If there are no special reasons for supposing that the difference between the magnetic moments of the sublattices is small (as, for example, in the case of Fe_2O_3, according to Refs. 7 and 8), then on carrying out calculations similar to those above we easily find that the magnetic part of the spectrum consists of two branches (if there are two sublattices). One of these has a large activation energy, of the order of Θ_C and is of course not excited at low temperatures (this is the analog of the optical branch in the vibrations of compound lattices). The other one is analogous to the ordinary Bloch spin waves. For the case of two moments and at not too low temperatures it has the form (supposing $M_1 > M_2$):

$$\omega = \frac{2gM_1M_2}{M_1 - M_2}(\beta - \beta_{12})k^2. \tag{19}$$

Therefore the magnetic part of the specific heat must be proportional to $T^{3/2}$, as it is for ordinary ferromagnetic substances.

In conclusion the writers take this occasion to thank L. D. Landau and I. M. Lifshitz for valuable discussions on the questions considered here.

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SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 1 JULY, 1958

SCATTERING OF DIRAC PARTICLES BY A SHORT-RANGE CENTER OF FORCE WITH DAMPING TAKEN INTO ACCOUNT

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Submitted to JETP editor July 10, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 110-112 (January, 1958)

The theory of radiation damping is used to investigate elastic scattering of Dirac particles by a stationary short-range center of force. An equation is obtained for the scattering cross section A relation [Eq. (5)] is established between the scattering phase shifts predicted by the theory of radiation damping for Dirac particles and relativistic spinless particles.

In the present work the theory of radiation damping is used to study elastic scattering of Dirac particles by an arbitrary short-range center of force. This has already been done[1] only for spinless particles (hereinafter we shall use the notation developed in that article and denote it by SK). In the present case we shall divide the wave functions into two groups according to the z component of the spin ($m_s = \pm\frac{1}{2}$), rather than according to the component of the spin in the direction of motion, as was done in SK. Then the fundamental integral equation [see Eq. (21) of SK] of the theory of radiation damping for elastic scattering of spin-$\frac{1}{2}$ particles becomes

$$(\varepsilon_n(l) - 1) H^{(n)}_{\mathbf{k}'\mathbf{k}}(l, m_{s'}, m_s) = \frac{kK}{8\pi^2 c\hbar i} \sum_{l', n', m_{s''}} \varepsilon_{n'}(l') \oint d\Omega'' H^{(n)}_{\mathbf{k}'\mathbf{k}''}(l, m_{s'}, m_{s''}) H^{(n')}_{\mathbf{k}''\mathbf{k}}(l', m_{s''}, m_s), \tag{1}$$

where $\hbar\mathbf{k}$, $\hbar\mathbf{k}'$, and $\hbar\mathbf{k}''$ are the momenta of the particles in the initial, final, and intermediate states, respectively. The matrix element of the transition from the state $\mathbf{k}''$, $m_{s''}$ to the state $\mathbf{k}'$, $m_{s'}$ is given by

$$H_{\mathbf{k}'\mathbf{k}''}(m_{s'}, m_{s''}) = \sum_{l,\,n} H^{(n)}_{\mathbf{k}'\mathbf{k}''}(l, m_{s'}, m_{s''}) = \sum_{l=0}^{\infty} \frac{4\pi b_l}{2l+1} \sum_{m=-l}^{l} Y_l^m(\mathbf{k}') Y_l^{m*}(\mathbf{k}'') b'^{+}(m_{s'}) b''(m_{s''}), \tag{2}$$

where $Y_\ell^m(\mathbf{k}')$ is a spherical function normalized to unity which depends on the spherical coordinate angles θ' and φ' of the vector $\mathbf{k}'$, b_ℓ is the amplitude in the expansion of the external potential, and n and $n' = 1$ and 2 take account of the fact that for a given ℓ, the value of j may be $\ell + \frac{1}{2}$ (when $n = 1$) or $\ell - \frac{1}{2}$ (when $n = 2$).

The spin part of the transition matrix element is

$$b'^{+}(m_{s'}) b''(m_{s''}) = \begin{cases} \frac{1}{2}\left(1+\frac{k_0}{K}\right) + \frac{1}{2}\left(1-\frac{k_0}{K}\right)[\cos\theta'\cos\theta'' + \sin\theta'\sin\theta''\exp\{2i(\varphi''-\varphi')m_{s'}\}] & \text{when } m_{s''} = m_{s'} \\ m_{s'}\left(1-\frac{k_0}{K}\right)[\cos\theta'\sin\theta'' e^{-2i\varphi'' m_{s'}} - \sin\theta'\cos\theta''\exp\{-2i\varphi' m_{s'}\}] & \text{when } m_{s''} = -m_{s'}. \end{cases} \tag{3}$$

In what follows we shall use recursion relations which relate $\cos\theta Y_\ell^m(\mathbf{k})$ and $\sin\theta e^{\pm i\varphi} Y_\ell^m(\mathbf{k})$ with the functions $Y_{\ell\pm1}^{m\pm1}(\mathbf{k})$. This makes it possible to eliminate $\cos\theta$ and $\sin\theta e^{\pm i\varphi}$ from the matrix elements. After some mathematical manipulation we obtain the following expression for the components of matrix element of Eq. (2):

$$H^{(n)}_{\mathbf{k}'\mathbf{k}''}(l, m_{s'}, m_{s''}) = \frac{8\pi^2 c\hbar}{kK} c_{nl} \sum_{m=-(j+1/2)}^{j+1/2} \Omega^{(n)}_{lm}(\mathbf{k}', m_{s'}) \Omega^{*(n)}_{lm}(\mathbf{k}'', m_{s''}), \tag{4}$$

where ($n = 1$ or 2)

$$c_{1l} = {}^1/_2(1 + k_0/K)c_l + {}^1/_2(1 - k_0/K)c_{l+1}, \quad c_{2l} = {}^1/_2(1 + k_0/K)c_l + {}^1/_2(1 - k_0/K)c_{l-1}, \tag{5}$$

and c_ℓ gives the scattering phase shift of a spinless particle. According to Eq. (52), of SK, the latter is related to the potential $V(r)$ by

$$c_l = \frac{kK}{2\pi c\hbar}\frac{b_l}{2l+1} = \frac{\pi K}{c\hbar}\int_0^\infty rV(r) J^2_{l+1/2}(kr)\,dr. \tag{6}$$

If we now set the components of the spherical spinor $\Omega^{(n)}_{\ell m}$ in Eq. (4) corresponding to the two values of the z component of the spin ($m = \pm\frac{1}{2}$) equal to

$$\Omega^{(n)}_{lm}(\mathbf{k}, m_s) = A^{(n),\,m_s}_{lm} Y_l^{m-m_s-1/2}(\mathbf{k}), \qquad A^{(1),\,1/2}_{lm} = A^{(2),\,-1/2}_{lm} = \sqrt{\frac{l+m}{2l+1}}, \quad -A^{(1),\,-1/2}_{lm} = A^{(2),\,1/2}_{lm} = \sqrt{\frac{l-m+1}{2l+1}},$$

we obtain the orthogonality of the components of the matrix elements, which is necessary in the theory of damping, namely

$$\sum_{m_{s''}} \oint d\Omega'' H^{(n)}_{\mathbf{k}'\mathbf{k}''}(l, m_{s'}, m_{s''}) H^{(n')}_{\mathbf{k}''\mathbf{k}}(l', m_{s''}, m_s) = \frac{8\pi^2 c\hbar}{kK} c_{nl}\delta_{nn'}\delta_{ll'} H^{(n)}_{\mathbf{k}'\mathbf{k}}(l', m_{s'}, m_s). \tag{7}$$

From (7) and (1) we obtain the following expression for the transition amplitudes $C'(m_{s'}) \equiv C(\mathbf{k}', m_{s'}, t)$ (see Eq. (15) of SK):

$$C'(m_{s'}) = \frac{1 - e^{ic(K'-K)t}}{c\hbar L^3(K'-K)} \sum_{l,\,n} \varepsilon_n(l) H^{(n)}_{\mathbf{k}'\mathbf{k}}(l, m_{s'}, m_s),$$

where

$$\varepsilon_n(l) = 1/(1 + ic_{nl}),$$

and we simply do not take into account the coefficients β_1 and β_2, which are of order L^{-3}. It is necessary to include β_1 and β_2 in order to prove that

$$C^+(m_s)C(m_s) + \sum_{\mathbf{k}', m_{s'}}' C'^+(m_{s'})C'(m_{s'}) = 1.$$

The differential cross section for elastic scattering is

$$d\sigma_{m_{s'}, m_s} = \frac{L^3 K}{ck} \sum_{\mathbf{k}'} \frac{\partial}{\partial t} C'^+(m_{s'})C'(m_{s'}) = |f(m_s, m_{s'}, \theta', \varphi')|^2 d\Omega'. \tag{8}$$

If the incident Dirac particle is directed along the z axis ($\cos\theta = 1$), the scattering amplitude is given by

$$f(m_s, m_{s'}, \theta', \varphi') = \begin{cases} -\frac{1}{k}\sum_{l=0}^{\infty}\left[(l+1)\frac{c_{1l}}{1+ic_{1l}} + l\frac{c_{2l}}{1+ic_{2l}}\right]P_l(\cos\theta') & \text{when } m_{s'} = m_s, \\ -\frac{1}{k}\sum_{l=0}^{\infty}\left[\frac{c_{1l}}{1+ic_{1l}} - \frac{c_{2l}}{1+ic_{2l}}\right]2m_{s'}e^{-2i\varphi' m_{s'}}P_l^1(\cos\theta') & \text{when } m_{s'} = -m_s. \end{cases} \tag{9}$$

The total effective elastic scattering cross section is

$$\sigma = \frac{1}{2}\sum_{m_{s'}, m_s}\oint |f(m_s, m_{s'}, \theta', \varphi')|^2 d\Omega' = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}\left[(l+1)\frac{c_{1l}^2}{1+c_{1l}^2} + l\frac{c_{2l}^2}{1+c_{2l}^2}\right] = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}[(l+1)\sin^2\eta_l^{(1)} + l\sin^2\eta_l^{(2)}]. \tag{10}$$

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SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 1 JULY, 1958

ON THE DESTRUCTION AND THE ONSET OF SUPERCONDUCTIVITY IN A MAGNETIC FIELD

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Submitted to JETP editor July 11, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 113-125 (January, 1958)

Transition from the superconducting state to the normal and vice versa in the presence of an external magnetic field is considered. Critical magnetic field strengths H_c, H_{c1} and H_{c2}, which correspond to equilibrium transition and to the boundaries of the supercooled and superheated regions respectively, are computed. Cases of small samples and of bulk metals are considered.

THE destruction and the onset of superconductivity in the presence of an external magnetic field proceed in entirely different ways, depending on the dimensions, shape and internal condition of the sample (its purity, its homogeneity, etc.). In the simplest case of a bulk sample of cylindrical shape subjected to a field parallel to the axis of the cylinder, assuming that no intermediate state occurs,