible to employ the quasi-classical approximation. For the region near such a point (where the trajectory can be represented by two hyperbolas, which degenerate into straight lines upon exact self-intersection), it is necessary to write the exact solution. A similar problem was solved in Ref. 3. It turns out that near the point of self-intersection the exact solution is expressed in terms of degenerate hypergeometric functions, similar to the manner in which the solution near the point of the classical turn is expressed in terms of the Airy functions. The "joining" of the quasi-classical solution (away from the point of self-intersection) and the exact solution (in the vicinity of this point) gives a quantization condition in the form

$$S = 4\pi\alpha_0^{-2}(n + 1/2 + \gamma_{1,2}(\lambda)); \tag{1}$$

here S is the total area of the curve, $\gamma_{1,2}(\lambda)$ are functions, to be determined below, of the quantity

$$\lambda = (\varkappa_0/\varepsilon)^2 \sqrt{R/\varkappa_0}, \tag{2}$$

where $\kappa_0 = \kappa_{10} a_1$, κ_{10} is the value of κ_1 at the boundary (for exact self-intersection $\kappa_{10} = 0$), $\epsilon = a/\alpha_0$, R is the radius of curvature of the trajectory at the point of self-intersection ($\sqrt{\kappa_0/R}$ is the slope of the tangent at the point of self-intersection for $\kappa_{10} = 0$, or the slope of the asymptote of the hyperbola in the case of inexact self-intersection). The quantities γ_1 and γ_2 are determined in the following manner

$$\gamma_1 = -\frac{1}{8} - \varphi_1 - \frac{\lambda}{4}\ln\left|\frac{\lambda}{4e}\right|, \quad \gamma_2 = -\frac{1}{8} - \varphi_2 - \frac{\lambda}{4}\ln\left|\frac{\lambda}{4e}\right|,$$
$$R_1 e^{i\varphi_1} = 1/\Gamma\left(\frac{1}{4} + i\frac{\lambda}{4}\right), \quad R_2 e^{i\varphi_2} = e^{-i\pi/4}/\Gamma\left(\frac{3}{4} + i\frac{\lambda}{4}\right) \tag{3}$$

In order to gain an idea of the splitting of the energy levels upon gradual deformation of the "figure eight," it is enough to consider the following cases.

(1) When $\lambda < 0$ and $|\lambda| \gg 1$, corresponding to two individual regions (κ_{10} is imaginary), $\gamma_1 = \gamma_2 = 0$. Then the area of each region is determined by the usual equation $S = 2\pi\alpha_0^{-2}(n + 1/2)$, and the total area is

$$S = 4\pi\alpha_0^{-2}(n + 1/2) \tag{4}$$