nucleonic charge) and the second as the electric current. From (7) we obtain the nucleon electric-charge operator

$$Q = -\varepsilon(T_3 - 1/2), \tag{8}$$

which commutes with the Hamiltonian and with the spin operator. The eigenvalues of Q are equal to zero for the neutron states and ϵ for the proton states.

It should be mentioned that (2) is not invariant under improper Lorentz transformations. It follows from the pseudoscalar character of γ^5 that a reflection of the space coordinates changes a proton into a neutron and vice versa. This means that (2) is invariant under simultaneous reflection of space and isospin coordinates.

Equation (2) can also be used to describe the doublet of the Ξ hyperons. One then has to take $m_0 = 2588\ m_e$ while a has the same value as for the nucleons. This yields a mass of 2586 m_e for the Ξ^- and 2590 m_e for the Ξ^0.

It is interesting to note that one can also find for the operators Γ^ν, T_k, and I a twelve by twelve irreducible representation which allows a description of the triplet of Σ hyperons.

*A more detailed derivation of (2) and a generalization for the case of weak interactions will be published in the Transactions (Trudy) of the Institute for Physics and Astronomy of the Academy of Sciences, Estonian S.S.R.

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CAPTURE OF ELECTRONS IN BETATRONS

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THE process of electron capture into betatron orbits has been studied by many authors, with various accelerators, and sufficient experimental material has consequently been compiled. A number of theoretical papers have been devoted to the subject of electron capture.[1-5] Although they were not fully successful in giving a complete explanation of the capture process, they still allow an estimate of the contributions of the different capture mechanisms and a comparison with experiment. However, no attention has been paid so far to the following process, which can contribute to the capture of electrons into betatron orbits. Consider the motion the electrons in a coordinate system moving along the equilibrium orbit with a velocity equal to the azimuthal injection velocity of the electrons. In this coordinate system the electrons will move towards each other. Since the electron velocities corresponding to radial motion are small, the collision probability will be sufficiently large. As a result of multiple scattering, electrons starting out with equal oscillation amplitudes will later acquire a gaussian amplitude distribution. Thus, the conditions for strong damping of the oscillations will statistically be fulfilled for a certain fraction of the injected electrons.

A rough estimate of this effect can be made in the following manner. If an electron makes an elastic collision at the time when its velocity is at maximum, then the amplitude of oscillation will decrease as

$$\rho_0' = \rho_0 \cos\psi, \tag{1}$$

where ψ is the scattering angle. The probability of single scattering at an angle ψ in traversing a distance dx through an electron gas of density N_0 is given by

$$P(\psi)\,d\psi = \pi \frac{e^4}{W_\rho^2} \frac{\cos(\psi/2)}{\sin^3(\psi/2)}\, d\psi N_0\, dx, \tag{2}$$

where W_ρ is the energy of the radial oscillations:

$$W_\rho = W_\theta (1-n)(\rho_0 / R_0)^2. \tag{3}$$

The number of electrons missing the injector is given by

$$N_\gamma = NN_0\, dx\, 2\pi \frac{e^4}{W^2_{\rho\,\mathrm{av}}} \int\limits_{\psi_{\min}}^{\pi-\psi_{\min}} \frac{\cos(\psi/2)}{\sin^3(\psi/2)} d\psi = NN_0\, dx\, 2\pi \frac{e^4}{W^2_{\rho\,\mathrm{av}}} \left(\frac{1}{\sin^2(\psi_{\min}/2)} - \frac{1}{\cos^2(\psi_{\min}/2)}\right), \tag{4}$$

where N is the number of injected electrons.

We now consider a numerical example. We assume: injection energy W_θ = 10 kev; injection current i_{in} = 0.4 amp, injection duration τ = 1.5 μsec, radius of the equilibrium orbit R_0 = 20 cm, instantaneous radius $R_t = R_0$, amplitude of radial oscillations ρ_0 = 1 cm, distance from the filament to the edge of the anode of the injector Δ = 0.1 cm, field index n = 0.65, and area of the injected beam S = 1 cm^2. We then obtain number of injected electrons

$$N = i_{\mathrm{in}} \tau / e = 3.75 \cdot 10^{12};$$

velocity of the injected electrons

$$v_\theta = \sqrt{\frac{2W_\theta}{m}} = 5.92 \cdot 10^9 \text{ cm/sec};$$

electron density in the beam

$$N_0 = N / v_\theta \tau S = 4.22 \cdot 10^8;$$

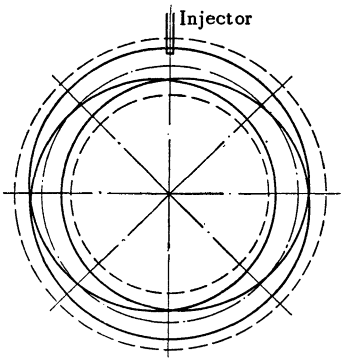
energy of the radial oscillations

$$W_\rho = 8.75 \text{ ev};$$

minimum scattering angle necessary to miss the injector, $\cos \psi_{\min} = 0.9$.

For $R_t = R_0$ and n = 0.65, an electron leaving the injector travels radially a distance $8\rho_0$ before returning (see figure). Then the number of electrons missing the injector will equal $N_\gamma = 0.8 \times 10^9$, which corresponds, at the end of the acceleration, to an electron current in the betatron doughnut of

$$i_\gamma = (c / 2\pi)\, e N_\gamma / R_0 = 0.03 \text{ amp}.$$



We note that the number of electrons missing the injector increases as the square of the emission current of the injector. Taking into account the influence of the Coulomb repulsion, the number of captured electrons will be proportional to the fourth power of the emission current as the result of the decrease of the energy of the radial oscillations. This agrees with the experimental results.

In a more accurate treatment, one has to take it into account that the multiple scattering process is the predominant one. Furthermore, the amplitude of the radial oscillations can also be increased in collisions involving electrons of different velocities.

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