EQUALITY OF THE NUCLEON AND ANTINUCLEON TOTAL INTERACTION CROSS SECTION AT HIGH ENERGIES

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The dispersion relations are used to show that the total interaction cross sections are equal for particles and antiparticles at high energies.

FROM the dispersion relations for elastic scattering of nucleons through angle zero[1-3] one can establish that the total interaction cross sections for nucleons and antinucleons must be equal at sufficiently high energies.

To be specific, let us consider scattering of protons and antiprotons through the angle zero. The dispersion relations for the scattered amplitudies, averaged over spins, are of the form

$$D_+(E) = \frac{1}{2}\left(1+\frac{E}{M}\right)D_+(M) + \frac{1}{2}\left(1-\frac{E}{M}\right)D_-(M) + \frac{p^2}{4\pi^2}\int_M^\infty \frac{dE'}{p'}\left[\frac{\sigma_+(E')}{E'-E} + \frac{\sigma_-(E')}{E'+E}\right] + \frac{f^2}{\mu^2}\frac{p^2}{M-\mu^2/2M-E} + \frac{p^2}{4\pi^2}\int_0^M \frac{i\,dE'}{p'}\left[\frac{\sigma_+(E')}{E'-E} + \frac{\sigma_-(E')}{E'+E}\right]; \quad (1)$$

$$D_-(E) = \frac{1}{2}\left(1+\frac{E}{M}\right)D_-(M) + \frac{1}{2}\left(1-\frac{E}{M}\right)D_+(M) + \frac{p^2}{4\pi^2}\int_M^\infty \frac{dE'}{p'}\left[\frac{\sigma_-(E')}{E'-E} + \frac{\sigma_+(E')}{E'+E}\right] + \frac{f^2}{\mu^2}\frac{p^2}{M-\mu^2/2M+E} + \frac{p^2}{4\pi^2}\int_0^M \frac{i\,dE'}{p'}\left[\frac{\sigma_+(E')}{E'+E} + \frac{\sigma_-(E')}{E'-E}\right]. \quad (1')$$

Here $D_+(E)$ is the real part of the elastic scattered amplitude, averaged over spins, for a proton of energy E scattered by a proton at rest, $D_-(E)$ is the real part of the elastic scattered amplitude for an antiproton of energy E scattered by a proton at rest, $\sigma_+(E)$ is the total scattering cross section for a proton with energy E, and $\sigma_-(E)$ is the total scattering cross section for an antiproton with energy E. For energies from zero to M, the function $\sigma_+(E)$ is the analytic continuation of $\sigma_+(E)$ for energies $E > M$, and $\sigma_-(E)$ is the analytic continuation of $\sigma_-(E)$ into the "nonphysical" region $E < M$.

As the energy E approaches infinity, $\sigma_+(E)$ and $\sigma_-(E)$ approach the constant values $\sigma_+(\infty)$ and $\sigma_-(\infty)$. This follows simply from the fact that all strong interactions approach zero exponentially for large values of the impact parameter ρ (here the form of the factor multiplying the exponential is entirely insignificant). Had we wished to consider the electromagnetic interaction of a proton or antiproton with a proton, we could have taken into account the screening of the target proton charge at large distances by the atomic electrons, and also obtained a constant total cross section at sufficiently high energies. It is true that in this way a constant cross section would be obtained only at energies of the order of $E \sim M^2/me^2 \sim 10^{14}$ ev. If, however, we were not to take into account the weak electromagnetic interaction, then up to a quantity of order $e^2 \ln(E/M)$ the total cross section would reach a constant value even at an energy of the order of 10^{10} ev. Allowing E to approach infinity and maintaining only the largest terms, Eqs. (1) and (1′) lead to

$$D_+(E) = \frac{E}{2M}[D_+(M) - D_-(M)] - \frac{f^2}{\mu^2}E + \frac{E}{4\pi^2}\int_0^M \frac{i\,dE'}{p'}[\sigma_-(E') - \sigma_+(E')] + \frac{p^2}{4\pi^2}\int_M^\infty \frac{dE'}{p'}\left[\frac{\sigma_+(E')}{E'-E} + \frac{\sigma_-(E')}{E'+E}\right]; \quad (2)$$

$$D_-(E) = \frac{E}{2M}[D_-(M) - D_+(M)] + \frac{f^2 E}{\mu^2} + \frac{E}{4\pi^2}\int_0^M \frac{i\,dE'}{p'}[\sigma_+(E') - \sigma_-(E')] + \frac{p^2}{4\pi^2}\int_M^\infty \frac{dE'}{p'}\left[\frac{\sigma_-(E')}{E'-E} + \frac{\sigma_+(E')}{E'+E}\right]. \quad (2')$$

In these equations let us first consider the integral from M to ∞. Starting at a certain energy ϵ, both cross sections σ_+ and σ_- may be considered constant and equal to $\sigma_+(\infty)$ and

$\sigma_{-}(\infty)$. We may therefore write ($E \gg \epsilon$)*

$$\frac{E^2}{4\pi^2}\int_M^\infty \left[\frac{\sigma_+(E')}{E'-E} + \frac{\sigma_-(E')}{E'+E}\right] dE' \frac{1}{\sqrt{E'^2-M^2}}$$

$$= \frac{E^2}{4\pi^2}\int_M^\varepsilon \left[\frac{\sigma_+(E')}{E'-E} + \frac{\sigma_-(E')}{E'+E}\right]\frac{dE'}{p'} \tag{3}$$

$$+\frac{E^2}{4\pi^2}\int_\varepsilon^\infty \frac{dE'}{E'}\left[\frac{\sigma_+(\infty)}{E'-E} + \frac{\sigma_-(\infty)}{E'+E}\right] = \frac{E}{4\pi^2}\int_M^\varepsilon [\sigma_-(E') - \sigma_+(E')]\frac{dE'}{p'}$$

$$+\frac{E^2}{4\pi^2}\int_\varepsilon^\infty \frac{dE'}{E'}\left[\frac{\sigma_+(\infty)}{E'-E} + \frac{\sigma_-(\infty)}{E'+E}\right].$$

When $E \gg \epsilon$, the asymptotic value of (3) is

$$\frac{E}{4\pi^2}\left(\ln\frac{E}{\varepsilon}\right)[\sigma_-(\infty) - \sigma_+(\infty)] \tag{4}$$

[where we have assumed that $\sigma_-(\infty) \neq \sigma_+(\infty)$]. The integral from M to ϵ is, with our conditions, proportional to E:

$$\frac{E}{4\pi^2}\int_M^\varepsilon [\sigma_-(E') - \sigma_+(E')]\frac{dE'}{p'}. \tag{5}$$

By combining (3), (4), and (5), Eqs. (2) and (2′) become

$$D_+(E) = E\left\{\frac{D_+(M) - D_-(M)}{2M} - \frac{f^2}{\mu^2} + \frac{1}{4\pi^2}\int_0^M \frac{idE'}{p'}[\sigma_-(E') - \sigma_+(E')]\right. \tag{6}$$

$$\left.+\frac{1}{4\pi^2}\int_M^\varepsilon \frac{dE'}{p'}[\sigma_-(E') - \sigma_+(E')] + \frac{1}{4\pi^2}[\sigma_-(\infty) - \sigma_+(\infty)]\ln\frac{E}{\varepsilon}\right\},$$

$$D_-(E) = E\left\{\frac{D_-(M) - D_+(M)}{2M} + \frac{f^2}{\mu^2} + \frac{1}{4\pi^2}\int_0^M \frac{idE'}{p'}[\sigma_+(E') - \sigma_-(E')]\right. \tag{6'}$$

$$\left.+\frac{1}{4\pi^2}\int_M^\varepsilon \frac{dE'}{p'}[\sigma_+(E') - \sigma_-(E')] + \frac{1}{4\pi^2}[\sigma_+(\infty) - \sigma_-(\infty)]\ln\frac{E}{\varepsilon}\right\}.$$

We see that if $\sigma_+(\infty) \neq \sigma_-(\infty)$, the principal term which determines $D_+(E)$ and $D_-(E)$ increases faster than E, and is proportional to $E \times \ln(E/\epsilon)$. This result contradicts the conclusion that the interaction decays exponentially for large distances. Let us write out the general expression for the elastic scattered amplitude A(0) for the angle zero. For simplicity, we shall not take account of spin, noting that the orbital quantum number ℓ is very large at high energies, so that the replacement of ℓ by $\ell \pm 1$, which must be made in order to account for spin, will change nothing in the following considerations.

At high energies, when $\ell \gg 1$, we have

$$A(0) = \frac{1}{2E}\sum_l (e^{2i\eta_l} - 1)\,l, \tag{7}$$

where η_ℓ is the phase of the wave with orbital angular momentum ℓ. For large ℓ we may use the semiclassical impact parameter ℓ/E. When ℓ/E is much larger than the radius of the interaction ρ (which is of the order of the Compton wavelength of the π meson), η_ℓ decays exponentially with ℓ. It follows from this that the effective upper limit in Eq. (7) is a quantity of order $E\rho$. (This result is independent of the form of the factor multiplying the exponential of the interaction.)

Since the modulus of $e^{2i\eta_\ell} - 1$ is no greater than 2, the order of magnitude of the modulus of A(0) is

$$|A(0)| \leqslant CE\rho^2, \quad C \sim 1. \tag{8}$$

It follows from this that $D_\pm(E)$ cannot contain terms proportional to $[\sigma_+(\infty) - \sigma_-(\infty)]E\ln(E/\epsilon)$. Therefore*

$$\sigma_+(\infty) = \sigma_-(\infty). \tag{9}$$

Thus at large energies the total antiproton and proton cross sections are equal. When we recall that at energies of several hundreds of Mev these cross sections are very different (σ_- is much greater[4,5] than σ_+), it becomes evident that at large energies we should expect one or both of these cross sections to be highly energy dependent.

An equation similar to (9) should hold also for the asymptotic values of the total cross sections $\sigma_n(\infty)$ and $\sigma_{\bar{n}}(\infty)$ for neutrons and antineutrons on protons, namely

$$\sigma_{\bar{n}}(\infty) = \sigma_n(\infty). \tag{10}$$

From statistical considerations and the requirement of isotopic invariance,[6] we may establish the equality of $\sigma_n(\infty)$ and $\sigma_+(\infty)$, as well as that of $\sigma_{\bar{n}}(\infty)$ and $\sigma_-(\infty)$. At high energies, therefore, the proton, the neutron, the antiproton, and the antineutron should all have the same lim-

*Let us bear in mind that we take the principal part of the integral at the point E′ = E. The same is true with respect to the singularity at E = M (where σ_- approaches ∞ as 1/v, with v being the velocity). In treating this singularity, one must treat simultaneously the intervals from zero to M and from M to ε.

*Equation (9) has been derived in a different way by B. V. Medvedev and D. V. Shirkov.

iting values for their cross sections.

Similar considerations hold also for K mesons. This means that at sufficiently high energies (of the order of 10^{10} ev, if electromagnetic interactions are not taken into account) the K^+, K^-, K^0, and $\overline{K}^0$ mesons should all have the same total cross section. Similarly, the π^+ and π^- cross sections should also approach equality as $E \to \infty$.[7] If we apply our result to hyperons, we obtain the following.

(1) The Λ and anti-Λ (i.e., the $\overline{\Lambda}$) have the same cross section $\sigma_\Lambda(\infty)$.

(2) The Σ^+, Σ^-, Σ^0, $\overline{\Sigma}^+$, $\overline{\Sigma}^-$, and $\overline{\Sigma}^0$ cross sections approach the common value $\sigma_\Sigma(\infty)$ as $E \to \infty$.

(3) The Ξ^-, Ξ^0, $\overline{\Xi}^-$, $\overline{\Xi}^0$ have the same cross section $\sigma_\Xi(\infty)$.

We note also that from (2), (2′), and (9) it follows that at large energies the main contributions to $D_+(E)$ and $D_-(E)$ are proportional to E and differ only in sign. This together with (9) shows that the differential cross sections for elastic scattering of nucleons and antinucleons by nucleons through the angle zero approach equality as $E \to \infty$.

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CALCULATION OF COORDINATE PROBABILITIES BY GIBBS METHOD

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Gibbs' statistical method is used to derive general formula which permits one to determine fully the stationary probability density as well as the transition probability density in a nonstationary process for an arbitrary generalized coordinate, provided the behavior of the mean value of the latter is known in the presence of (or after turning on) additional forces acting in the direction of this coordinate.

IT is well known that, using general methods of statistical mechanics, we can derive exact relationships which enable us to reduce a calculation of fluctuations and correlations of various quantities (among them time correlations) to a determination of average values of these quantities in the presence of (or after turning on) additional constant forces.[1-4]

It was shown in Ref. 5 that all the principal momenta determined in fluctuation theory, as well as in the theory of Brownian motion, can be computed by this method.