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SKIN EFFECT AND FERROMAGNETIC RESONANCE

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We consider the normal and the anomalous skin effect in ferromagnetic metals under conditions of ferromagnetic resonance. The constant magnetic field is taken to be perpendicular to the surface of the sample. The influence of exchange effects on the resonance line width and on the shift of the resonance frequency is taken into account.

I. Ferromagnetic resonance absorption is observed when the frequency of the electromagnetic field incident on the surface of the ferromagnetic is nearly equal to the eigenfrequency of the precession of the magnetization vector **M** around the direction of the magnetic field **H**. This effect is described by the equation of motion of the magnetization vector **M** (Ref. 1) in a certain effective magnetic field,

$$\frac{1}{\gamma}\frac{\partial \mathbf{M}}{\partial t} = \mathbf{M}\times\left(\mathbf{H} + \frac{2A}{M_s^2}\nabla^2\mathbf{M} - \frac{\beta}{M_s}\,\mathbf{M}\times\mathbf{H}\right). \tag{1}$$

The first term on the right-hand side of (1) is the true magnetic field inside the specimen. The second term is the effective field due to exchange forces caused by the inhomogeneity of the magnetic moment **M**, and the third term is a relaxation term that describes the approach of the magnetization **M** to an equilibrium position along the field **H**; γ is the gyromagnetic ratio, and M_s the saturation value of the magnetization.

The relaxation term was introduced by Landau and Lifshitz[1] for a phenomenological description of damping processes. Up to the present the physical meaning of this term is not completely clear, since all existing theories lead to values of the dimensionless constant β that are rather low compared with experiment. In most cases, however, such a term gives a fairly good description of the experimentally observed effects that are connected with relaxation.

In ferromagnetic metals, however, there exists one mechanism which leads to a finite resonance line width. It is connected with the second term on the right hand side of Eq. (1) and plays an important role when this term is large, i.e., when the magnetization M is appreciably inhomogeneous in the skin layer. Ament and Rado[2] and Macdonald[3] have given the theory of this phenomenon for the case where there is a constant magnetic field parallel to the surface of the sample, and Rado and Weertman[4] observed experimentally an exchange broadening of the resonance line of this kind and determined in this way the value of the constant A for the substance investigated by them. In the present paper we consider the case where the constant magnetic field is perpendicular to the surface of the ferromagnetic metal.

It is clear that for pure metals effects connected with exchange broadening of the resonance lines will occur only at low temperatures. Lowering the temperature increases the mean free path ℓ_0 of the conduction electrons and with it the electrical conductivity σ; the inhomogeneity of the magnetic moment in the skin layer increases thus since the depth of the skin layer itself decreases. One can show that if the temperature of the ferromagnetic substance is sufficiently low the mean free path of the conduction electrons will become of the same order of magnitude as, or larger than the depth of the skin layer. In that case we cannot use the classical theory of the skin effect: the skin effect becomes anomalous.[5] We consider therefore in the present paper both the normal and the anomalous skin effect under conditions of ferromagnetic resonance.

An investigation of the skin effect in ferromagnetic metals makes it possible to determine for them the value of the constant A. Another way to determine this constant is to consider the magnetization of ferromagnetic substances at low temperatures.[6] From Bloch's theory it follows that

$$M_s = M_s^0 (1 - CT^{3/2}),$$

and knowing the constant C we can calculate A. Skin effect experiments, however, do not require such low temperatures as are necessary for Bloch's law to be valid. In addition, there are metals (for instance, cobalt) which show a phase transition and lose their ferromagnetism at temperatures where the Bloch law cannot be applied. Skin effect experiments allow us to determine the constant A also for high-temperature modifications of such ferromagnetic substances.

Experiments on the anomalous skin effect in ferromagnets enable us to determine for them the ratio ℓ_0/σ and to find out in this way to what extent we can apply in this case the normal representation of the electron theory of metals[7] which we are using in the present paper. One must note that the anomalous skin effect in ferromagnetic substances can easily be observed at higher temperatures than in other metals. This is connected with the fact that the expression for the skin depth δ contains the magnetic permeability, which is large, especially when the constant magnetic field H_z is much smaller than $4\pi M_s$, and when the frequency of the electromagnetic field is nearly equal to the resonance frequency.

We have also considered[8] the anomalous skin effect in a magnetic field which is parallel to the surface of the ferromagnet, but in that paper we did not take into account the role of exchange effects.

2. Our problem is described by Eq. (1), the Maxwell equations, and the transport equation for the conduction electrons in the metal:

$$\operatorname{curl} \mathbf{e} = -i(\omega/c)(\mathbf{h} + 4\pi\mathbf{m}), \tag{2a}$$

$$\operatorname{curl} \mathbf{h} = (4\pi/c)\,\mathbf{j}, \tag{2b}$$

$$\frac{1+i\omega\tau}{\tau} + v_z \frac{\partial f}{\partial z} + \frac{eH_0}{c}\left(v_x \frac{\partial f}{\partial p_y} - v_y \frac{\partial f}{\partial p_x}\right) = e\mathbf{v}\mathbf{e}\,\frac{\partial F}{\partial \varepsilon}. \tag{3}$$

In these equations the magnetic field H consists of a constant term $H_z \mathbf{i}_z$ ($\mathbf{i}_z$ is a unit vector along the z-axis which is directed into the metal, perpendicular to its surface), and of the field **h** of the electromagnetic wave, which is of frequency ω and which is propagated along the z-axis. The magnetization M is the sum of two terms of similar kind. We shall assume that $h/H_z \ll 1$ and that $m/M_s \ll 1$. We neglect the displacement current in Eq. (2b).

In Eq. (3), F is the equilibrium distribution function of the conduction electrons, f the small non-equilibrium part of the distribtuion function, **v** the electron velocity, and τ their relaxation time. We shall assume that the electron energy ϵ is an isotropic function of their quasi-momentum **p**. H_0 is the resultant constant magnetic field in which the conduction electrons of the ferromagnetic are placed. Wannier[9] has considered the problem of the magnitude of this field. The reflection coefficient of the electrons q at the surface of the metal gives us the boundary condition for Eq. (3); q = 1 corresponds to specular reflection and $q = 0$ to diffuse reflection; the last case is, apparently, actually realized.

We must solve Eq. (1) under the boundary conditions given in Ref. 2. In the case where the magnetic field is perpendicular to the metal surface, the boundary conditions are

$$dm_x / dz = 0, \quad dm_y / dz = 0 \quad \text{for } z = 0.$$

We replace further the components e_x and e_y of the electric field intensity by the quantities $e_\pm = e_x \pm ie_y$ and similar expressions for the components of the other vectors in (1) and (2). These equations will then be of the following form

$$\{\pm\omega/\gamma + H_z(1 \mp i\beta) - (2A/M_s)\, d^2/dz^2\}\, m_\pm = M_s(1 \mp i\beta)\, h_\pm, \tag{4}$$

$$de_\pm / dz = \mp(\omega/c)(h_\pm + 4\pi m_\pm), \tag{5a}$$

$$dh_\pm / dz = \mp(4\pi i/c)\, j_\pm. \tag{5b}$$

In writing down (4) we have neglected products of the small quantities m and h.

Azbel' and Kaganov[10] obtained an expression for the electrical current density $j_\pm$ from the solution of the transport equation (3),

$$j_\pm(z) = \frac{2\pi e^2 p^2}{(2\pi\hbar)^3}\left\{q\int\limits_{-\infty}^{\infty} K\left(\frac{z-z'}{l_\pm}\right) e_\pm(z')\, dz' + (1-q)\int\limits_0^{\infty} K\left(\frac{z-z'}{l_\pm}\right) e_\pm(z')\, dz'\right\}, \tag{6}$$

where

$$l_\pm = l_0/[1 + i(\omega \mp \Omega)\tau], \quad l_0 = v\tau, \quad \Omega = evH_0/pc;$$

and where all quantities are evaluated on the Fermi surface. The kernel K is given by the equation

$$K\left(\frac{z}{l_\pm}\right) = \int\limits_0^{\pi/2} \frac{\sin^3\theta}{\cos\theta} \exp\{-|z|/l_\pm \cos\theta\}\, d\theta. \tag{7}$$

It is now convenient for us to introduce the functions

$$n_\pm(z') = \int\limits_{z'}^{\infty} m_\pm(z'')\, dz'',$$

which represent the components of the magnetic moment of that part of the sample which lies between the plane $z = z'$ and infinity. They are connected with the components of the electric field by the equation

$$[\pm\omega/\gamma + B_z(1 \mp i\beta) - (2A/M_s)\, d^2/dz^2]\, n_\pm(z) = \mp(c/\omega)\, M_3(1 \pm i\beta)\, e_\pm(z), \tag{8}$$

which can easily be obtained by substituting into (5a) the relation between $b_\pm = h_\pm + 4\pi m_\pm$ and $m_\pm$ which follows from (4),

$$[\pm\omega/\gamma + B_z(1 \mp i\beta) - (2A/M_s)\, d^2/dz^2]\, m_\pm = M_s(1 \mp i\beta)\, b_\pm. \tag{9}$$

We now introduce the dimensionless parameter $\xi = z/l_0$ and the new functions $g_\pm(\xi) = n_\pm(l\xi)$. Substituting (8) and (4) into (5) and defining $g_\pm(\xi)$ as an even function for negative values of ξ $[g_\pm(-\xi) = g_\pm(\xi)]$, we obtain for the functions $g_\pm$ the following equations

$$\frac{d^4 g_\pm}{d\xi^4} - P_\pm \frac{d^2 g_\pm}{d\xi^2} = i\zeta\left[q\int\limits_{-\infty}^{\infty} K\left(\frac{z-z'}{l_\pm}\right)\left(\frac{d^2 g_\pm}{d\xi'^2} - Q_\pm g_\pm\right) d\xi' + (1-q)\int\limits_0^{\infty} K\left(\frac{z-z'}{l_\pm}\right)\left(\frac{d^2 g_\pm}{d\xi'^2} - Q_\pm g_\pm\right) d\xi'\right]. \tag{10}$$

where

$$P_\pm = [H_z(1 \mp i\beta) \pm \omega/\gamma]\, M_s l_0^2/2A; \quad Q_\pm = [B_z(1 \mp i\beta) \pm \omega/\gamma]\, M_z l_0^2/2A; \quad \zeta = 3l_0^2/2\delta_0^2, \quad \delta_0^2 = c^2/2\pi\omega\sigma,$$

and where σ is the electrical conductivity.

3. Solving Eq. (10) we can calculate the surface impedances,

$$Z_\pm = \pm(4\pi i/c)\, e_\pm(0)/h_\pm(0),$$

i.e., the quantities which are directly measured in ferromagnetic-resonance experiments. We shall assume that $q = 1$, since that case is mathematically the simplest one, while the expression for the impedances does not depend on q at all in the limit of the normal skin effect and depends only weakly on q in the case of the anomalous skin effect. We must solve Eq. (10) (for the case of $q = 1$) in such a way that $g_\pm(\xi) \to 0$ as $\xi \to \infty$. Following Reuter and Sondheimer[5] we introduce thereto the functions

$$\varphi_\pm(t) = \int_{-\infty}^{\infty} g_\pm(\xi) e^{-i\xi t} dt, \tag{11}$$

$$\varkappa_\pm(t) = \frac{1}{l_0}\int_{-\infty}^{\infty} K\left(\frac{z}{l_\pm}\right) e^{-izt/l_0} dz. \tag{12}$$

Performing respectively four and two integrations by parts, we get

$$\int_{-\infty}^{\infty} g_\pm^{\mathrm{IV}}(\xi) e^{-i\xi t} dt = t^4\varphi_\pm(t) + 2t^2 g_1^\pm - 2g_3^\pm, \qquad \int_{-\infty}^{\infty} g''_\pm(\xi) e^{-i\xi t} dt = -t^2\varphi_\pm(t) - 2g_1^\pm, \tag{13}$$

where

$$g_3^\pm = g'''_\pm(0), \quad g_1^\pm = g'_\pm(0).$$

From Eqs. (10) to (13) we find the solution

$$g_\pm(\xi) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\xi t}\frac{g_1^\pm[t^2 + P_\pm + i\zeta\varkappa_\pm(t)] - g_3}{t^4 + P_\pm t^2 + i\zeta\varkappa_\pm(t)(t^2 + Q_\pm)}. \tag{14}$$

If we assume that the first derivative $g_1^\pm$ is given, we can express $g_3^\pm$ in terms of it, using the boundary condition $n''_\pm(0) = 0$,

$$\eta_\pm = \frac{g_3^\pm}{g_1^\pm} = -i\zeta Q_\pm \int_0^{\infty}\frac{\varkappa_\pm(t)\,dt}{R_\pm(t)} \Big/ \int_0^{\infty}\frac{t^2 dt}{R_\pm(t)}, \tag{15}$$

where

$$R_\pm(t) = t^4 + P_\pm t^2 + i\zeta\varkappa_\pm(t)(t^2 + Q_\pm).$$

The values of $e_\pm(0)$ and $h_\pm(0)$ on the metal surface are determined by Eqs. (8) and (4), while the surface impedances are given by

$$Z_\pm = \frac{8i\omega l_0}{c^2} Q_\pm\left\{\frac{1}{P_\pm - \eta_\pm}\int_0^{\infty}\frac{t^2 + i\zeta\varkappa(t)}{R_\pm(t)} + \int_0^{\infty}\frac{dt}{R_\pm(t)}\right\}. \tag{16}$$

We see immediately that the impedance Z_- depends on the magnetic field H_Z in resonance fashion. It increases considerably when $H_Z \approx \omega/\gamma$ (resonance) and decreases strongly when $H_Z \approx \omega/\gamma - 4\pi M_S$ (anti-resonance). At the same time the impedance Z_+ does not show resonance behavior and is a monotonic function of the field. We shall therefore be mainly interested henceforth in the behavior of the quantity Z_- as a function of H_Z.

We must note that in most cases we can neglect in the right-hand side of Eq. (10) the second derivatives of $g_\pm$ compared with the terms $Q_\pm g_\pm$. Indeed,

$$d^2 g_\pm / d\xi^2 \sim g_\pm l_0^2/\delta^2,$$

where δ is the skin depth. At the same time, of the two coefficients Q, the smallest value is reached by Q_- in the anti-resonance region, where it is of the order of $\beta \dot{B}_Z M_S l_0^2/A$. Typical values of the quantities which enter in that expression are, for pure ferromagnetic metals, $\beta = 5\times 10^{-2}$, $B_Z \sim 10^4$ oersted, $M_S \sim 10^3$ oersted, $A \sim 10^{-6}$ erg-cm^{-1}, and $\delta \sim 10^{-5}$ cm. One can easily verify that even in the anti-resonance region the second quantity exceeds the first by one or two orders of magnitude. Their ratio becomes even larger away from anti-resonance. If we neglect the first term on the left-hand side of Eq. (10), the expression for the impedances simplifies slightly:

$$Z_{\pm} = \frac{8i\omega l_0}{c^2} Q_{\pm} \left\{ \frac{1}{P_{\pm} - \eta_{\pm}} \int_0^\infty \frac{t^2 dt}{r_{\pm}(t)} + \int_0^\infty \frac{dt}{r_{\pm}(t)} \right\}, \qquad (16a)$$

where $r_{\pm}(t) = t^4 + P_{\pm}t^2 + i\zeta\kappa_{\pm}(t)Q$, and

$$\eta_{\pm} = -i\zeta Q_{\pm} \int_0^\infty \frac{\kappa_{\pm}(t)\,dt}{r_{\pm}(t)} \Big/ \int_0^\infty \frac{t^2 dt}{r_{\pm}(t)}. \qquad (15a)$$

We shall make this simplification in the following.

4. Let the distance over which the electric field intensity varies significantly be much larger than the electron mean free path ℓ_0. We can then, following Ref. 5, substitute $\kappa_{\pm}(0) = \frac{4}{3}[1 + i(\omega \mp \Omega)\tau]$ for $\kappa_{\pm}(t)$ in evaluating the integrals which occur in (15) and (16). This is the limiting case of the normal skin effect, and the surface impedance is given by

$$Z_{-} = -\frac{4\pi\omega l_0}{c^2} \frac{Q_{-}}{P_{-} - t_1 t_2} \left(\frac{1}{t_1} + \frac{1}{t_2} \right), \qquad (17)$$

where t_1 and t_2 are the roots of the equation

$$t^4 + P_{-}t^2 + 2il^2 Q_{-} / \delta_0^2 \, [1 + i(\omega + \Omega)\tau] = 0, \qquad (18)$$

satisfying the condition $\mathrm{Im}\, t > 0$. The frequencies and magnetic field intensities used in ferromagnetic resonance experiments mean, in the region of the normal skin effect, that $(\omega + \Omega)\tau \ll 1$. If $(\omega + \Omega)\tau$ can be neglected compared to unity, Eq. (18) can be slightly simplified and becomes

$$t^4 + P_{-}t^2 + 2il_0^2 Q_{-}/\delta_0^2 = 0, \qquad (19)$$

In the case where the exchange line broadening is not appreciable, it is possible to simplify Eq. (17) considerably. It is necessary for this to let the constant A tend formally to zero. One of the roots of (19) then tends to infinity as $A^{-1/2}$, but the other one remains finite. We get for the surface impedance Z_{-} the well known expression

$$Z_{-} = \frac{(4\pi i\omega\mu_{-})^{1/2}}{c\sigma^{1/2}} = \frac{(4\pi\omega|\mu_{-}|)^{1/2}}{c\sigma^{1/2}} e^{i\vartheta/2}, \qquad (20)$$

where

$$\mu_{-} = P_{-} / Q_{-} = |\mu_{-}|\, e^{i(3\pi/2 + \vartheta)}.$$

Since $\mathrm{Im}\,\mu_{-} < 0$, $-\pi/2 < \vartheta < \pi/2$.

One sees easily that exchange effects play an important part only near resonance. Far from resonance, in particular near anti-resonance, they are unimportant and we can then use expression (20). To clarify Eq. (17) for the case where it includes these effects, we introduce, following Ament and Rado,[2] an equivalent magnetic permeability μ. We define μ as that quantity which, if substituted instead of μ_{-} into expression (17), leads to expression (20) without having to consider explicity exchange effects. If $H_Z \ll 4\pi M_S$ and the frequency ω is close to resonance, we have

$$\mu = \frac{4\pi M_s [H_z(1 + i\beta) - \omega/\gamma + 4(1 + i)(2\pi A / \delta_0^2)^{1/2}]}{[H_z(1 + i\beta) - \omega/\gamma + 2(1 + i)(2\pi A / \delta_0^2)^{1/2}]^2}. \qquad (21)$$

From Eq. (21) it is clear that the exchange width of the resonance line ΔH is of the order $A^{1/2}/\delta_0$ and that exchange effects cause a shift of the resonance field of the same order of magnitude. Moreover, the ratio $\Delta H/H_Z$ increases when the field decreases so that the limiting case $H_Z \ll 4\pi M_S$, which we have considered in most detail, is the most interesting one from the point of view of studying exchange effects.

Finally we give in this section the expression for Z_{-} for the case where the line width is determined by the exchange effects, and the magnetic field is $H_Z = \omega/\gamma$.

$$Z_{-} = (32\pi^2 M_z^2 \omega / c^2 A\sigma^3)^{1/4} e^{i\pi/8}. \qquad (22)$$

In this case $Z_{-} \sim \omega^{1/4}$.

5. We shall now consider the opposite limiting case of the ultra-anomalous skin effect. Let the distance over which the value of the electrical field strength $e(z)$ changes appreciably be much less than the electron mean free path ℓ_0. In the integrals occurring in Eqs. (15) and (16) we have then essentially $t \gg 1$. We can thus replace the functions $\kappa_\pm(t)$ in these integrals by asymptotic expressions valid for large values of t, viz.: $\kappa_\pm(t) \sim \pi/t$, as was also done by Reuter and Sondheimer.[5] We get then

$$Z_\pm = \frac{8i\omega l_0}{C^2} Q_\pm \left\{ \frac{1}{P_\pm - \eta_\pm} \int_0^\infty \frac{t^3 dt}{t^5 + P_\pm t^3 + i\pi\zeta Q_\pm} + \int_0^\infty \frac{t dt}{t^5 + P_\pm t^3 + i\pi\zeta Q_\pm} \right\}, \quad (23)$$

$$\eta_\pm = -i\pi\zeta Q_\pm \int_0^\infty \frac{dt}{t^5 + P_\pm t^3 + i\pi\zeta Q_\pm} \Bigg/ \int_0^\infty \frac{t^3 dt}{t^5 + P_\pm t^3 + i\pi\zeta Q_\pm}. \quad (24)$$

The surface impedances $Z_\pm$ do not depend on the electron mean free path ℓ_0 in the case of the ultra-anomalous skin effect. This can be verified by making in the integrals occuring in (23) and (24) the substitution $t = \ell_0 s$.

In the general case, however, Eqs. (23) and (24) are rather complicated, and we shall therefore again consider only two limiting cases. The first takes place when the exchange effects are not important, in other words, if the second term on the left-hand side of Eq. (10) is much larger than the first one. In order that this case be realized it is necessary to have $|P_-| \gg (\pi\xi|Q|)^{2/5}$, or, stated in clear notation,

$$\left| H_z(1+i\beta) - \frac{\omega}{\gamma} \right| \gg \left[\frac{72\pi^4\omega^2 A^3\sigma^2}{c^4 M_s^3 l_0^2} \left| B_z(1+i\beta) - \frac{\omega}{\gamma} \right|^2 \right]^{1/5}. \quad (25)$$

This inequality is satisfied if either the field H_Z is sufficiently far from the value ω/γ, or the line width βH_Z is much larger than the exchange width. If inequality (25) is satisfied, the first term in Eq. (23) will be small compared to the second one and we get for the impedance the expression

$$Z_- = \frac{8i\omega l_0}{c^2} \int_0^\infty \frac{tdt}{t^3 + i\pi\zeta\mu_-} = \frac{16}{9} \left(\frac{\pi\sqrt{3}|\mu_-|^2\omega^2 l_0}{c^4\sigma} \right)^{1/3} e^{i2\vartheta/3}. \quad (26)$$

In the case where the opposite inequality $|P_-| \ll (\pi\xi|Q_-|)^{2/5}$ holds, we have

$$Z_- = \theta (M_z^4 l_0^3 \omega^2 / c^4 A^2 \sigma^3)^{1/5} e^{i\pi/5}, \quad (27)$$

where the dimensionless constant is given by

$$\theta = {}^8/_5 (4\pi/27)^{1/5} (b_0 + b_1) = 5.51,$$

with

$$b_n = \left[\sin\frac{3\pi(4-n)}{5} + 3\sin\frac{\pi(4-n)}{5} \right] \Big/ \left(1 - \cos\frac{2\pi}{5}\right)\left(1 + \cos\frac{\pi}{5}\right).$$

The exchange width of the resonance line is of the order of magnitude $\Delta H \sim 10 A^3\sigma^2\omega^2/c^4 M_S \ell_0^2$. For sufficiently low temperatures ΔH is thus proportional to $\omega^{2/5}$ and does not depend on the temperature; this is a refinement of the statement by Kittel and Herring.[11] The shift of the resonance field is of the same order of magnitude. As in the case of the normal skin effect, the shift is in the direction of smaller fields.

6. The dependence of the impedance Z_+ on the magnetic field has a non-resonant character. The role of the terms responsible for the exchange effects in the expression for Z_+ is negligibly small since the inequality $|P_+| \gg (\pi\xi|Q_+|)^{2/5}$ is always satisfied. We have thus

$$Z_+ = \frac{(4\pi\mu_+\omega)^{1/2}}{c\sigma^{1/2}} e^{i\pi/4} \quad (28)$$

in the limiting case of the normal skin effect and

$$Z_+ = \frac{16}{9} \left(\frac{\pi\sqrt{3}\mu_+^2\omega^2 l_0}{c^4\sigma} \right)^{1/3} e^{i\pi/3} \quad (29)$$

in the case of the ultra-anomalous skin effect. These equations differ from the corresponding expressions of Ref. 10 for the impedance Z_+ only by the factor $\mu_+ = (\gamma B_z + \omega)/(\gamma H_z + \omega)$.

7. We shall now consider the assumptions on which the present paper is based. In the equation of motion for the magnetization vector (1) we did not include explicity the effective field of the magnetic anisotropy. This can be done in two cases. Firstly, in the case where the magnetic field H_z is much larger than the effective field, the latter can simply be neglected. Secondly, if, to a sufficiently good approximation, the magnetic field is directed along an axis of symmetry of the ferromagnetic cyrstal, one can consider the field H_z to consist of two terms, the ordinary magnetic field H_i and the effective field of the magnetic anisotropy H_a. Such axes of symmetry are, for instance, the hexagonal axis in a hexagonal crystal and the tetragonal or trigonal axes in a cubic crystal. We must note, by the way, that the magnetic field H_i in the ferromagnet is connected with the external magnetic field H_e through the relation $H_i = H_e - 4\pi M_s$.

As was pointed out by Karplus and Luttinger[13] the current density in a ferromagnetic metal is not completely determined by expression (6). In ferromagnetic substances there is also a stationary current which is proportional to the vector product $[\mathbf{M} \times \mathbf{e}]$, with a constant of proportionality which is practically independent of temperature. This current gives rise to the anomalous Hall effect in ferromagnetic materials.[14] However, at the frequencies and fields ordinarily used in ferromagnetic resonance experiments, and at low temperatures, this current plays a relatively small role. At high temperatures it can be taken into account by a formal change of the quantity Ω in (18). The product $\Omega\tau$ is also usually still small compared to unity.

If the value of the gyromagnetic ratio γ is known, it is convenient to use Eqs. (22) or (27) to evaluate the constant A. These are valid for the limiting cases of the normal and the ultra-anomalous skin effect, respectively, for a field $H_z = \omega/\gamma$. If the values of the constants γ and A and of the ratio ℓ_0/σ for the given substance are determined experimentally by investigating the behavior of the impedances in the corresponding limiting cases, we can evaluate all the parameters in expression (16). One can then calculate by numerical integration the impedances $Z_\pm$ in the intermediate case and thus compare in that case the theory with experiment.

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