LETTERS TO THE EDITOR

THE ELECTRODISINTEGRATION OF NUCLEI AT HIGH ENERGIES

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In this letter we calculate the cross section for the electrodisintegration of nuclei under conditions when it would be meaningless to expand the external field, which is given in Møller potentials, in terms of multipoles; this differs from the hypothesis in an earlier article of the author.[1] Such a case can occur when $|\mathbf{q}| = |\mathbf{q}_i - \mathbf{q}_f| > \hbar c/R$, where $\mathbf{q}_i/c$ and $\mathbf{q}_f/c$ are the momentum of the electron before and after scattering, and R is the nuclear radius.

The ground state of a nucleus (Z, A) will be described by an orbital wave function[2] and the final state by a plane wave, i.e., we shall use a Born approximation for the neutron ejected from the nucleus. The interaction of the nuclear system (core + neutron) will be described by a potential well with symmetrical symmetry. The radial wave functions are then expressible in terms of spherical Bessel functions and the matrix elements are easily calculated.

The calculation gives the following result for the cross section of the electrodisintegration of a nucleus[3] when the electron is scattered within the solid angle $d\Omega$ while transferring the energy $\Delta E = \epsilon_i - \epsilon_f$ to the nucleus, and the neutron escapes from the nucleus with momentum $\hbar \mathbf{k}_n = \mathbf{q}_n/c$ and energy $\Delta E - \epsilon_n$, where ϵ_n is the binding energy of the neutron in the nucleus, within the solid angle $d\Omega_n$:

$$\frac{d\sigma}{d\Omega d\Omega_n} = \frac{16}{2j+1}\left(\frac{e^2}{mc^2}\right)^2 \frac{|\eta|}{x_0^3(1+\beta_\alpha^2)}\left\{\frac{2j+1}{4\pi}\left[g_n^2 S_1 \Phi_l'^2 + \left(\frac{2Z}{A-1}\right)^2 S_2 \Phi_l^2\right] - \frac{2g_n Z}{A-1}\Phi_l \Phi_l' S_3^{(k)} a_l^{(k)}\right\} k_{n\alpha}^2 dk_{n\alpha},$$

$$\Phi_l(k_\alpha' x_0) = \frac{x_0^2}{\beta_\alpha^2 - k_\alpha'^2}\left[k_\alpha' j_l(\beta_\alpha x_0) j_{l-1}(k_\alpha' x_0) - \beta_\alpha j_{l-1}(\beta_\alpha x_0) j_l(k_\alpha' x_0)\right] + \frac{x_0^2}{1+k_\alpha'^2}\left[k_\alpha' k_l(x_0) j_{l-1}(k_\alpha' x_0) + k_{l-1}(x_0) j_l(k_\alpha' x_0)\right], \quad (1)$$

$$\Phi_l' = \Phi_l(k_\alpha'' x_0), \quad x_0 = \alpha R, \quad \beta = \hbar^{-1}\sqrt{2\mu(V_0 - \varepsilon_n)}, \quad \eta = -\left[j_{l+1}(\beta_\alpha x_0) j_{l-1}(\beta_\alpha x_0)\right]^{-1}, \quad g_n = -1{,}91.$$

Here the subscript α denotes division by $\alpha = \sqrt{2\mu\epsilon_n}/\hbar$ (for example, $\beta_\alpha = \beta/\alpha$); μ and m are the reduced mass of the nuclear system and the mass of the nucleon, respectively; V_0 and R are the depth and width of the potential well; $j_l(z)$ and $k_l(z)$ are spherical Bessel functions.[4] We have furthermore

$$S_1 = \frac{q_f}{q_i(q^2-\Delta E^2)^2}\left[q^2(\varepsilon_i\varepsilon_f - \mu_e^2) - \frac{1}{4}(q^4 - \Delta E^4) + \Delta E^2 \varepsilon_i \varepsilon_f\right],$$

$$S_2 = \frac{\Delta E^{-2}}{2(q^2-\Delta E^2)^2}\frac{q_f}{q_i}\left[2(\mathbf{q}'\mathbf{q})^2\varepsilon_i\varepsilon_f + \frac{1}{2}\left(Q_1^2 - (\mathbf{q}'\mathbf{q})^2\right)(q^2 - \Delta E^2) + 2(\mathbf{Q}_1\mathbf{q}_i)(\mathbf{Q}_1\mathbf{q}_f) - 2\mathbf{Q}_1(\mathbf{q}'\mathbf{q})(\varepsilon_f\mathbf{q}_i + \varepsilon_i\mathbf{q}_f)\right],$$

$$S_3^{(k)} = \frac{\Delta E^{-1}}{(q^2-\Delta E^2)^2}\frac{q_f}{q_i}\left\{\left[(\varepsilon_i+\varepsilon_f)\mathbf{q}'\mathbf{q} + (-1)^k \mathbf{Q}_1(\mathbf{q}_i + \mathbf{q}_f)\right](\mathbf{q}_i \times \mathbf{q}_f)_k + \frac{(-1)^k}{2}(q^2 - \Delta E^2)(\mathbf{q}\times\mathbf{Q}_1)_k\right\},$$

where μ_e is the rest energy of the electron,

$$\mathbf{q}' = \hbar c\mathbf{k}' = \mathbf{q}_n - \mathbf{q}/A \quad \text{and} \quad \mathbf{Q}_1 = \Delta E\mathbf{q}' + (\mathbf{q}'\mathbf{q})\mathbf{q}/2mc^2.$$

The functions $a_l^{(k)}$, which depend on the angles, are given by

$$a_l^{(1)} = \pm\frac{1}{\pi}\sum_{m_j=-j}^{j}\left\{\frac{(l+1/2)^2 - m_j^2}{(2l+1)^2}\Theta_{lm_j-1/2}(\theta\varphi)\,\Theta_{lm_j+1/2}(\theta'\varphi')\sin\left[m_j(\varphi-\varphi') - \frac{1}{2}(\varphi+\varphi')\right]\right\},$$

$$a_l^{(2)} = \pm\frac{1}{\pi}\sum_{m_j=-j}^{j}\left\{\frac{(l+1/2)^2 - m_j^2}{(2l+1)^2}\Theta_{lm_j-1/2}(\theta\varphi)\,\Theta_{lm_j+1/2}(\theta'\varphi')\cos\left[m_j(\varphi-\varphi') - \frac{1}{2}(\varphi+\varphi')\right]\right\},$$

$$a_l^{(3)} = \frac{1}{\pi} \sum_{m_j=-j}^{j} \left\{ \frac{(l + 1/2 \pm m_j)^2}{(2l+1)^2} \Theta_{lm_j-1/2}(\theta\varphi)\, \Theta_{lm_j-1/2}(\theta'\varphi') \sin\left(m_j - \frac{1}{2}\right)(\varphi - \varphi') \right\},$$

where the plus sign is taken for $j = \ell + \frac{1}{2}$ and the minus sign for $j = \ell - \frac{1}{2}$; $\Theta_{\ell m}(\theta\varphi)$ are normalized associated Legendre polynomials, and the angles θ, φ and θ', φ' fix the direction of the vectors $\mathbf{q}'$ and $\mathbf{q}'' = \mathbf{q}' + \mathbf{q} = \hbar\mathbf{c}\mathbf{k}''$. When $q \ll q_n$, q' and q'' are almost independent of the angles and are given by

$$q' \approx q_n, \quad q'' \approx q_n, \quad q'q \approx -q^2/2. \tag{2}$$

In this case Φ_ℓ and Φ'_ℓ are also independent of the angles and the integration in Eq. (1) is greatly simplified. In this approximation the second term in the expression for Q_1 can also be neglected.

We note that when the neutron is ejected from an s state $a_\ell^{(k)} = 0$. Then in the energy region defined by (2) we obtain for the angular distribution of the neutrons the formula

$$d\sigma/d\Omega_n = A\cos^2\theta_n + B\cos\theta_n + C,$$

where θ_n is the angle between $\mathbf{q}_n$ and $\mathbf{q}_i$ and the coefficients A, B and C, after integration with respect to Ω and $k_{n\alpha}$, depend only on the initial energy ϵ_i of the electron.

In conclusion, the author considers it his pleasant duty to thank Professor V. S. Mamasakhlisov for his interest.

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[3] Thie, Mullin, and Guth, Phys. Rev. **87**, 962 (1952).
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LOCALIZATION OF NUCLEONS IN THE O_8^{16} NUCLEUS

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CERTAIN problems related to the structure of the nucleus call for a determination of the localization of the nucleons in the nucleus.[1,2] In the present work, an attempt is made to find the localization of the nucleons in the O_8^{16} nucleus by a method developed by Daudel.[3]

Let us consider a system of nucleons in a volume V in a state with a definite value of spin projection; p is the number of protons (neutrons) with spin $\frac{1}{2}$. We break up the space V into p volumes v_i such that one can determine in each of these the probability P_i of meeting a proton (neutron) with spin $\frac{1}{2}$. The volumes v_i are called "boxes" following Ref. 3. The same determination is also applied to spin $-\frac{1}{2}$. The quantity

$$\eta = \left(p - \sum_i P_i\right)/p$$

characterizes the absence of localization of nucleons for two chosen divisions of the space V into boxes. The smaller the value of η for a given volume, the better the division into boxes.

Let $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ be the wave function of the nucleus, then

$$P_i = C_p^A\, p \int_{v_i} d\mathbf{r}_1 \int_{V-v_i} d\mathbf{r}_2 \ldots \int_{V-v_i} d\mathbf{r}_p \int_V d\mathbf{r}_{p+1} \ldots \int_V d\mathbf{r}_A \psi^*\psi.$$

According to the oscillator model, the nucleus O^{16} is in an $(s_{1/2})^4\,(1p_{3/2})^8\,(1p_{1/2})^4$ state. The wave function of the nucleus is $\psi = \det\varphi_i \det\varphi_k$; φ_i is the wave function of a single proton in the oscillator