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## SPACE-ENERGY DISTRIBUTION OF NEUTRONS IN A HEAVY GASEOUS MODERATOR

## M. V. KAZARNOVSKII

P. N. Lebedev Institute of Physics, Academy of Sciences, U.S.S.R.

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THE theory of neutron thermalization in a heavy (atomic weight $M \gg 1$ ) monoatomic gas with constant mean free path $\lambda$ and constant neutron lifetime $\tau$ has been discussed in a number of papers. ${ }^{1-3}$ However, the majority of the results refers to the energy distribution only. The space-energy distribution function has been found only in the region of relatively large energies. ${ }^{2}$ In the case of weak absorption this problem can be solved exactly.

The equation for the space-energy distribution function ${ }^{2}$ may be written as follows:

$$
\begin{equation*}
-\alpha \psi(\mathbf{r}, x)+\beta x \nabla^{2} \psi(\mathbf{r}, x)+\left(3-2 x^{2}\right) \frac{\partial \psi(\mathbf{r}, x)}{\partial x}+x \frac{\partial^{2} \psi(\mathbf{r}, x)}{\partial x^{2}}=0, \quad \alpha=(2 M \lambda / \tau) \sqrt{m / 2 k T}, \quad \beta=2 M \lambda^{2} / 3 \tag{1}
\end{equation*}
$$

where $x^{2}$ is the neutron energy in units of $k T$ ( $T$ - temperature of the moderator), $\psi(r, x)$ is the spaceenergy distribution function divided by $x^{2} e^{-x^{2}}$, and $m$ is the neutron mass.

For a moderator of finite dimensions one may obtain a solution of Eq. (1) in the form of an expansion in a complete set of orthonormal functions $R_{\ell}(r)$ of the Laplacian for the corresponding boundary value problem $\left[\nabla^{2} R_{\ell}(\mathbf{r})+\Omega_{\ell} R_{\ell}(\mathbf{r})=0\right]$, i.e.,

$$
\begin{equation*}
\psi(\mathbf{r}, x)=\sum_{l} R_{l}(\mathbf{r}) n_{l}(x) \tag{2}
\end{equation*}
$$

then each of the functions $\mathrm{n}_{\boldsymbol{\ell}}(\mathrm{x})$ should satisfy the equation

$$
\begin{equation*}
x d^{2} n_{l} / d x^{2}+\left(3-2 x^{2}\right) d n_{l} / d x-\left(\alpha+\beta x \Omega_{l}\right) n_{l}=0 \tag{3}
\end{equation*}
$$

Making use of the requirement that $n_{\ell}(x)$ be finite as $x \rightarrow 0$, this equation may be transformed into an integral equation of the Volterra type

$$
\begin{gather*}
n_{l}(x)=C_{l} \Phi\left(a, 2, x^{2}\right)+\alpha \int_{0}^{x} n_{l}(t) K(x, t) d t  \tag{4}\\
K(x, t)=\frac{1}{2} \Gamma(a) t^{2} e^{-t^{2}}\left[\Psi\left(a, 2, t^{2}\right) \Phi\left(a, 2, x^{2}\right)-\Phi\left(a, 2, t^{2}\right) \Psi\left(a, 2, x^{2}\right)\right]
\end{gather*}
$$

the solution of which, as is well known, is of the form

$$
\begin{equation*}
n_{l}(x)=C_{l} \sum_{m=0}^{\infty} \alpha^{m} \varphi_{m}(x), \quad \varphi_{0}(x)=\Phi\left(a, 2, x^{2}\right) ; \quad \varphi_{m+1}(x)=\int_{0}^{x} \varphi_{m}(t) K(x, t) d t . \tag{5}
\end{equation*}
$$

Here $\Phi(\mathrm{a}, \mathrm{b}, \mathrm{z})$ is confluent hypergeometric function and

$$
\Psi(a, b, z)=\frac{1}{\Gamma(a)} \int_{0}^{\infty} e^{--z t}+a-1(1+t)^{b-a-1} d t
$$

is the other linearly independent solution of the hypergeometric equation; $a=\beta \Omega_{\ell} / 4$.
The normalization constants $C_{\ell}$ may be obtained by comparing the assymptotic form of $n_{\ell}(x)$ for large $x$ with the results of a calculation in the Fermi Age approximation. Thus, neglecting absorption for the sake of simplicity, one has for the case of a unit intensity source of neutrons at the point $\mathbf{r}=\mathbf{r}_{0}$ in the age approximation, the following

$$
\begin{equation*}
\Psi_{\mathrm{Age}}(\mathrm{r}, x)=\frac{\lambda M}{2} \sqrt{\frac{m}{2 k T}} \cdot \frac{x^{2}}{x^{4}} \sum_{l} \quad R_{l}(\mathbf{r})\left(\frac{x}{x_{0}}\right)^{\beta \Omega_{l} / 2}, \tag{6}
\end{equation*}
$$

where $x_{0}$ is the source neutrons speed. Hence

$$
\begin{equation*}
\psi(\mathbf{r}, x)=\frac{\lambda M}{2} \sqrt{\frac{m}{2 k T}} \sum_{l} R_{l}\left(\mathbf{r}_{0}\right) R_{l}(\mathbf{r}) x_{0}^{-\beta \Omega_{l} / 2} \Gamma\left(\frac{\beta \Omega_{l}}{4}\right) \Phi\left(\frac{\beta \Omega_{l}}{4}, 2, x^{2}\right) \tag{7}
\end{equation*}
$$

In the case of a source located in an infinite homogeneous medium the sum over $\ell$ must be replaced by the corresponding integral.

A detailed discussion of applications of the above results to various special cases will be published later.

In conclusion I express deep gratitude to F. L. Shapiro for valuable discussions in the process of this work.

[^0]Translated by A. Bincer
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## INFLUENCE OF FINITE NUCLEAR SIZE ON EFFECTS CONNECTED WITH PARITY NONCONSERVATION IN BETA DECAY

## B. V. GESHKENBEIN

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For $\beta$-decay, particularly for forbidden transitions, the action of the nuclear field is of importance. As in Ref. 1, we take for the electron wave function

$$
\psi_{e}=\binom{\varphi_{e}}{\chi_{e}}, \begin{align*}
& \varphi_{e}=\left[\alpha_{0}+i \operatorname{pr} \alpha_{1}+i(\sigma \mathrm{r})(\sigma \mathrm{n}) \beta_{c}\right] u_{\xi},  \tag{1}\\
& \chi_{e}=\left[\beta_{0}+i \operatorname{pr} \beta_{1}+i(\sigma \mathrm{r})(\sigma \mathrm{n}) \alpha_{c}\right](\sigma \mathrm{n}) u_{\xi},
\end{align*}
$$

where

$$
\begin{gathered}
\mathbf{n}=\frac{\mathbf{p}}{p}, \quad \alpha_{0}=\sqrt{\frac{\pi}{2 p \varepsilon}} i e^{-i \delta_{-1}} g_{-1}, \quad \beta_{0}=\sqrt{\frac{\pi}{2 p \varepsilon}} f_{1} e^{-i \delta_{1}}, \quad \alpha_{1}=\sqrt{\frac{\pi}{2 p \varepsilon}} \frac{3}{p r} e^{-i \delta_{-1}} g_{-2}, \quad \beta_{1}=\sqrt{\frac{\pi}{2 p \varepsilon} \frac{3}{i p r}} e^{-i \delta_{\delta_{2}}} f_{2}, \\
\beta_{c}=\sqrt{\frac{\pi}{2 p \varepsilon}} \frac{1}{r}\left(e^{-i \delta_{1}} g_{1}-e^{\left.-i \delta_{-1} g_{-2}\right), \quad \alpha_{c}=\sqrt{\frac{\pi}{2 p \varepsilon} \frac{1}{i r}\left(e^{-i \delta_{-1}} f_{-1}-e^{-i \delta_{1}} f_{2}\right)}}\right. \text {, }
\end{gathered}
$$

( $m_{e}=c=\hbar=1$ ), $g_{\kappa}, f_{\kappa}$ are the inside-the-nucleus solutions of the radial Dirac equation joined with the outside solutions at $r=r_{0} ; \delta_{\kappa}$ is the phase. ${ }^{2}$ We write the $\beta$-interaction Hamiltonian as follows

$$
\begin{equation*}
H=\sum\left\{g_{i}\left(\bar{\psi}_{2} O_{i} \psi_{1}\right)\left(\bar{\psi}_{e} O_{i} \frac{1-\gamma_{5}}{2} \psi_{v}\right)+g_{i}^{\prime}\left(\bar{\psi}_{2} O_{i} \psi_{1}\right)\left(\bar{\psi}_{e} O_{i} \frac{1+\gamma_{5}}{2} \psi_{v}\right)\right\} \tag{2}
\end{equation*}
$$

(summation over $\mathrm{i}=\mathrm{S}, \mathrm{T}, \mathrm{V}, \mathrm{A}, \mathrm{P}$ ). If the two-component neutrino theory ${ }^{3-5}$ holds, then emission of an antineutrino together with an electron corresponds to $\mathrm{g}_{i}^{\prime}=0$, whereas emission of a neutrino corresponds to $\mathrm{g}_{\mathrm{i}}=0$.


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